Class XI : Maths
Chapter 8 : Sequence And Series

Questions and Solutions | Exercise 8.1 - NCERT Books

Question 1:

Write the first five terms of the sequences whose n[th] term is $a_n = n(n+2)$

Answer

$a_n = n(n+2)$

Substituting n = 1, 2, 3, 4, and 5, we obtain

$a_1 = 1(1+2) = 3$

$a_2 = 2(2+2) = 8$

$a_3 = 3(3+2) = 15$

$a_4 = 4(4+2) = 24$

$a_5 = 5(5+2) = 35$

Therefore, the required terms are 3, 8, 15, 24, and 35.

Question 2:

Write the first five terms of the sequences whose n[th] term is $a_n = \frac{n}{n+1}$

Answer

$a_n = \frac{n}{n+1}$

Substituting n = 1, 2, 3, 4, 5, we obtain

$a_1 = \frac{1}{1+1} = \frac{1}{2},\ a_2 = \frac{2}{2+1} = \frac{2}{3},\ a_3 = \frac{3}{3+1} = \frac{3}{4},\ a_4 = \frac{4}{4+1} = \frac{4}{5},\ a_5 = \frac{5}{5+1} = \frac{5}{6}$

Therefore, the required terms are $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$, and $\frac{5}{6}$.

Question 3:

Write the first five terms of the sequences whose n[th] term is $a_n = 2^n$

Answer

$a_n = 2^n$

Substituting n = 1, 2, 3, 4, 5, we obtain

$a_1 = 2^1 = 2$
$a_2 = 2^2 = 4$
$a_3 = 2^3 = 8$
$a_4 = 2^4 = 16$
$a_5 = 2^5 = 32$

Therefore, the required terms are 2, 4, 8, 16, and 32.

Question 4:

Write the first five terms of the sequences whose n^{th} term is $a_n = \frac{2n-3}{6}$

Answer

Substituting n = 1, 2, 3, 4, 5, we obtain

$$a_1 = \frac{2\times1-3}{6} = \frac{-1}{6}$$
$$a_2 = \frac{2\times2-3}{6} = \frac{1}{6}$$
$$a_3 = \frac{2\times3-3}{6} = \frac{3}{6} = \frac{1}{2}$$
$$a_4 = \frac{2\times4-3}{6} = \frac{5}{6}$$
$$a_5 = \frac{2\times5-3}{6} = \frac{7}{6}$$

Therefore, the required terms are $\frac{-1}{6}, \frac{1}{6}, \frac{1}{2}, \frac{5}{6}$, and $\frac{7}{6}$.

Question 5:

Write the first five terms of the sequences whose n^{th} term is $a_n = (-1)^{n-1} 5^{n+1}$

Answer

Substituting n = 1, 2, 3, 4, 5, we obtain

$a_1 = (-1)^{1-1} 5^{1+1} = 5^2 = 25$

$a_2 = (-1)^{2-1} 5^{2+1} = -5^3 = -125$

$a_3 = (-1)^{3-1} 5^{3+1} = 5^4 = 625$

$a_4 = (-1)^{4-1} 5^{4+1} = -5^5 = -3125$

$a^5 = (-1)^{5-1} 5^{5+1} = 5^6 = 15625$

Therefore, the required terms are 25, –125, 625, –3125, and 15625.

Question 6:

Write the first five terms of the sequences whose n^{th} term is $a_n = n\dfrac{n^2+5}{4}$

Answer

Substituting *n* = 1, 2, 3, 4, 5, we obtain

$$a_1 = 1\cdot\frac{1^2+5}{4} = \frac{6}{4} = \frac{3}{2}$$

$$a_2 = 2\cdot\frac{2^2+5}{4} = 2\cdot\frac{9}{4} = \frac{9}{2}$$

$$a_3 = 3\cdot\frac{3^2+5}{4} = 3\cdot\frac{14}{4} = \frac{21}{2}$$

$$a_4 = 4\cdot\frac{4^2+5}{4} = 21$$

$$a_5 = 5\cdot\frac{5^2+5}{4} = 5\cdot\frac{30}{4} = \frac{75}{2}$$

Therefore, the required terms are $\frac{3}{2}, \frac{9}{2}, \frac{21}{2}, 21, \text{ and } \frac{75}{2}.$

Question 7:

Find the 17th term in the following sequence whose n^{th} term is $a_n = 4n - 3; a_{17}, a_{24}$

Answer

Substituting *n* = 17, we obtain

$a_{17} = 4(17) - 3 = 68 - 3 = 65$

Substituting *n* = 24, we obtain

$a_{24} = 4(24) - 3 = 96 - 3 = 93$

Question 8:

Find the 7th term in the following sequence whose *n*th term is $a_n = \frac{n^2}{2n}; a_7$

Answer

Substituting *n* = 7, we obtain

$$a_7 = \frac{7^2}{2^7} = \frac{49}{128}$$

Question 9:

Find the 9th term in the following sequence whose *n*th term is $a_n = (-1)^{n-1} n^3; a_9$

Answer

Substituting *n* = 9, we obtain

$$a_9 = (-1)^{9-1}(9)^3 = (9)^3 = 729$$

Question 10:

Find the 20th term in the following sequence whose *n*th term is $a_n = \frac{n(n-2)}{n+3}; a_{20}$

