CLASS X: MATHS
Chapter 6: Triangles

Questions and Solutions | Exercise 6.1 - NCERT Books

Q1. Fill in the blanks using the correct word given in brackets :

(i) All circles are ______. (congruent, similar)

(ii) All squares are _______. (similar, congruent)

(iii) All _______ triangles are similar. (isosceles, equilateral)

(iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are _______ and (b) their corresponding sides are _______. (equal, proportional)

Sol. (i) All circles are similar.

(ii) All squares are similar.

(iii) All equilateral triangles are similar.

(iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are equal and (b) their corresponding sides are proportional.

Q2. Give two different examples of pair of

(i) Similar figures.

(ii) Non-similar figures.

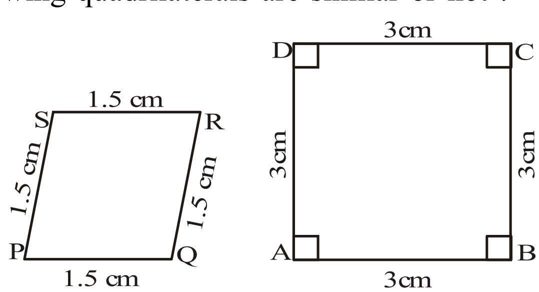
Sol. (i) 1. Pair of equilateral triangles are similar figures.

2. Pair of squares are similar figures.

(ii) 1. One equilateral triangle and one isosceles triangle are non-similar.

2. Square and rectangle are non-similar.

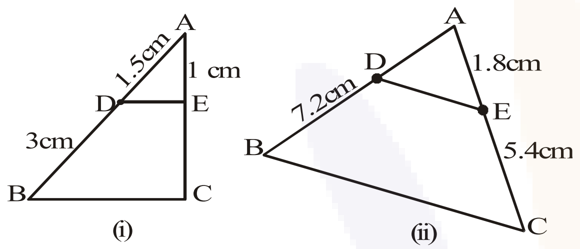
Q3. State whether the following quadrilaterals are similar or not :



Sol. The two quadrilateral in figure are not similar because their corresponding angles are not equal.

Questions and Solutions | Exercise 6.2 - NCERT Books

Q1. In figure, (i) and (ii), DE || BC. Find EC in (i) and AD in (ii).



Sol. (i) In figure, (i) DE || BC (Given)

$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$ (By Basic Proportionality Theorem)

$\Rightarrow \frac{1.5}{3} = \frac{1}{EC}$

{$\because$ AD = 1.5 cm, DB = 3 cm and AE = 1 cm}

$\Rightarrow$ EC = $\frac{3}{1.5}$ = 2 cm

(ii) In fig. (ii) $DE \| BC$ (given)

So, $\frac{AD}{BD} = \frac{AE}{CE} \Rightarrow \frac{AD}{7.2} = \frac{1.8}{5.4}$

{$\because$ BD = 7.2, AE = 1.8 cm and CE = 5.4 cm}

AD = 2.4 cm

Q2. E and F are points on the sides PQ and PR respectively of a ΔPQR. For each of the following cases, State whether EF || QR :

(i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm.

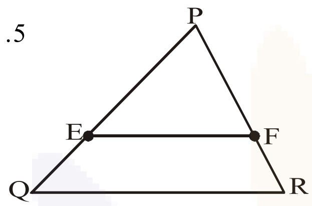
(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm.

(iii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm.

Sol. (i) In figure,

$$\frac{PE}{EQ} = \frac{3.9}{3} = 1.3,$$

$$\frac{PF}{FR} = \frac{3.6}{2.4} = \frac{3}{2} = 1.5$$



$\Rightarrow \frac{PE}{EQ} \neq \frac{PF}{FR}$

$\Rightarrow$ EF is not || QR

(ii) In figure,

$$\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9} \text{ and } \frac{PF}{FR} = \frac{8}{9}$$

