CLASS X: MATHS
Chapter 8: Introduction to Trigonometry

Questions and Solutions | Exercise 8.1 - NCERT Books

Q1. In ΔABC, right angled at B, AB = 24 cm, BC = 7 cm. Determine : (i) sin A, cos A (ii) sin C, cos C.

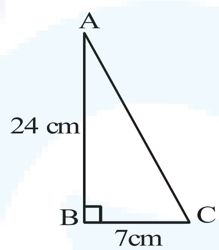
Sol. By Pythagoras Theorem,

$$AC^2 = AB^2 + BC^2 = (24)^2 + (7)^2 = 625$$

$$\Rightarrow AC = \sqrt{625} = 25 \text{ cm.}$$

(i) $\sin A = \frac{BC}{AC} \left\{\text{i.e., } \frac{\text{side opposite to angle A}}{\text{Hyp.}}\right\}$

$= \frac{7}{25}$ ($\because$ BC = 7 cm and AC = 25 cm)



$\cos A = \frac{AB}{AC} \left\{\text{i.e., } \frac{\text{side adjacent to angle A}}{\text{Hyp.}}\right\}$

$= \frac{24}{25}$ ($\because$ AB = 24 cm and AC = 25 cm)

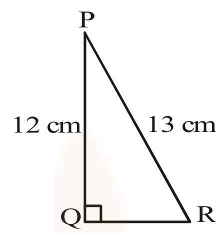
(ii) $\sin C = \frac{AB}{AC} \left\{\text{i.e., } \frac{\text{side opposite to angle C}}{\text{Hyp.}}\right\}$

$= \frac{24}{25}$

$\cos C = \frac{BC}{AC} \left\{\text{i.e., } \frac{\text{side adjacent to angle C}}{\text{Hyp.}}\right\}$

$= \frac{7}{25}$

Q2. In fig, find tan P – cot R.



Sol. In figure, by the Pythagoras Theorem,

$QR^2 = PR^2 - PQ^2 = (13)^2 - (12)^2 = 25$

$\Rightarrow$ $QR = \sqrt{25} = 5$ cm

In ΔPQR right angled at Q, QR = 5 cm is side opposite to the angle P and PQ = 12 cm is side adjacent to the angle P.

Therefore, $\tan P = \frac{QR}{PQ} = \frac{5}{12}$.

Now, QR = 5 cm is side adjacent to the angle R and PQ = 12 cm is side opposite to the angle R.

Therefore, $\cot R = \frac{QR}{PQ} = \frac{5}{12}$

Hence, $\tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$

Q3. If $\sin A = \frac{3}{4}$, calculate cos A and tan A.

Sol. In figure,

$\sin A = \frac{3}{4}$

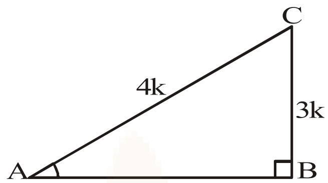
$\Rightarrow \frac{BC}{AC} = \frac{3}{4}$

$\Rightarrow$ BC = 3k

and AC = 4k

where k is the constant of proportionality.

By Pythagoras Theorem,



$AB^2 = AC^2 - BC^2 = (4k)^2 - (3k)^2 = 7k^2$

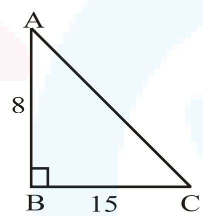
$\Rightarrow$ $AB = \sqrt{7}\,k$

So, $\cos A = \frac{AB}{AC} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$

and $\tan A = \frac{BC}{AB} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}$

Q4. Given 15 cot A = 8, find sin A and sec A.