Answer

Substituting *n* = 20, we obtain

$$a_{20} = \frac{20(20-2)}{20+3} = \frac{20(18)}{23} = \frac{360}{23}$$

Question 11:

Write the first five terms of the following sequence and obtain the corresponding series:

$a_1 = 3, a_n = 3a_{n-1} + 2 \text{ for all } n > 1$

Answer

$a_1 = 3, a_n = 3a_{n-1} + 2 \text{ for all } n > 1$

$$\Rightarrow a_2 = 3a_1 + 2 = 3(3) + 2 = 11$$
$$a_3 = 3a_2 + 2 = 3(11) + 2 = 35$$
$$a_4 = 3a_3 + 2 = 3(35) + 2 = 107$$
$$a_5 = 3a_4 + 2 = 3(107) + 2 = 323$$

Hence, the first five terms of the sequence are 3, 11, 35, 107, and 323.

The corresponding series is 3 + 11 + 35 + 107 + 323 + …

Question 12:

Write the first five terms of the following sequence and obtain the corresponding series:

$$a_1 = -1, a_n = \frac{a_{n-1}}{n}, n \geq 2$$

Answer

$$a_1 = -1, a_n = \frac{a_{n-1}}{n}, n \geq 2$$
$$\Rightarrow a_2 = \frac{a_1}{2} = \frac{-1}{2}$$
$$a_3 = \frac{a_2}{3} = \frac{-1}{6}$$
$$a_4 = \frac{a_3}{4} = \frac{-1}{24}$$
$$a_5 = \frac{a_4}{4} = \frac{-1}{120}$$

Hence, the first five terms of the sequence are $-1, \frac{-1}{2}, \frac{-1}{6}, \frac{-1}{24}$, and $\frac{-1}{120}$.

The corresponding series is $(-1) + \left(\frac{-1}{2}\right) + \left(\frac{-1}{6}\right) + \left(\frac{-1}{24}\right) + \left(\frac{-1}{120}\right) + \ldots$

Question 13:

Write the first five terms of the following sequence and obtain the corresponding series:

$$a_1 = a_2 = 2, a_n = a_{n-1} - 1, n > 2$$

Answer

$$a_1 = a_2 = 2, a_n = a_{n-1} - 1, n > 2$$

$\Rightarrow a_3 = a_2 - 1 = 2 - 1 = 1$

$a_4 = a_3 - 1 = 1 - 1 = 0$

$a_5 = a_4 - 1 = 0 - 1 = -1$

Hence, the first five terms of the sequence are 2, 2, 1, 0, and −1.

The corresponding series is 2 + 2 + 1 + 0 + (−1) + …

Question 14:

The Fibonacci sequence is defined by

$1 = a_1 = a_2 \text{ and } a_n = a_{n-1} + a_{n-2}, n > 2$

Find $\frac{a_{n+1}}{a_n}$, for n = 1, 2, 3, 4, 5

Answer

$1 = a_1 = a_2$

$a_n = a_{n-1} + a_{n-2}, n > 2$

$\therefore a_3 = a_2 + a_1 = 1 + 1 = 2$

$a_4 = a_3 + a_2 = 2 + 1 = 3$

$a_5 = a_4 + a_3 = 3 + 2 = 5$

$a_6 = a_5 + a_4 = 5 + 3 = 8$

$\therefore$ For n = 1, $\frac{a_n + 1}{a_n} = \frac{a_2}{a_1} = \frac{1}{1} = 1$

For n = 2, $\frac{a_n + 1}{a_n} = \frac{a_3}{a_2} = \frac{2}{1} = 2$

For n = 3, $\frac{a_n + 1}{a_n} = \frac{a_4}{a_3} = \frac{3}{2}$

For n = 4, $\frac{a_n + 1}{a_n} = \frac{a_5}{a_4} = \frac{5}{3}$

For n = 5, $\frac{a_n + 1}{a_n} = \frac{a_6}{a_5} = \frac{8}{5}$

Class XI : Maths
Chapter 8 : Sequence And Series

Questions and Solutions | Exercise 8.2 - NCERT Books

Question 1:

Find the 20th and nth terms of the G.P.

Answer

The given G.P. is $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \ldots$

Here, a = First term = $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \ldots$

r = Common ratio = $\frac{5}{2}$

$$a_{20} = ar^{20-1} = \frac{5}{2}\left(\frac{1}{2}\right)^{19} = \frac{5}{(2)(2)^{19}} = \frac{5}{(2)^{20}}$$

$$a_n = ar^{n-1} = \frac{5}{2}\left(\frac{1}{2}\right)^{n-1} = \frac{5}{(2)(2)^{n-1}} = \frac{5}{(2)^n}$$

Question 2:

Find the 12th term of a G.P. whose 8th term is 192 and the common ratio is 2.

Answer

Common ratio, r = 2

Let a be the first term of the G.P.

$\therefore a_8 = ar^{8-1} = ar^7$

$\Rightarrow ar^7 = 192$

$a(2)^7 = 192$

$a(2)^7 = (2)^6\ (3)$

$$\Rightarrow a=\frac{(2)^6\times 3}{(2)^7}=\frac{3}{2}$$

$$\therefore a_{12}=a\,r^{12-1}=\left(\frac{3}{2}\right)(2)^{11}=(3)(2)^{10}=3072$$

Question 3:

The 5th, 8th and 11th terms of a G.P. are p, q and s, respectively. Show that $q^2 = ps$.