$\Rightarrow \frac{PE}{EQ} = \frac{PF}{FR} \Rightarrow$ EF || QR

(iii) In figure,

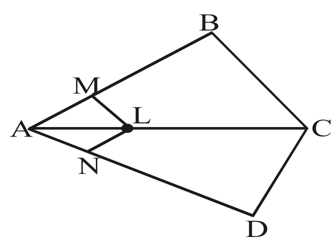
$$\frac{PE}{QE} = \frac{0.18}{PQ - PE} = \frac{0.18}{1.28 - 0.18} = \frac{0.18}{1.10}$$

$$= \frac{18}{110} = \frac{9}{55} = \frac{PF}{FR} = \frac{0.36}{PR - PF}$$

$$= \frac{0.36}{2.56 - 0.36} = \frac{0.36}{2.20} = \frac{9}{55} = \frac{PE}{QE} = \frac{PF}{FR}$$

$\therefore$ $EF \| QR$ (By converse of Basic Proportionality Theorem)

Q3. In figure, if LM || CB and LN || CD, prove that $\frac{AM}{AB} = \frac{AN}{AD}$.



Sol. In ΔACB (see figure), LM || CB (Given)

$\Rightarrow \quad \frac{AM}{MB} = \frac{AL}{LC} \quad ...(1)$

(Basic Proportionality Theorem)

In ΔACD (see figure), LN || CD(Given)

$\Rightarrow \quad \frac{AN}{ND} = \frac{AL}{LC} \quad ...(2)$

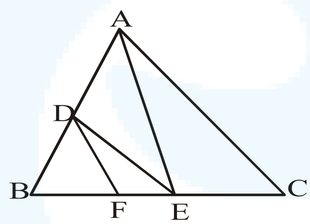
(Basic Proportionality Theorem)

From (1) and (2), we get

$$\frac{AM}{MB} = \frac{AN}{ND}$$

$$\Rightarrow \frac{AM}{AM + MB} = \frac{AN}{AN + ND} \Rightarrow \frac{AM}{AB} = \frac{AN}{AD}$$

Q4. In figure, DE || AC and DF || AE. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$.



Sol. In ΔABE,

$DF \| AE$ (Given)

$\frac{BD}{DA} = \frac{BF}{FE}$ ...(i) (Basic Proportionality Theorem)

In ΔABC,

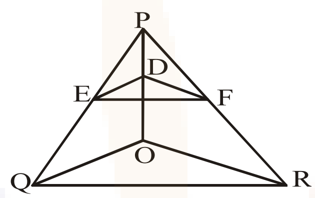
$DE \| AC$ (Given)

$\frac{BD}{DA} = \frac{BE}{EC}$ ......(ii) (Basic Proportionality Theorem)

From (i) and (ii), we get

$\frac{BF}{FE} = \frac{BE}{EC}$ Hence proved.

Q5. In figure, DE || OQ and DF || OR. Show that EF || QR.



Sol. In figure, DE || OQ and DF || OR, then by Basic Proportionality Theorem,

We have $\frac{PE}{EQ} = \frac{PD}{DO}$ ...(1)

and $\frac{PF}{FR} = \frac{PD}{DO}$ ...(2)

From (1) and (2), $\frac{PE}{EQ} = \frac{PF}{FR}$

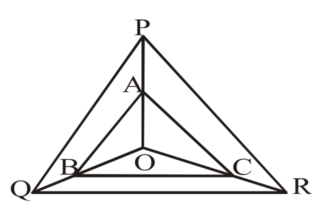
Now, in ΔPQR, we have proved that

$\Rightarrow \frac{PE}{EQ} = \frac{PF}{FR}$

EF || QR

(By converse of Basic Proportionality Theorem)

Q6. In figure, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.