Sol. $\cot A = \frac{8}{15}$



$\Rightarrow$ $\frac{AB}{BC} = \frac{8}{15}$

$\Rightarrow$ $AB = 8k$

and $BC = 15\,k$

Now, $AC = \sqrt{(8\,k)^2 + (15\,k)^2} = 17\,k$

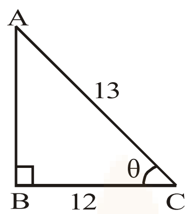
$\sin A = \frac{BC}{AC} = \frac{15\,k}{17\,k} = \frac{15}{17}$, $\sec A = \frac{AC}{AB} = \frac{17\,k}{8\,k} = \frac{17}{8}$

Q5. Given $\sec\theta = \frac{13}{12}$, calculate all other trigonometric ratios.

Sol. $\sec\theta = \frac{13}{12}$

$\Rightarrow \frac{AC}{BC} = \frac{13}{12}$

By Pythagoras Theorem,



$AC^2 = AB^2 + BC^2$

$(13k)^2 = AB^2 + (12k)^2$

$AB^2 = 169k^2 - 144\ k^2$

$AB = \sqrt{25\,k^2} = 5\ k$

$\sin\theta = \frac{AB}{AC} = \frac{5k}{13k} = \frac{5}{13}$

$\cos\theta = \frac{BC}{AC} = \frac{12k}{13k} = \frac{12}{13}$

$\tan\theta = \frac{AB}{BC} = \frac{5k}{12k} = \frac{5}{12}$

$\cot\theta = \frac{BC}{AB} = \frac{12k}{5k} = \frac{12}{5}$

$\text{cosec}\,\theta = \frac{AC}{AB} = \frac{13k}{5k} = \frac{13}{5}$

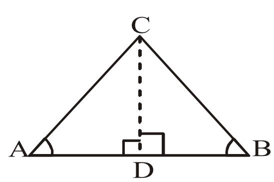
Q6. If ∠A and ∠B are acute angles such that cos A = cos B, then show that ∠A = ∠B.

Sol. In figure ∠A and ∠B are acute angles of ΔABC.

Draw CD ⊥ AB.

We are given that cos A = cos B

$\Rightarrow \frac{AD}{AC} = \frac{BD}{BC}$



$\Rightarrow \quad \frac{AD}{BD} = \frac{AC}{BC} \quad \left(\text{Each} = \frac{CD}{CD}\right)$

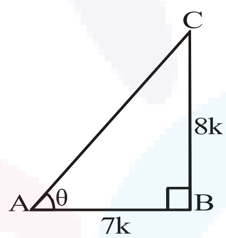
$\Rightarrow \quad \Delta ADC \sim \Delta BDC$ (SSS similarity criterion) $\Rightarrow \angle A = \angle B$

($\because$ all the corresponding angles of two similar triangles are equal)

Q7. If $\cot\theta = \frac{7}{8}$, evaluate :

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ (ii) $\cot^2\theta$

Sol. In figure,



$\cot\theta = \frac{7}{8}$

$\Rightarrow \quad \frac{AB}{BC} = \frac{7}{8}$

$\Rightarrow \quad AB = 7\ k$

and $\quad BC = 8\ k$

Now, $AC^2 = AB^2 + BC^2 = (7k)^2 + (8k)^2$

$= 113\ k^2$

$\Rightarrow \quad AC = \sqrt{113}\ k$

Then $\sin\theta = \frac{BC}{AC} = \frac{8k}{\sqrt{113}\,k} = \frac{8}{\sqrt{113}}$

and $\quad \cos\theta = \frac{AB}{AC} = \frac{7k}{\sqrt{113}\,k} = \frac{7}{\sqrt{113}}$.