Answer

Let a be the first term and r be the common ratio of the G.P.

According to the given condition,

$a_5 = a\,r^{5-1} = a\,r^4 = p$ … (1)

$a_8 = a\,r^{8-1} = a\,r^7 = q$ … (2)

$a_{11} = a\,r^{11-1} = a\,r^{10} = s$ … (3)

Dividing equation (2) by (1), we obtain

$$\frac{a\,r^7}{a\,r^4}=\frac{q}{p}$$

$$r^3=\frac{q}{p} \qquad \ldots(4)$$

Dividing equation (3) by (2), we obtain

$$\frac{a\,r^{10}}{a\,r^7}=\frac{s}{q}$$

$$\Rightarrow r^3=\frac{s}{q} \qquad \ldots(5)$$

Equating the values of r^3 obtained in (4) and (5), we obtain

$$\frac{q}{p}=\frac{s}{q}$$

$$\Rightarrow q^2=ps$$

Thus, the given result is proved.

Question 4:

The 4th term of a G.P. is square of its second term, and the first term is –3. Determine its 7th term.

Answer

Let a be the first term and r be the common ratio of the G.P.

$\therefore a = -3$

It is known that, $a_n = ar^{n-1}$

$\therefore a_4 = ar^3 = (-3)\ r^3$

$a_2 = a\ r^1 = (-3)\ r$

According to the given condition,

$(-3)\ r^3 = [(-3)\ r]^2$

$\Rightarrow -3r^3 = 9\ r^2$

$\Rightarrow r = -3$

$a_7 = a\ r^{7-1} = a\ r^6 = (-3)\ (-3)^6 = -\ (3)^7 = -2187$

Thus, the seventh term of the G.P. is –2187.

Question 5:

Which term of the following sequences:

(a) (b) $\sqrt{3}, 3, 3\sqrt{3}, \ldots$ is 729? (c) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$ is $\frac{1}{19683}$?

Answer

(a) The given sequence is $2, 2\sqrt{2}, 4, \ldots$

Here, a = 2 and $r = \frac{2\sqrt{2}}{2} = \sqrt{2}$

Let the n^{th} term of the given sequence be 128.

$a_n = ar^{n-1}$

$\Rightarrow (2)(\sqrt{2})^{n-1} = 128$

$\Rightarrow (2)(2)^{\frac{n-1}{2}} = (2)^7$

$\Rightarrow (2)^{\frac{n-1}{2}+1} = (2)^7$

$\therefore \frac{n-1}{2} + 1 = 7$

$\Rightarrow \frac{n-1}{2} = 6$

$\Rightarrow n-1 = 12$

$\Rightarrow n = 13$

Thus, the 13th term of the given sequence is 128.

(b) The given sequence is $\sqrt{3}, 3, 3\sqrt{3}, \ldots$

Here, $a = \sqrt{3}$ and $r = \frac{3}{\sqrt{3}} = \sqrt{3}$

Let the n^{th} term of the given sequence be 729.

$a_n = ar^{n-1}$

$\therefore ar^{n-1} = 729$

$\Rightarrow (\sqrt{3})(\sqrt{3})^{n-1} = 729$

$\Rightarrow (3)^{\frac{1}{2}}(3)^{\frac{n-1}{2}} = (3)^6$

$\Rightarrow (3)^{\frac{1}{2}+\frac{n-1}{2}} = (3)^6$

$\therefore \frac{1}{2} + \frac{n-1}{2} = 6$

$\Rightarrow \frac{1+n-1}{2} = 6$

$\Rightarrow n = 12$

Thus, the 12th term of the given sequence is 729.

(c) The given sequence is $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$

Here, $a=\frac{1}{3}$ and $r=\frac{1}{9}\div\frac{1}{3}=\frac{1}{3}$

Let the n^{th} term of the given sequence be $\frac{1}{19683}$.

$$a_n = ar^{n-1}$$
$$\therefore ar^{n-1}=\frac{1}{19683}$$
$$\Rightarrow\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)^{n-1}=\frac{1}{19683}$$
$$\Rightarrow\left(\frac{1}{3}\right)^{n}=\left(\frac{1}{3}\right)^{9}$$
$$\Rightarrow n=9$$

Thus, the 9^{th} term of the given sequence is $\frac{1}{19683}$.

Question 6:

For what values of x, the numbers $\frac{2}{7},x,-\frac{7}{2}$ are in G.P?

Answer

The given numbers are $\frac{-2}{7},x,\frac{-7}{2}$.

Common ratio = $\frac{x}{\frac{-2}{7}}=\frac{-7x}{2}$

Also, common ratio = $\frac{\frac{-7}{2}}{x}=\frac{-7}{2x}$

$$\therefore \frac{-7x}{2} = \frac{-7}{2x}$$

$$\Rightarrow x^2 = \frac{-2\times 7}{-2\times 7} = 1$$

$$\Rightarrow x = \sqrt{1}$$

$$\Rightarrow x = \pm 1$$

Thus, for $x = \pm 1$, the given numbers will be in G.P.