Sol. In ΔPOQ,

$AB \parallel PQ$ (given)

$\frac{OB}{BQ} = \frac{OA}{AP}$ ...(i) (Basic Proportionality Theorem)

In ΔPOR,

$AC \parallel PR$ (given)

$\frac{OA}{AP} = \frac{OC}{CR}$ ...(ii) (Basic Proportionality Theorem)

From (i) and (ii), we get

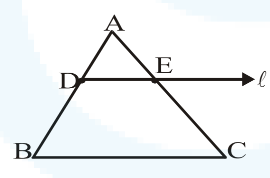
$\frac{OB}{BQ} = \frac{OC}{CR}$

$\therefore$ By converse of Basic Proportionality Theorem,

$BC \parallel QR$

Q7. Using Theorem 6.1, prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side.

Sol. In ΔABC, D is mid point of AB (see figure)



i.e., $\frac{AD}{DB} = 1$ ...(1)

Straight line $\ell \parallel BC$.

Line ℓ is drawn through D and it meets AC at E.

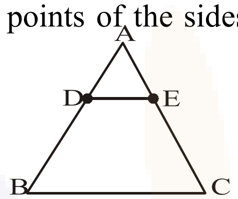
By Basic Proportionality Theorem

$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{AE}{EC} = 1$ [From (1)]

$\Rightarrow$ AE = EC $\Rightarrow$ E is mid point of AC.

Q8. Using Theorem 6.2, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side.

Sol. In ΔABC, D and E are mid points of the sides AB and AC respectively.



$\Rightarrow \frac{AD}{DB} = 1$

and $\frac{AE}{EC} = 1$ (see figure)

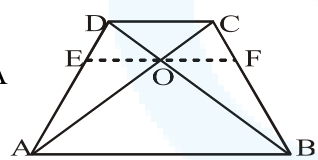
$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC} \quad \Rightarrow \quad DE \parallel BC$

(By Converse of Basic Proportionality Theorem)

Q9. ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Sol. We draw EOF || AB(also || CD) (see figure)

In ΔACD, OE || CD



$\Rightarrow \frac{AE}{ED} = \frac{AO}{OC}$ ...(1)

In ΔABD, OE || BA

$\Rightarrow \frac{DE}{EA} = \frac{DO}{OB}$

$\Rightarrow \frac{AE}{ED} = \frac{OB}{OD}$ ...(2)

From (1) and (2)

$\frac{AO}{OC} = \frac{OB}{OD}$,

i.e., $\frac{AO}{BO} = \frac{CO}{DO}$.

Q10. The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium.

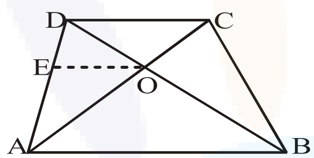
Sol. In figure $\frac{AO}{BO} = \frac{CO}{DO}$

$\Rightarrow \quad \frac{AO}{OC} = \frac{BO}{OD}$ ...(1) (given)

Through O, we draw

OE || BA

OE meets AD at E.



From ΔDAB,

EO || AB

$\Rightarrow \quad \frac{DE}{EA} = \frac{DO}{OB}$ (by Basic Proportionality Theorem)

$\Rightarrow \quad \frac{AE}{ED} = \frac{BO}{OD}$ ...(2)

From (1) and (2),

$\frac{AO}{OC} = \frac{AE}{ED} \quad \Rightarrow \quad$ OE || CD

(by converse of basic proportionality theorem)