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{\left(1+\frac{8}{\sqrt{113}}\right)\left(1-\frac{8}{\sqrt{113}}\right)}{\left(1+\frac{7}{\sqrt{113}}\right)\left(1-\frac{7}{\sqrt{113}}\right)}$

$\frac{(\sqrt{113}+8)(\sqrt{113}-8)}{(\sqrt{113}+7)(\sqrt{113}-7)} = \frac{(\sqrt{113})^2-(8)^2}{(\sqrt{113})^2-(7)^2}$

$\{\because (a+b)(a-b) = a^2 - b^2\}$

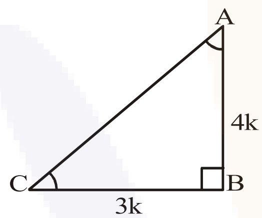
$= \frac{113-64}{113-49} = \frac{49}{64}$

(ii) $\cot\theta = \frac{7}{8} \quad \Rightarrow \cot^2\theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$

Q8. If 3 cot A = 4, check whether

$\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

Sol. In figure,



$3 \cot A = 4$

$\Rightarrow \cot A = \frac{4}{3}$

$\Rightarrow \frac{AB}{BC} = \frac{4}{3}$

$\Rightarrow$ AB = 4k and BC = 3k

Now, $AC = \sqrt{(4k)^2 + (3k)^2} = 5k$

Then $\sin A = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$,

$\cos A = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$

and $\tan A = \frac{BC}{AB} = \frac{3k}{4k} = \frac{3}{4}$

$$LHS = \frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2}$$

$$= \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{16-9}{16+9} = \frac{7}{25}$$

$$RHS = \cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

Therefore, LHS = RHS,

i.e., $\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$

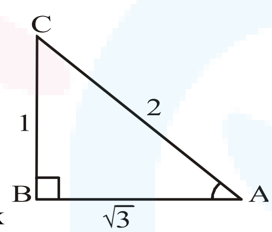
$\left(\because \text{Each side} = \frac{7}{25}\right)$

Q9. In triangle ABC right angled at B, if $\tan A = \frac{1}{\sqrt{3}}$, find the value of :

(i) sin A cos C + cos A sin C

(ii) cos A cos C – sin A sin C.

Sol. $\tan A = \frac{1}{\sqrt{3}}$



$\frac{BC}{BA} = \frac{1}{\sqrt{3}}$

BC = k and BA = $\sqrt{3}k$

$AC^2 = BC^2 + BA^2$

$= k^2 + (\sqrt{3}k)^2 = k^2 + 3k^2 = 4k^2$

$AC = \sqrt{4k^2} = 2k$

(i) sin A.cos C + cos A sin C

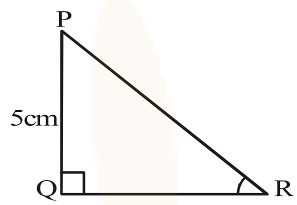
$$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = 1$$

(ii) cos A . cos C – sin A . sin C

$$= \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

Q10. In ΔPQR, right angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.

Sol. In figure,



PQ = 5 cm

PR + QR = 25 cm

i.e., PR = 25 cm – QR

Now, $PR^2 = PQ^2 + QR^2$

$\Rightarrow (25 - QR)^2 = (5)^2 + QR^2$

$\Rightarrow 625 - 50 \times QR + QR^2 = 25 + QR^2$

$\Rightarrow 50 \times QR = 600 \Rightarrow QR = 12$ cm

and PR = 25 cm – 12cm = 13 cm

We find $\sin P = \frac{QR}{PR} = \frac{12}{13}$, $\cos P = \frac{PQ}{PR} = \frac{5}{13}$

and $\tan P = \frac{QR}{PQ} = \frac{12}{5}$

Q11. State whether the following are true or false. Justify your answer.

(i) The value of tan A is always less than 1.

(ii) $\sec A = \frac{12}{5}$ for some value of angle A.

(iii) cos A is the abbreviation used for the cosecant of angle A.

(iv) cot A is the product of cot and A.

(v) $\sin\theta = \frac{4}{3}$ for some angle θ.

Sol. (i) False.

We know that 60° = $\sqrt{3} > 1$.

(ii) True.

We know that value of sec A is always ≥ 1.