Question 7:

Find the sum to 20 terms in the geometric progression 0.15, 0.015, 0.0015 …

Answer

The given G.P. is 0.15, 0.015, 0.00015, …

Here, a = 0.15 and $r = \frac{0.015}{0.15} = 0.1$

$$S_n = \frac{a\left(1-r^n\right)}{1-r}$$

$$\therefore S_{20} = \frac{0.15\left[1-(0.1)^{20}\right]}{1-0.1}$$

$$= \frac{0.15}{0.9}\left[1-(0.1)^{20}\right]$$

$$= \frac{15}{90}\left[1-(0.1)^{20}\right]$$

$$= \frac{1}{6}\left[1-(0.1)^{20}\right]$$

Question 8:

Find the sum to n terms in the geometric progression $\sqrt{7}, \sqrt{21}, 3\sqrt{7} \ldots$

Answer

The given G.P. is $\sqrt{7}, \sqrt{21}, 3\sqrt{7}, \ldots$

Here, $a = \sqrt{7}$

$$r=\frac{\sqrt{21}}{\sqrt{7}}=\sqrt{3}$$

$$S_n=\frac{a\left(1-r^n\right)}{1-r}$$

$$\therefore S_n=\frac{\sqrt{7}\left[1-\left(\sqrt{3}\right)^n\right]}{1-\sqrt{3}}$$

$$=\frac{\sqrt{7}\left[1-\left(\sqrt{3}\right)^n\right]}{1-\sqrt{3}}\times\frac{1+\sqrt{3}}{1+\sqrt{3}} \quad \text{(By rationalizir}$$

$$=\frac{\sqrt{7}\left(1+\sqrt{3}\right)\left[1-\left(\sqrt{3}\right)^n\right]}{1-3}$$

$$=\frac{-\sqrt{7}\left(1+\sqrt{3}\right)}{2}\left[1-(3)^{\frac{n}{2}}\right]$$

$$=\frac{\sqrt{7}\left(1+\sqrt{3}\right)}{2}\left[(3)^{\frac{n}{2}}-1\right]$$

Question 9:

Find the sum to *n* terms in the geometric progression $1, -a, a^2, -a^3 \ldots \ (\text{if } a \neq -1)$

Answer

The given G.P. is $1, -a, a^2, -a^3, \ldots\ldots\ldots\ldots$

Here, first term = a_1 = 1

Common ratio = r = − a

$$S_n=\frac{a_1\left(1-r^n\right)}{1-r}$$

$$\therefore S_n=\frac{1\left[1-(-a)^n\right]}{1-(-a)}=\frac{\left[1-(-a)^n\right]}{1+a}$$

Question 10:

Find the sum to *n* terms in the geometric progression $x^3, x^5, x^7 \ldots \ (\text{if } x \neq \pm 1)$

Answer

The given G.P. is $x^3, x^5, x^7, \ldots$

Here, $a = x^3$ and $r = x^2$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{x^3\left[1-(x^2)^n\right]}{1-x^2} = \frac{x^3(1-x^{2n})}{1-x^2}$$

Question 11:

Evaluate $\sum_{k=1}^{11}(2+3^k)$

Answer

$$\sum_{k=1}^{11}(2+3^k) = \sum_{k=1}^{11}(2) + \sum_{k=1}^{11}3^k = 2(11) + \sum_{k=1}^{11}3^k = 22 + \sum_{k=1}^{11}3^k \quad \ldots(1)$$

$$\sum_{k=1}^{11}3^k = 3^1 + 3^2 + 3^3 + \ldots + 3^{11}$$

The terms of this sequence 3, 3^2, 3^3, … forms a G.P.

$$S_n = \frac{a(r^n-1)}{r-1}$$

$$\Rightarrow S_{11} = \frac{3\left[(3)^{11}-1\right]}{3-1}$$

$$\Rightarrow S_{11} = \frac{3}{2}(3^{11}-1)$$

$$\therefore \sum_{k=1}^{11}3^k = \frac{3}{2}(3^{11}-1)$$

Substituting this value in equation (1), we obtain

$$\sum_{k=1}^{11}(2+3^k) = 22 + \frac{3}{2}(3^{11}-1)$$

Question 12:

The sum of first three terms of a G.P. is $\frac{39}{10}$ and their product is 1. Find the common ratio and the terms.

Answer

Let $\frac{a}{r}, a, ar$ be the first three terms of the G.P.

$$\frac{a}{r}+a+ar=\frac{39}{10} \qquad ...(1)$$

$$\left(\frac{a}{r}\right)(a)(ar)=1 \qquad ...(2)$$

From (2), we obtain

$a^3 = 1$

⇒ a = 1 (Considering real roots only)

Substituting a = 1 in equation (1), we obtain

$$\frac{1}{r}+1+r=\frac{39}{10}$$

$$\Rightarrow 1+r+r^2=\frac{39}{10}r$$

$$\Rightarrow 10+10r+10r^2-39r=0$$

$$\Rightarrow 10r^2-29r+10=0$$

$$\Rightarrow 10r^2-25r-4r+10=0$$

$$\Rightarrow 5r(2r-5)-2(2r-5)=0$$

$$\Rightarrow (5r-2)(2r-5)=0$$

$$\Rightarrow r=\frac{2}{5} \text{ or } \frac{5}{2}$$

Thus, the three terms of G.P. are $\frac{5}{2}, 1, \text{and } \frac{2}{5}$.