Now, we have BA || OE

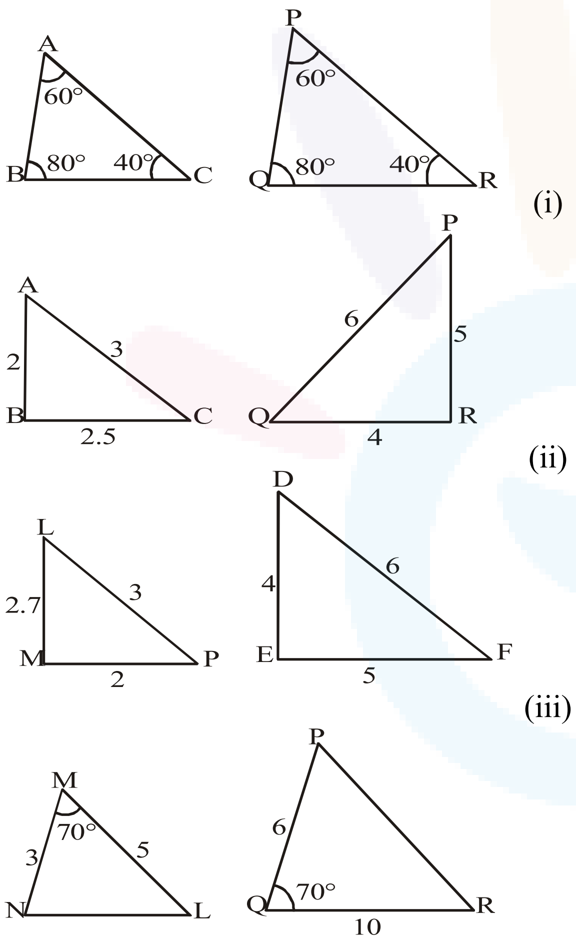
and OE || CD

$\Rightarrow$ AB || CD

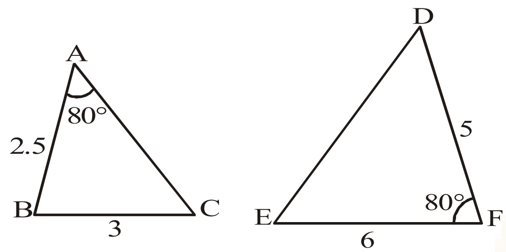
$\Rightarrow$ Quadrilateral ABCD is a trapezium.

Questions and Solutions | Exercise 6.3 - NCERT Books

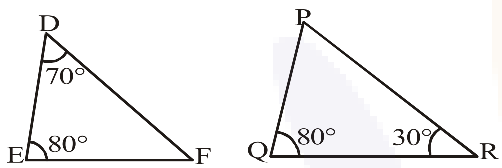
Q1. State which pairs of triangles in figure, are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form :



(iv)



(v)



(vi)

Sol. (i) Yes. $\angle A = \angle P = 60°$, $\angle B = \angle Q = 80°$,

$\angle C = \angle R = 40°$

Therefore, $\Delta ABC \sim \Delta PQR$.

By AAA similarity criterion

(ii) Yes.

$$\frac{AB}{QR} = \frac{2}{4} = \frac{1}{2}, \frac{BC}{RP} = \frac{2.5}{5} = \frac{1}{2}, \frac{CA}{PQ} = \frac{3}{6} = \frac{1}{2}$$

Therefore, $\Delta ABC \sim \Delta QRP$.

By SSS similarity criterion.

(iii) No.

$$\frac{MP}{DE} = \frac{2}{4} = \frac{1}{2}, \frac{LP}{DF} = \frac{3}{6} = \frac{1}{2}, \frac{LM}{EF} = \frac{2.7}{5} \neq \frac{1}{2}$$

i.e., $\frac{MP}{DE} = \frac{LP}{DF} \neq \frac{LM}{EF}$

Thus, the two triangles are not similar.

(iv) Yes,

$$\frac{MN}{QP} = \frac{ML}{QR} = \frac{1}{2}$$

and $\angle NML = \angle PQR = 70°$

By SAS similarity criterion

$\Delta NML \sim \Delta PQR$

(v) No,

$\frac{AB}{FD} \neq \frac{AC}{FE}$

Thus, the two triangles are not similar

(vi) In triangle DEF $\angle D + \angle E + \angle F = 180°$

$70° + 80° + \angle F = 180°$

$\angle F = 30°$

In triangle PQR

$\angle P + 80° + 30° = 180°$

$\angle P = 70°$

$\angle E = \angle Q = 80°$

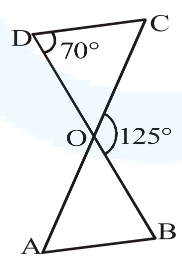
$\angle D = \angle P = 70°$

$\angle F = \angle R = 30°$

By AAA similarity criterion,

$\Delta DEF \sim \Delta PQR$.