(iii) False.

Because cos A is abbreviation used for cosine A.

(iv) False, because cot A is not the product of cot and A.

(v) False, because value of sin cannot be more than 1.

Questions and Solutions | Exercise 8.2 - NCERT Books

Q1. Evaluate :

(i) $\sin 60° \cos 30° + \sin 30° \cos 60°$

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

(iii) $\dfrac{\cos 45°}{\sec 30° + \text{cosec } 30°}$

(iv) $\dfrac{\sin 30° + \tan 45° - \text{cosec } 60°}{\sec 30° + \cos 60° + \cot 45°}$

(v) $\dfrac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

Sol. (i) $\sin 60° \cos 30° + \sin 30° \cos 60°$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)=\left(\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2$$

$$= \frac{3}{4}+\frac{1}{4}=1$$

(ii) $2 \tan^2 45° + \cos^2 30° - \sin^2 60°$

$$= 2 \times (1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 2 + \frac{3}{4} - \frac{3}{4} = 2$$

(iii) $\dfrac{\cos 45°}{\sec 30° + \operatorname{cosec} 30°}$

$$= \frac{1/\sqrt{2}}{\frac{2}{\sqrt{3}}+2} = \frac{1/\sqrt{2}}{2\left(\frac{1+\sqrt{3}}{\sqrt{3}}\right)} = \frac{1(\sqrt{3})}{2\sqrt{2}\,(1+\sqrt{3})}$$

$$= \frac{\sqrt{3}}{2(\sqrt{2})} \times \frac{(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{\sqrt{3}(\sqrt{3}-1)}{2\sqrt{2}\times 2}$$

$$= \frac{(3-\sqrt{3})}{4\sqrt{2}}$$

(iv) $\dfrac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° - \cos 60° + \cot 45°}$

$$= \frac{\frac{1}{2}+1-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}+1} = \frac{\frac{\sqrt{3}+2\sqrt{3}-4}{2\sqrt{3}}}{\frac{4+\sqrt{3}+2\sqrt{3}}{2\sqrt{3}}}$$

$$= \frac{(3\sqrt{3}-4)}{(4+3\sqrt{3})}\ \frac{(4-3\sqrt{3})}{(4-3\sqrt{3})}$$

$$= \frac{12\sqrt{3}-27-16+12\sqrt{3}}{16-9\times 3} = \frac{24\sqrt{3}-43}{-11} = \frac{43-24\sqrt{3}}{11}$$

(v) $\dfrac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

$= \dfrac{5(\cos 60°)^2 + 4(\sec 30°)^2 - (\tan 45°)^2}{(\sin 30°)^2 + (\cos 30°)^2}$

$= \dfrac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \dfrac{\frac{5}{4} + 4 \times \frac{4}{3} - 1}{\frac{1}{4} + \frac{3}{4}}$

$= \dfrac{\frac{5}{4} + \frac{16}{3} - 1}{\frac{1}{4} + \frac{3}{4}} = \frac{5}{4} + \frac{16}{3} - 1$

$= \dfrac{15 + 64 - 12}{12} = \dfrac{67}{12}$

Q2. Choose the correct option and justify your choice:

(i) $\dfrac{2\tan 30°}{1 + \tan^2 30°} =$

(A) sin 60° (B) cos 60° (C) tan 60° (D) sin 30°

(ii) $\dfrac{1 - \tan^2 45°}{1 + \tan^2 45°} =$

(A) tan 90° (B) 1 (C) sin 45° (D) 0

(iii) sin 2 A = 2 sin A is true when A =

(A) 0° (B) 30° (C) 45° (D) 60°

(iv) $\dfrac{2\tan 30°}{1 - \tan^2 30°} =$

(A) cos 60° (B) sin 60 (C) tan 60 ° (D) sin 30°

Sol. (i) Option (A) is correct.