Question 13:

How many terms of G.P. 3, 3^2, 3^3, … are needed to give the sum 120?

Answer

The given G.P. is 3, 3^2, 3^3, …

Let n terms of this G.P. be required to obtain the sum as 120.

$$S_n=\frac{a(r^n-1)}{r-1}$$

Here, a = 3 and r = 3

$$\therefore S_n = 120 = \frac{3(3^n - 1)}{3 - 1}$$

$$\Rightarrow 120 = \frac{3(3^n - 1)}{2}$$

$$\Rightarrow \frac{120 \times 2}{3} = 3^n - 1$$

$$\Rightarrow 3^n - 1 = 80$$

$$\Rightarrow 3^n = 81$$

$$\Rightarrow 3^n = 3^4$$

$\therefore n = 4$

Thus, four terms of the given G.P. are required to obtain the sum as 120.

Question 14:

The sum of first three terms of a G.P. is 16 and the sum of the next three terms is 128. Determine the first term, the common ratio and the sum to n terms of the G.P.

Answer

Let the G.P. be a, ar, ar^2, ar^3, …

According to the given condition,

$a + ar + ar^2 = 16$ and $ar^3 + ar^4 + ar^5 = 128$

$\Rightarrow a(1 + r + r^2) = 16$ … (1)

$ar^3(1 + r + r^2) = 128$ … (2)

Dividing equation (2) by (1), we obtain

$$\frac{ar^3(1 + r + r^2)}{a(1 + r + r^2)} = \frac{128}{16}$$

$$\Rightarrow r^3 = 8$$

$$\therefore r = 2$$

Substituting $r = 2$ in (1), we obtain

$a(1 + 2 + 4) = 16$

$\Rightarrow a(7) = 16$

$\Rightarrow a=\frac{16}{7}$

$S_n=\frac{a\left(r^n-1\right)}{r-1}$

$\Rightarrow S_n=\frac{16}{7}\frac{\left(2^n-1\right)}{2-1}=\frac{16}{7}\left(2^n-1\right)$

Question 15:

Given a G.P. with a = 729 and 7^{th} term 64, determine S_7.

Answer

a = 729

a_7 = 64

Let r be the common ratio of the G.P.

It is known that, $a_n = a\, r^{n-1}$

$a_7 = ar^{7-1} = (729)r^6$

$\Rightarrow 64 = 729\, r^6$

$\Rightarrow r^6=\frac{64}{729}$

$\Rightarrow r^6=\left(\frac{2}{3}\right)^6$

$\Rightarrow r=\frac{2}{3}$

Also, it is known that, $S_n=\frac{a\left(1-r^n\right)}{1-r}$

$$\therefore S_7 = \frac{729\left[1-\left(\frac{2}{3}\right)^7\right]}{1-\frac{2}{3}}$$

$$= 3\times 729\left[1-\left(\frac{2}{3}\right)^7\right]$$

$$= (3)^7\left[\frac{(3)^7-(2)^7}{(3)^7}\right]$$

$$= (3)^7-(2)^7$$

$$= 2187-128$$

$$= 2059$$

Question 16:

Find a G.P. for which sum of the first two terms is –4 and the fifth term is 4 times the third term.

Answer

Let a be the first term and r be the common ratio of the G.P.

According to the given conditions,

$$S_2 = -4 = \frac{a\left(1-r^2\right)}{1-r} \qquad ...(1)$$

$a_5 = 4 \times a_3$

$ar^4 = 4ar^2$

$\Rightarrow r^2 = 4$

$\therefore r = \pm 2$

From (1), we obtain

$$-4=\frac{a\left[1-(2)^2\right]}{1-2} \text{ for } r=2$$

$$\Rightarrow -4=\frac{a(1-4)}{-1}$$

$$\Rightarrow -4=a(3)$$

$$\Rightarrow a=\frac{-4}{3}$$

$$\text{Also, } -4=\frac{a\left[1-(-2)^2\right]}{1-(-2)} \text{ for } r=-2$$

$$\Rightarrow -4=\frac{a(1-4)}{1+2}$$

$$\Rightarrow -4=\frac{a(-3)}{3}$$

$$\Rightarrow a=4$$

Thus, the required G.P. is

$\frac{-4}{3}, \frac{-8}{3}, \frac{-16}{3}, \ldots$ or 4, −8, 16, −32, …

Question 17:

If the 4^{th}, 10^{th} and 16^{th} terms of a G.P. are *x, y* and *z*, respectively. Prove that *x*, *y*, *z* are in G.P.

Answer

Let *a* be the first term and *r* be the common ratio of the G.P.

According to the given condition,

$a_4 = a\,r^3 = x$ … (1)

$a_{10} = a\,r^9 = y$ … (2)

$a_{16} = a\,r^{15} = z$ … (3)

Dividing (2) by (1), we obtain

$$\frac{y}{x}=\frac{ar^9}{ar^3} \Rightarrow \frac{y}{x}=r^6$$

Dividing (3) by (2), we obtain

$$\frac{z}{y}=\frac{ar^{15}}{ar^{9}} \Rightarrow \frac{z}{y}=r^{6}$$

$$\therefore \frac{y}{x}=\frac{z}{y}$$

Thus, x, y, z are in G. P.