Q2. In figure, $\Delta ODC \sim \Delta OBA$, $\angle BOC = 125°$ and $\angle CDO = 70°$. Find $\angle DOC$, $\angle DCO$ and $\angle OAB$.



Sol. From figure,

$\angle DOC + 125° = 180°$

$\Rightarrow \angle DOC = 180° - 125° = 55°$

$\angle DCO + \angle CDO + \angle DOC = 180°$

(Sum of three angles of ΔODC)

$\Rightarrow \angle DCO + 70° + 55° = 180°$

$\Rightarrow \angle DCO + 125° = 180°$

$\Rightarrow \angle DCO = 180° - 125° = 55°$

Now, we are given that $\Delta ODC \sim \Delta OBA$

$\Rightarrow \angle OCD = \angle OAB$

$\Rightarrow \angle OAB = \angle OCD = \angle DCO = 55°$

i.e., $\angle OAB = 55°$

Hence, we have

$\angle DOC = 55°, \angle DCO = 55°, \angle OAB = 55°$

Q3. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$.

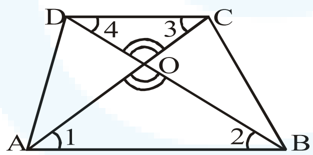
Sol. In figure, AB || DC

$\Rightarrow \angle 1 = \angle 3, \angle 2 = \angle 4$

(Alternate interior angles)

Also $\angle DOC = \angle BOA$

(Vertically opposite angles)

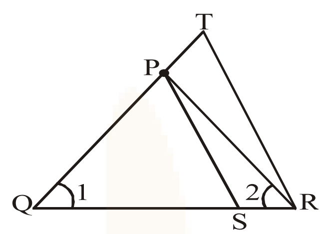


$\Rightarrow \Delta OCD \sim \Delta OAB \quad \Rightarrow \frac{OC}{OA} = \frac{OD}{OB}$

(Ratios of the corresponding sides of the similar triangle)

$\Rightarrow \frac{OA}{OC} = \frac{OB}{OD}$ (Taking reciprocals)

Q4. In figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$. Show that $\Delta PQS \sim \Delta TQR$.



Sol. In figure, $\angle 1 = \angle 2$ (Given)

$\Rightarrow$ PQ = PR

(Sides opposite to equal angles of ΔPQR)

We are given that

$$\frac{QR}{QS} = \frac{QT}{PR}$$

$$\Rightarrow \frac{QR}{QS} = \frac{QT}{PQ} \quad (\because PQ = PR \text{ proved})$$

$$\Rightarrow \frac{QS}{QR} = \frac{PQ}{QT} \quad \text{(Taking reciprocals)} \ldots(1)$$

Now, in ΔPQS and ΔTQR, we have

$\angle PQS = \angle TQR$ (Each $= \angle 1$)

and $\frac{QS}{QR} = \frac{PQ}{QT}$ (By (1))

Therefore, by SAS similarity criterion, we have

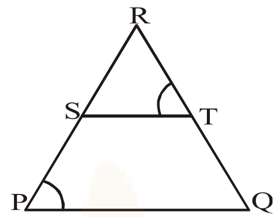
$\Delta PQS \sim \Delta TQR$.

Q5. S and T are points on sides PR and QR of ΔPQR such that $\angle P = \angle RTS$. Show that $\Delta RPQ \sim \Delta RTS$.

Sol. In figure, We have ΔRPQ and ΔRTS in which

$\angle RPQ = \angle RTS$ (Given)

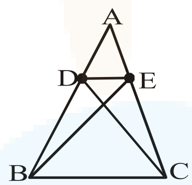
$\angle PRQ = \angle SRT$ (Each $= \angle R$)



Then by AA similarity criterion, we have

$\Delta RPQ \sim \Delta RTS$

Q6. In figure, if $\Delta ABE \cong \Delta ACD$, show that $\Delta ADE \sim \Delta ABC$.