$\dfrac{2\tan 30°}{1 + \tan^2 30°} = \dfrac{2\left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} = \dfrac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}}$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{\sqrt{3}}{2} = \sin 60°$$

(ii) Option (D) is correct.

$$\frac{1 - \tan^2 45°}{1 + \tan^2 45°} = \frac{1-1}{1+1} = 0$$

(iii) Option (A) is correct.

$\sin 2A = 2\sin A$

$\Rightarrow 2\sin A \,.\, \cos A = 2\sin A$

$\Rightarrow \cos A = 1$

$\Rightarrow A = 0°$

(iv) Option (C) is correct.

$$\frac{2 \tan 30°}{1 - \tan^2 30°}$$

$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$= \tan 60°$

Q3. If $\tan (A + B) = \sqrt{3}$ and $\tan (A - B) = \frac{1}{\sqrt{3}}$;

$0° < A + B \leq 90°$; $A > B$, find A and B.

Sol. $\tan (A + B) = \sqrt{3} \quad \Rightarrow A + B = 60° \quad$ ...(1)

$\tan (A - B) = \frac{1}{\sqrt{3}} \quad \Rightarrow A - B = 30° \quad$ ...(2)

Adding (1) and (2),

$2A = 90° \Rightarrow A = 45°$

Then from (1), $45° + B = 60° \Rightarrow B = 15°$

Q4. State whether the following are true or false. Justify your answer.

(i) $\sin (A + B) = \sin A + \sin B$

(ii) The value of $\sin \theta$ increases as θ increases.

(iii) The value of $\cos \theta$ increases as θ increases.

(iv) $\sin \theta = \cos \theta$ for all values of θ.

(v) $\cot A$ is not defined for $A = 0°$.

Sol. (i) False.

When $A = 60°, B = 30°$

LHS $= \sin (A + B) = \sin (60° + 30°)$

$= \sin 90° = 1$

RHS $= \sin A + \sin B$

$= \sin 60° + \sin 30°$

$= \frac{\sqrt{3}}{2} + \frac{1}{2} \neq 1$

i.e., LHS $\neq$ RHS

(ii) True.

Note that $\sin 0° = 0$, $\sin 30° = \frac{1}{2} = 0.5$,

$\sin 45° = \frac{1}{\sqrt{2}} = 0.7$ (approx.),

$\sin 60° = \frac{\sqrt{3}}{2} = 0.87$ (approx.)

and $\sin 90° = 1$

i.e., value of $\sin \theta$ increases as θ increases from $0°$ to $90°$.

(iii) False.

Note that $\cos 0° = 1$,

$\cos 30° = \frac{\sqrt{3}}{2} = 0.87$ (approx.)

$\cos 45° = \frac{1}{\sqrt{2}} = 0.7$ (approx.),

$\cos 60° = \frac{1}{2} = 0.5$ and $\cos 90° = 0$

i.e., value of cos θ decreases as θ increases from 0° to 90°.

(iv) False, it is true for only $\theta = 45°$

(v) True, $\cot A = \frac{1}{0}$ = not defined.

Questions and Solutions | Exercise 8.3 - NCERT Books

Q1. Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.

Sol. We have $\text{cosec}^2 A - \cot^2 A = 1$

$\Rightarrow \text{cosec}^2 A = 1 + \cot^2 A$

$\Rightarrow (\text{cosec } A)^2 = \cot^2 A + 1$

$$\Rightarrow \left(\frac{1}{\sin A}\right)^2 = \cot^2 A + 1$$

$$\Rightarrow (\sin A)^2 = \frac{1}{\cot^2 A + 1}$$

$$\Rightarrow \sin A = \pm \frac{1}{\sqrt{\cot^2 A + 1}}$$

We reject negative value of sin A for acute angle

A. Therefore, $\sin A = \frac{1}{\sqrt{\cot^2 A + 1}}$ $\tan A = \frac{1}{\cot A}$

We have $\sec^2 A - \tan^2 A = 1$

$\Rightarrow \sec^2 A = 1 + \tan^2 A$

$$= 1 + \frac{1}{\cot^2 A} = \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\Rightarrow \sec A = \frac{\sqrt{\cot^2 A + 1}}{\cot A}$$

Q2. Write all the other trigonometric ratios of ∠A in terms of sec A.