Question 18:

Find the sum to n terms of the sequence, 8, 88, 888, 8888…

Answer

The given sequence is 8, 88, 888, 8888…

This sequence is not a G.P. However, it can be changed to G.P. by writing the terms as

S_n = 8 + 88 + 888 + 8888 + …………….. to n terms

$$=\frac{8}{9}\left[9+99+999+9999+\ldots\ldots\ldots\text{to } n \text{ terms}\right]$$

$$=\frac{8}{9}\left[(10-1)+(10^2-1)+(10^3-1)+(10^4-1)+\ldots\ldots\text{to } n \text{ terms}\right]$$

$$=\frac{8}{9}\left[(10+10^2+\ldots\ldots n \text{ terms})-(1+1+1+\ldots. n \text{ terms})\right]$$

$$=\frac{8}{9}\left[\frac{10(10^n-1)}{10-1}-n\right]$$

$$=\frac{8}{9}\left[\frac{10(10^n-1)}{9}-n\right]$$

$$=\frac{80}{81}(10^n-1)-\frac{8}{9}n$$

Question 19:

Find the sum of the products of the corresponding terms of the sequences 2, 4, 8, 16, 32 and 128, 32, 8, 2, $\frac{1}{2}$.

Answer

Required sum = $2\times128+4\times32+8\times8+16\times2+32\times\frac{1}{2}$

$$= 64\left[4+2+1+\frac{1}{2}+\frac{1}{2^2}\right]$$

Here, 4, 2, 1, $\frac{1}{2}, \frac{1}{2^2}$ is a G.P.

First term, a = 4

Common ratio, r = $\frac{1}{2}$

It is known that, $S_n = \frac{a(1-r^n)}{1-r}$

$$\therefore S_5 = \frac{4\left[1-\left(\frac{1}{2}\right)^5\right]}{1-\frac{1}{2}} = \frac{4\left[1-\frac{1}{32}\right]}{\frac{1}{2}} = 8\left(\frac{32-1}{32}\right) = \frac{31}{4}$$

∴Required sum = $64\left(\frac{31}{4}\right) = (16)(31) = 496$

Question 20:

Show that the products of the corresponding terms of the sequences

$a, ar, ar^2, \ldots ar^{n-1}$ and $A, AR, AR^2, \ldots AR^{n-1}$ form a G.P, and find the common ratio.

Answer

It has to be proved that the sequence, aA, $arAR$, ar^2AR^2, …$ar^{n-1}AR^{n-1}$, forms a G.P.

$$\frac{\text{Second term}}{\text{First term}} = \frac{arAR}{aA} = rR$$

$$\frac{\text{Third term}}{\text{Second term}} = \frac{ar^2AR^2}{arAR} = rR$$

Thus, the above sequence forms a G.P. and the common ratio is rR.

Question 21:

Find four numbers forming a geometric progression in which third term is greater than the first term by 9, and the second term is greater than the 4th by 18.

Answer

Let a be the first term and r be the common ratio of the G.P.

$a_1 = a$, $a_2 = ar$, $a_3 = ar^2$, $a_4 = ar^3$

By the given condition,

$a_3 = a_1 + 9$

$\Rightarrow ar^2 = a + 9$ … (1)

$a_2 = a_4 + 18$

$\Rightarrow ar = ar^3 + 18$ … (2)

From (1) and (2), we obtain

$a(r^2 - 1) = 9$ … (3)

$ar(1 - r^2) = 18$ … (4)

Dividing (4) by (3), we obtain

$$\frac{ar(1-r^2)}{a(r^2-1)} = \frac{18}{9}$$

$$\Rightarrow -r = 2$$

$$\Rightarrow r = -2$$

Substituting the value of r in (1), we obtain

$4a = a + 9$

$\Rightarrow 3a = 9$

$\therefore a = 3$

Thus, the first four numbers of the G.P. are 3, 3(− 2), $3(-2)^2$, and $3(-2)^3$ i.e., 3¸−6, 12, and −24.

Question 22:

If the p^{th}, q^{th} and r^{th} terms of a G.P. are a, b and c, respectively. Prove that

$$a^{q-r} b^{r-p} c^{p-q} = 1$$

Answer

Let A be the first term and R be the common ratio of the G.P.

According to the given information,

$AR^{p-1} = a$

$AR^{q-1} = b$

$AR^{r-1} = c$

$a^{q-r} b^{r-p} c^{p-q}$

$= A^{q-r} \times R^{(p-1)(q-r)} \times A^{r-p} \times R^{(q-1)(r-p)} \times A^{p-q} \times R^{(r-1)(p-q)}$

$= Aq^{-r+r-p+p-q} \times R^{(pr-pr-q+r)+(rq-r+p-pq)+(pr-p-qr+q)}$

$= A^0 \times R^0$

$= 1$

Thus, the given result is proved.

Question 23:

If the first and the n^{th} term of a G.P. are a ad b, respectively, and if P is the product of n terms, prove that $P^2 = (ab)^n$.

Answer

The first term of the G.P is a and the last term is b.

Therefore, the G.P. is $a, ar, ar^2, ar^3, \ldots ar^{n-1}$, where r is the common ratio.

$b = ar^{n-1}$ … (1)

P = Product of n terms

$= (a)(ar)(ar^2) \ldots (ar^{n-1})$

$= (a \times a \times \ldots a)(r \times r^2 \times \ldots r^{n-1})$

$= a^n r^{1+2+\ldots(n-1)}$ … (2)

Here, 1, 2, …$(n - 1)$ is an A.P.