Sol. In figure,

$\Delta ABE \cong \Delta ACD$ (Given)

$\Rightarrow$ $AB = AC$ and $AE = AD$ (CPCT)

$\Rightarrow$ $\frac{AB}{AC} = 1$ and $\frac{AD}{AE} = 1$

$\Rightarrow$ $\frac{AB}{AC} = \frac{AD}{AE}$ (Each = 1)

Now, in ΔADE and ΔABC, we have

$\frac{AD}{AE} = \frac{AB}{AC}$ (proved)

i.e., $\frac{AD}{AB} = \frac{AE}{AC}$

and also $\angle DAE = \angle BAC$ (Each = $\angle A$)

$\Rightarrow$ $\Delta ADE \sim \Delta ABC$ (By SAS similarity criterion)

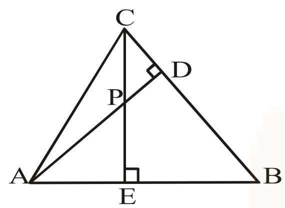
Q7. In figure, altitudes AD and CE of ΔABC intersect each other at the point P. Show that :

(i) $\Delta AEP \sim \Delta CDP$

(ii) $\Delta ABD \sim \Delta CBE$

(iii) $\Delta AEP \sim \Delta ADB$

(iv) $\Delta PDC \sim \Delta BEC$



Sol. (i) In ΔAEP and ΔCDP,

$\angle APE = \angle CPD$ (vertically opposite angles)

$\angle AEP = \angle CDP = 90°$

$\therefore$ By AA similarity

$\Delta AEP \sim \Delta CDP$

(ii) In ΔABD and ΔCBE,

$\angle ABD = \angle CBE$ (common)

$\angle ADB = \angle CEB = 90°$

$\therefore$ By AA similarity

$\Delta ABD \sim \Delta CBE$

(iii) In ΔAEP and ΔADB,

$\angle PAE = \angle DAB$ (common)

$\angle AEP = \angle ADB = 90°$

$\therefore$ By AA similarity

$\Delta AEP \sim \Delta ADB$

(iv) In ΔPDC and ΔBEC,

$\angle PCD = \angle BCE$ (common)

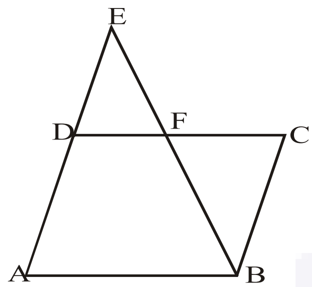
$\angle PDC = \angle BEC = 90°$

$\therefore$ By AA similarity

$\Delta PDC \sim \Delta BEC$

Q8. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that $\Delta ABE \sim \Delta CFB$.

Sol.



In ΔABE and ΔCFB,

$\angle EAB = \angle BCF$ (opp. angles of parallelogram)

$\angle AEB = \angle CBF$ (Alternate interior angles, As $AE \| BC$)

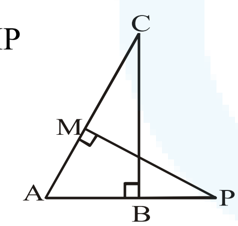
$\therefore$ By AA similarity

$\Delta ABE \sim \Delta CFB$

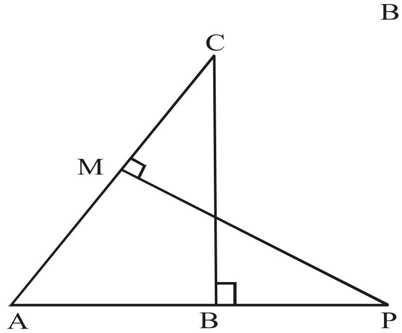
Q9. In figure, ABC and AMP are two right triangles, right angled at B and M respectively. Prove that:

(i) $\Delta ABC \sim \Delta AMP$

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$



Sol.