Sol. (i) $\sin A = \sqrt{1-\cos^2 A}$

$$= \sqrt{1-\frac{1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A-1}}{\sec A}$$

(ii) $\cos A = \frac{1}{\sec A}$

(iii) $\tan A = \sqrt{\sec^2 A-1}$

(iv) $\cot A = \frac{1}{\tan A} = \frac{1}{\sqrt{\sec^2 A-1}}$

(v) $\text{cosec } A = \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A-1}}$

Q3. Choose the correct option. Justify your choice :

(i) $9 \sec^2 A - 9 \tan^2 A =$

(A) 1 (B) 9 (C) 8 (D) 0

(ii) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \text{cosec}\,\theta) =$

(A) 0 (B) 1 (C) 2 (D) – 1

(iii) $(\sec A + \tan A)(1 - \sin A) =$

(A) sec A (B) sin A (C) cosec A (D) cos A

(iv) $\frac{1+\tan^2 A}{1+\cot^2 A} =$

(A) $\sec^2 A$ (B) – 1 (C) $\cot^2 A$ (D) $\tan^2 A$

Sol. (i) Correct option is (B).

$9 \sec^2 A - 9 \tan^2 A = 9 (\sec^2 A - \tan^2 A)$

$= 9 \times 1 = 9.$

(ii) Correct option is (C).

$(1 + \tan \theta + \sec \theta)(1 + \cot \theta - \operatorname{cosec} \theta)$

$$= \left\{1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right\} \times \left\{1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right\}$$

$$= \left\{\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right\} \times \left\{\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right\}$$

$$= \frac{\{(\cos\theta + \sin\theta) + 1\} \times \{(\cos\theta + \sin\theta) - 1\}}{\cos\theta \times \sin\theta}$$

$$= \frac{(\cos\theta + \sin\theta)^2 - (1)^2}{\cos\theta \times \sin\theta}$$

$\{\because (a + b)(a - b) = a^2 - b^2\}$

$$= \frac{\cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta - 1}{\cos\theta \times \sin\theta}$$

$$= \frac{1 + 2\cos\theta\sin\theta - 1}{\cos\theta\sin\theta} = 2.$$

(iii) Correct option is (D).

$(\sec A + \tan A)(1 - \sin A)$

$$= \sec A - \tan A + \tan A - \frac{\sin^2 A}{\cos A}$$

$$= \frac{1}{\cos A} - \frac{\sin^2 A}{\cos A} = \frac{1 - \sin^2 A}{\cos A}$$

$$= \frac{\cos^2 A}{\cos A} = \cos A$$

(iv) Correct option is (D).

$$\frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{\sec^2 A}{\operatorname{cosec}^2 A} = \tan^2 A$$

Q4. Prove the following identities, where the angles involved are acute angles for which the following expressions are defined.

(i) $(\text{cosec } \theta - \cot \theta)^2 = \frac{1 - \cos\theta}{1 + \cos\theta}$.

(ii) $\frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$.

(iii) $\frac{\tan\theta}{1 - \cot\theta} + \frac{\cot\theta}{1 - \tan\theta} = 1 + \sec\theta \text{ cosec } \theta$.

(iv) $\frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A}$.

(v) $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \text{cosec } A + \cot A$, using the identity $\text{cosec}^2 A = 1 + \cot^2 A$.

(vi) $\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$.

(vii) $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$.

(viii) $(\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$.

(ix) $(\text{cosec } A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$.