$\therefore 1 + 2 + \ldots\ldots\ldots + (n-1) = \frac{n-1}{2}\left[2+(n-1-1)\times 1\right] = \frac{n-1}{2}\left[2+n-2\right] = \frac{n(n-1)}{2}$

$$P = a^n r^{\frac{n(n-1)}{2}}$$

$$\therefore P^2 = a^{2n} r^{n(n-1)}$$

$$= \left[a^2 r^{(n-1)}\right]^n$$

$$= \left[a \times ar^{n-1}\right]^n$$

$$= (ab)^n \qquad [\text{Using } (1)]$$

Thus, the given result is proved.

Question 24:

Show that the ratio of the sum of first *n* terms of a G.P. to the sum of terms from

$(n+1)^{th}$ to $(2n)^{th}$ term is $\frac{1}{r^n}$.

Answer

Let *a* be the first term and *r* be the common ratio of the G.P.

$$\text{Sum of first n terms} = \frac{a(1-r^n)}{(1-r)}$$

Since there are *n* terms from $(n+1)^{th}$ to $(2n)^{th}$ term,

Sum of terms from $(n+1)^{th}$ to $(2n)^{th}$ term $= \frac{a_{n+1}(1-r^n)}{(1-r)}$

$a_{n+1} = ar^{n+1-1} = ar^n$

Thus, required ratio $= \frac{a(1-r^n)}{(1-r)} \times \frac{(1-r)}{ar^n(1-r^n)} = \frac{1}{r^n}$

Thus, the ratio of the sum of first *n* terms of a G.P. to the sum of terms from $(n+1)^{th}$ to $(2n)^{th}$ term is $\frac{1}{r^n}$.

Question 25:

If *a, b, c* and *d* are in G.P. show that $(a^2+b^2+c^2)(b^2+c^2+d^2)=(ab+bc+cd)^2$.

Answer

a, *b*, *c*, *d* are in G.P.

Therefore,

$bc = ad$ … (1)

$b^2 = ac$ … (2)

$c^2 = bd$ … (3)

It has to be proved that,

$(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (ab + bc - cd)^2$

R.H.S.

$= (ab + bc + cd)^2$

$= (ab + ad + cd)^2$ [Using (1)]

$= [ab + d(a + c)]^2$

$= a^2b^2 + 2abd(a + c) + d^2(a + c)^2$

$= a^2b^2 + 2a^2bd + 2acbd + d^2(a^2 + 2ac + c^2)$

$= a^2b^2 + 2a^2c^2 + 2b^2c^2 + d^2a^2 + 2d^2b^2 + d^2c^2$ [Using (1) and (2)]

$= a^2b^2 + a^2c^2 + a^2c^2 + b^2c^2 + b^2c^2 + d^2a^2 + d^2b^2 + d^2b^2 + d^2c^2$

$= a^2b^2 + a^2c^2 + a^2d^2 + b^2 \times b^2 + b^2c^2 + b^2d^2 + c^2b^2 + c^2 \times c^2 + c^2d^2$

[Using (2) and (3) and rearranging terms]

$= a^2(b^2 + c^2 + d^2) + b^2(b^2 + c^2 + d^2) + c^2(b^2 + c^2 + d^2)$

$= (a^2 + b^2 + c^2)(b^2 + c^2 + d^2)$

= L.H.S.

∴ L.H.S. = R.H.S.

$\therefore \left(a^2+b^2+c^2\right)\left(b^2+c^2+d^2\right)=\left(ab+bc+cd\right)^2$

Question 26:

Insert two numbers between 3 and 81 so that the resulting sequence is G.P.

Answer

Let G_1 and G_2 be two numbers between 3 and 81 such that the series, 3, G_1, G_2, 81, forms a G.P.

Let a be the first term and r be the common ratio of the G.P.

$\therefore 81 = (3)(r)^3$

$\Rightarrow r^3 = 27$

$\therefore r = 3$ (Taking real roots only)

For $r = 3$,

$G_1 = ar = (3)(3) = 9$

$G_2 = ar^2 = (3)(3)^2 = 27$

Thus, the required two numbers are 9 and 27.

Question 27:

Find the value of n so that $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the geometric mean between a and b.

Answer

G. M. of *a* and *b* is $\sqrt{ab}$.

By the given condition, $\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}=\sqrt{ab}$

Squaring both sides, we obtain

$$\frac{\left(a^{n+1}+b^{n+1}\right)^2}{\left(a^n+b^n\right)^2}=ab$$

$$\Rightarrow a^{2n+2}+2a^{n+1}b^{n+1}+b^{2n+2}=(ab)\left(a^{2n}+2a^nb^n+b^{2n}\right)$$

$$\Rightarrow a^{2n+2}+2a^{n+1}b^{n+1}+b^{2n+2}=a^{2n+1}b+2a^{n+1}b^{n+1}+ab^{2n+1}$$

$$\Rightarrow a^{2n+2}+b^{2n+2}=a^{2n+1}b+ab^{2n+1}$$

$$\Rightarrow a^{2n+2}-a^{2n+1}b=ab^{2n+1}-b^{2n+2}$$

$$\Rightarrow a^{2n+1}(a-b)=b^{2n+1}(a-b)$$

$$\Rightarrow \left(\frac{a}{b}\right)^{2n+1}=1=\left(\frac{a}{b}\right)^0$$

$$\Rightarrow 2n+1=0$$

$$\Rightarrow n=\frac{-1}{2}$$

Question 28:

The sum of two numbers is 6 times their geometric mean, show that numbers are in the ratio $\left(3+2\sqrt{2}\right):\left(3-2\sqrt{2}\right)$.