(i) In ΔABC and ΔAMP

$\angle CAB = \angle PAM$ (common)

$\angle ABC = \angle AMP = 90°$

$\therefore$ By AA similarity

$\Delta ABC \sim \Delta AMP$

(ii) As $\Delta ABC \sim \Delta AMP$ (Proved above)

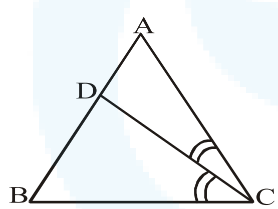
$$\therefore \quad \frac{CA}{PA} = \frac{BC}{MP}$$

Q10. CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGF$ such that D and H lie on sides AB and FE of ΔABC and ΔEFG respectively. If $\Delta ABC \sim \Delta FEG$, show that :

(i) $\frac{CD}{GH} = \frac{AC}{FG}$ (ii) $\Delta DCB \sim \Delta HGE$

(iii) $\Delta DCA \sim \Delta HGF$

Sol. $\Delta ABC \sim \Delta FEG$



$\Rightarrow \angle ACB = \angle EGF$

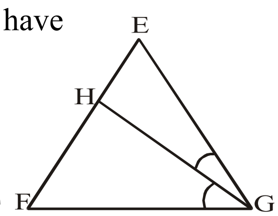
$\Rightarrow \frac{1}{2}\angle ACB = \frac{1}{2}\angle EGF$

$\Rightarrow \angle DCB = \angle HGE$ ...(1)

Also, $\angle B = \angle E$

$\Rightarrow \angle DBC = \angle HEG$ ...(2)

From (1) and (2), we have



$\Rightarrow \Delta DCB \sim \Delta HGE$

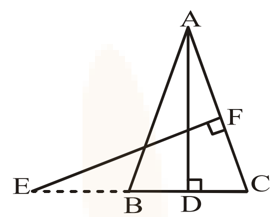
Similarly, we have

$\Delta DCA \sim \Delta HGF$

Now, $\Delta DCA \sim \Delta HGF$

$\Rightarrow \frac{DC}{HG} = \frac{CA}{GF} \Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$

Q11. In figure, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD $\perp$ BC and EF $\perp$ AC, prove that Δ ABD ~ ΔECF.



Sol. In figure,

We are given that ΔABC is isosceles.

and $\quad AB = AC$

$\Rightarrow \quad \angle B = \angle C$ ...(1)

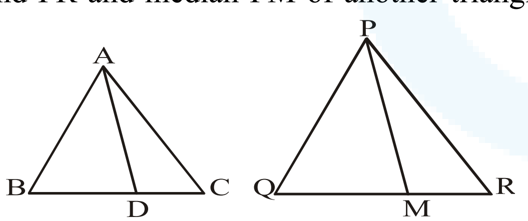
For triangles ABD and ECF,

$\angle ABD = \angle ECF \quad$ {from (1)}

and $\quad \angle ADB = \angle EFC \quad$ {each = 90°}

$\Rightarrow \quad \Delta ABD \sim \Delta ECF$ (AA similarity)

Q12. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that ΔABC ~ ΔPQR.

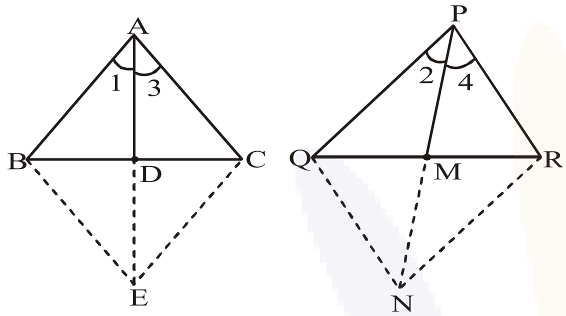


Sol. Given. ΔABC and ΔPQR. AD and PM are their medians respectively.

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM} \qquad ...(1)$$

To prove. $\Delta ABC \sim \Delta PQR$.

Construction : Produce AD to E such that AD = DE and produce PM to N such that PM = MN. Join BE, CE, QN, RN.



Proof : Quadrilaterals ABEC and PQNR are parallelograms because their diagonals bisect each other at D and M respectively.