(x) $\left(\frac{1 + \tan^2 A}{1 + \cot^2 A}\right) = \left(\frac{1 - \tan A}{1 - \cot A}\right)^2 = \tan^2 A$.

Sol. (i) LHS $= (\text{cosec } \theta - \cot \theta)^2$

$$= \left\{\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right\}^2 = \left(\frac{1 - \cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1 - \cos\theta)^2}{\sin^2\theta} = \frac{(1 - \cos\theta)^2}{1 - \cos^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{(1-\cos\theta)\times(1+\cos\theta)} = \frac{1-\cos\theta}{1+\cos\theta}$$

$\therefore$ LHS = RHS.

(ii) $\text{LHS} = \dfrac{\cos A}{1+\sin A} + \dfrac{1+\sin A}{\cos A}$

$$= \frac{\cos^2 A + (1+\sin A)^2}{\cos A(1+\sin A)}$$

$$= \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{\cos A(1+\sin A)}$$

$$= \frac{2(1+\sin A)}{\cos A(1+\sin A)}$$

= 2secA = R.H.S.

(iii) $\text{LHS} = \dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta}$

$$= \frac{\left(\frac{\sin\theta}{\cos\theta}\right)}{\left(1-\frac{\cos\theta}{\sin\theta}\right)} + \frac{\left(\frac{\cos\theta}{\sin\theta}\right)}{\left(1-\frac{\sin\theta}{\cos\theta}\right)}$$

$$= \frac{\left(\frac{\sin\theta}{\cos\theta}\right)}{\left(\frac{\sin\theta-\cos\theta}{\sin\theta}\right)} + \frac{\left(\frac{\cos\theta}{\sin\theta}\right)}{\left(\frac{\cos\theta-\sin\theta}{\cos\theta}\right)}$$

$$= \frac{\sin\theta\times\sin\theta}{\cos\theta\times(\sin\theta-\cos\theta)} + \frac{\cos\theta\times\cos\theta}{\sin\theta\times(\cos\theta-\sin\theta)}$$

$$= \frac{\sin^2\theta}{\cos\theta\times(\sin\theta-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta\times(\sin\theta-\cos\theta)}$$

$$= \frac{\sin\theta\times\sin^2\theta - \cos\theta\times\cos^2\theta}{\cos\theta\times\sin\theta\times(\sin\theta-\cos\theta)}$$

$$= \frac{\sin^3\theta - \cos^3\theta}{\cos\theta \times \sin\theta \times (\sin\theta - \cos\theta)}$$

$$= \frac{(\sin\theta - \cos\theta) \times (\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta)}{\cos\theta \times \sin\theta \times (\sin\theta - \cos\theta)}$$

$\{\because a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$$= \frac{\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta}{\cos\theta \times \sin\theta}$$

$$= \frac{1 + \sin\theta\cos\theta}{\cos\theta\sin\theta} = \frac{1}{\cos\theta\sin\theta} + 1$$

$$= 1 + \left(\frac{1}{\cos\theta}\right)\left(\frac{1}{\sin\theta}\right)$$

$= 1 + \sec\theta \operatorname{cosec}\theta$

$\therefore$ LHS = RHS

(iv) L.H.S. $= \dfrac{1 + \sec A}{\sec A} = \dfrac{1 + \frac{1}{\cos A}}{\frac{1}{\cos A}} = \dfrac{\cos A + 1}{1}$

R.H.S. $= \dfrac{\sin^2 A}{1 - \cos A} = \dfrac{1 - \cos^2 A}{1 - \cos A} = 1 + \cos A$

$\therefore$ L.H.S. = R.H.S.