$\Rightarrow$ BE = AC and QN = PR.

$\Rightarrow \frac{BE}{QN} = \frac{AC}{PR} \Rightarrow \frac{BE}{QN} = \frac{AB}{PQ}$ (By 1)

i.e., $\frac{AB}{PQ} = \frac{BE}{QN}$ ...(2)

From (1), $\frac{AB}{PQ} = \frac{AD}{PM} = \frac{2AD}{2PM} = \frac{AE}{PN}$

i.e., $\frac{AB}{PQ} = \frac{AE}{PN}$ ...(3)

From (2) and (3), we have

$$\frac{AB}{PQ} = \frac{BE}{QN} = \frac{AE}{PN}$$

$\Rightarrow \Delta ABE \sim \Delta PQN \Rightarrow \angle 1 = \angle 2$ ...(4)

Similarly, we can prove

$\Rightarrow \Delta ACE \sim \Delta PRN \Rightarrow \angle 3 = \angle 4$ ...(5)

Adding (4) and (5), we have

$\Rightarrow \angle 1 + \angle 3 = \angle 2 + \angle 4 \Rightarrow \angle A = \angle P$

$\Rightarrow \Delta ABC \sim \Delta PQR$ (SAS similarity criterion)

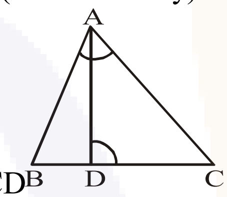
Q13. D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Show that CA^2 = CB. CD.

Sol. For ΔABC and ΔDAC, We have

$\angle BAC = \angle ADC$ (Given)

and $\angle ACB = \angle DCA$ (Each $= \angle C$)

$\Rightarrow \Delta ABC \sim \Delta DAC$ (AA similarity)



$\Rightarrow \frac{AC}{DC} = \frac{CB}{CA}$

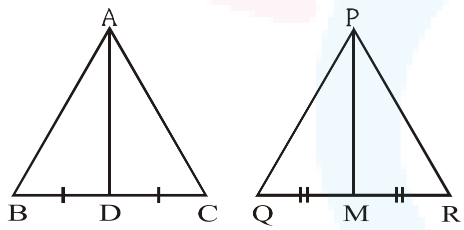
$\Rightarrow \frac{CA}{CD} = \frac{CB}{CA}$

$\Rightarrow CA \times CA = CB \times CD$

$\Rightarrow CA^2 = CB \times CD$

Q14. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR (see figure). Show that $\Delta ABC \sim \Delta PQR$.

Sol.



As, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$ (Given)

So, $\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

$$\left\{\because \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{BD}{QM}\right\}$$

$\therefore$ By SSS similarity,

$\Delta ABD \sim \Delta PQM$.

As, $\Delta ABD \sim \Delta PQM$.

$\therefore \quad \angle ABD = \angle PQM$

Now, In ΔABC and ΔPQR

$$\frac{AB}{PQ} = \frac{BC}{QR} \text{ (Given)}$$

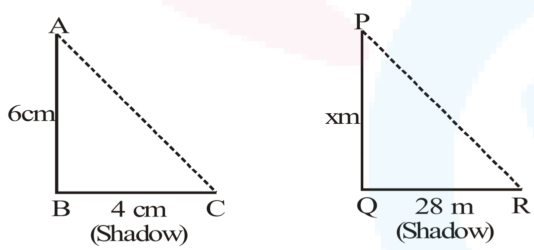
$\angle ABC = \angle PQR$ (Proved above)

$\therefore$ By SAS similarity

$\Delta ABC \sim \Delta PQR$.

Q15. A vertical stick of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Sol.



$\Delta ABC \sim \Delta PQR$

$$\therefore \quad \frac{AB}{PQ} = \frac{BC}{QR}$$

$$\frac{6}{x} = \frac{4}{28}$$

$\Rightarrow \quad x = 42$ m

Q16. If AD and PM are medians of triangles ABC and PQR, respectively where $\Delta ABC \sim \Delta PQR$, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.