(v) LHS $= \dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

$$= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} - \frac{1}{\sin A}}$$

(Dividing the numerator and denominator by sin A)

$$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A} = \frac{(\operatorname{cosec} A + \cot A) - 1}{\{1 + \cot A - \operatorname{cosec} A\}}$$

$$= \frac{(\operatorname{cosec} A + \cot A) - (\operatorname{cosec}^2 A - \cot^2 A)}{\{1 + \cot A - \operatorname{cosec} A\}}$$

$(\because \operatorname{cosec}^2 A = 1 + \cot^2 A$, i.e., $\operatorname{cosec}^2 A - \cot^2 A = 1)$

$$= \frac{(\operatorname{cosec} A + \cot A) - (\operatorname{cosec} A + \cot A) \times (\operatorname{cosec} A - \cot A)}{\{1 + \cot A - \operatorname{cosec} A\}}$$

$\{\because (a + b)(a - b) = a^2 - b^2\}$

$$= \frac{(\operatorname{cosec} A + \cot A) \times \{1 - (\operatorname{cosec} A - \cot A)\}}{\{1 + \cot A - \operatorname{cosec} A\}}$$

$$= \frac{(\operatorname{cosec} A + \cot A) \times \{1 + \cot A - \operatorname{cosec} A\}}{\{1 + \cot A - \operatorname{cosec} A\}}$$

$= \operatorname{cosec} A + \cot A$

= RHS

(vi) LHS $= \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sqrt{\frac{(1 + \sin A)(1 + \sin A)}{(1 - \sin A)(1 + \sin A)}}$

$$= \sqrt{\frac{(1 + \sin A)^2}{(1)^2 - (\sin A)^2}} = \sqrt{\frac{(1 + \sin A)^2}{1 - \sin^2 A}}$$

$$= \sqrt{\frac{(1 + \sin A)^2}{\cos^2 A}} = \frac{1 + \sin A}{\cos A}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \sec A + \tan A$$

$\therefore$ LHS = RHS.

(vii) L.H.S. $= \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

$= \dfrac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta-1)}$

$= \dfrac{\sin\theta(\sin^2\theta+\cos^2\theta-2\sin^2\theta)}{\cos\theta(2\cos^2\theta-\sin^2\theta-\cos^2\theta)}$

$= \dfrac{\tan\theta(\cos^2\theta-\sin^2\theta)}{(\cos^2\theta-\sin^2\theta)}$

$= \tan\theta$ = R.H.S.

(viii) L.H.S. = $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \operatorname{cosec}^2 A + 2 + \cos^2 A + \sec^2 A + 2$

$= 4 + 1 + 1 + \cot^2 A + 1 + \tan^2 A$

$= 7 + \tan^2 A + \cot^2 A$ = R.H.S.

(ix) LHS = $(\operatorname{cosec} A - \sin A)(\sec A - \cos A)$

$= \left(\dfrac{1}{\sin A} - \sin A\right) \times \left(\dfrac{1}{\cos A} - \cos A\right)$

$= \dfrac{1-\sin^2 A}{\sin A} \times \dfrac{1-\cos^2 A}{\cos A}$

$= \dfrac{\cos^2 A}{\sin A} \times \dfrac{\sin^2 A}{\cos A} = \sin A \cos A$

Now, RHS $= \dfrac{1}{\tan A + \cot A} = \dfrac{1}{\dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A}}$

$= \dfrac{1}{\dfrac{\sin^2 A + \cos^2 A}{\sin A \cos A}} = \dfrac{\sin A \cos A}{\sin^2 A + \cos^2 A}$

$= \dfrac{\sin A \cos A}{1}$

$\therefore$ LHS = RHS.

(x) L.H.S. $= \left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right) = \dfrac{\sec^2 A}{\operatorname{cosec}^2 A}$

$= \tan^2 A$ = R.H.S.

& $\left(\dfrac{1-\tan A}{1-\cot A}\right)^2 = \left(\dfrac{\dfrac{\cos A - \sin A}{\cos A}}{\dfrac{\sin A - \cos A}{\sin A}}\right)^2$

$= \dfrac{\sin^2 A}{\cos^2 A} = \tan^2 A$ = R.H.S.