Answer

Let the two numbers be *a* and *b*.

G.M. = $\sqrt{ab}$

According to the given condition,

$$a+b=6\sqrt{ab} \qquad \ldots(1)$$

$$\Rightarrow (a+b)^2=36(ab)$$

Also,

$$(a-b)^2=(a+b)^2-4ab=36ab-4ab=32ab$$
$$\Rightarrow a-b=\sqrt{32}\sqrt{ab}$$
$$=4\sqrt{2}\sqrt{ab} \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2a=\left(6+4\sqrt{2}\right)\sqrt{ab}$$
$$\Rightarrow a=\left(3+2\sqrt{2}\right)\sqrt{ab}$$

Substituting the value of *a* in (1), we obtain

$$b=6\sqrt{ab}-\left(3+2\sqrt{2}\right)\sqrt{ab}$$
$$\Rightarrow b=\left(3-2\sqrt{2}\right)\sqrt{ab}$$
$$\frac{a}{b}=\frac{\left(3+2\sqrt{2}\right)\sqrt{ab}}{\left(3-2\sqrt{2}\right)\sqrt{ab}}=\frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

Thus, the required ratio is $\left(3+2\sqrt{2}\right):\left(3-2\sqrt{2}\right)$.

Question 29:

If A and G be A.M. and G.M., respectively between two positive numbers, prove that the numbers are $A\pm\sqrt{(A+G)(A-G)}$.

Answer

It is given that *A* and *G* are A.M. and G.M. between two positive numbers. Let these two positive numbers be *a* and *b*.

$$\therefore AM=A=\frac{a+b}{2} \qquad ...(1)$$
$$GM=G=\sqrt{ab} \qquad ...(2)$$

From (1) and (2), we obtain

$a + b = 2A$ … (3)

$ab = G^2$ … (4)

Substituting the value of *a* and *b* from (3) and (4) in the identity $(a-b)^2 = (a+b)^2 - 4ab$, we obtain

$(a-b)^2 = 4A^2 - 4G^2 = 4(A^2-G^2)$

$(a - b)^2 = 4(A + G)(A - G)$

$$(a-b)=2\sqrt{(A+G)(A-G)} \quad ...(5)$$

From (3) and (5), we obtain

$$2a = 2A + 2\sqrt{(A+G)(A-G)}$$

$$\Rightarrow a = A + \sqrt{(A+G)(A-G)}$$

Substituting the value of a in (3), we obtain

$$b = 2A - A - \sqrt{(A+G)(A-G)} = A - \sqrt{(A+G)(A-G)}$$

Thus, the two numbers are $A \pm \sqrt{(A+G)(A-G)}$.

Question 30:

The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 2nd hour, 4th hour and nth hour?

Answer

It is given that the number of bacteria doubles every hour. Therefore, the number of bacteria after every hour will form a G.P.

Here, $a = 30$ and $r = 2$

$\therefore a_3 = ar^2 = (30)(2)^2 = 120$

Therefore, the number of bacteria at the end of 2nd hour will be 120.

$a_5 = ar^4 = (30)(2)^4 = 480$

The number of bacteria at the end of 4th hour will be 480.

$a_{n+1} = ar^n = (30)\, 2^n$

Thus, number of bacteria at the end of nth hour will be $30(2)^n$.

Question 31:

What will Rs 500 amounts to in 10 years after its deposit in a bank which pays annual interest rate of 10% compounded annually?

Answer

The amount deposited in the bank is Rs 500.

At the end of first year, amount = $\text{Rs}\,500\left(1+\frac{1}{10}\right)$ = Rs 500 (1.1)

At the end of 2nd year, amount = Rs 500 (1.1) (1.1)

At the end of 3rd year, amount = Rs 500 (1.1) (1.1) (1.1) and so on

∴Amount at the end of 10 years = Rs 500 (1.1) (1.1) … (10 times)

= Rs $500(1.1)^{10}$

Question 32:

If A.M. and G.M. of roots of a quadratic equation are 8 and 5, respectively, then obtain the quadratic equation.

Answer

Let the root of the quadratic equation be *a* and *b*.

According to the given condition,

$$\text{A.M.}=\frac{a+b}{2}=8 \Rightarrow a+b=16 \qquad \ldots(1)$$

$$\text{G.M.}=\sqrt{ab}=5 \Rightarrow ab=25 \qquad \ldots(2)$$

The quadratic equation is given by,

$x^2 - x$ (Sum of roots) + (Product of roots) = 0

$x^2 - x(a+b) + (ab) = 0$

$x^2 - 16x + 25 = 0$ [Using (1) and (2)]

Thus, the required quadratic equation is $x^2 - 16x + 25 = 0$

Class XI : Maths
Chapter 8 : Sequence And Series

Questions and Solutions | Miscellaneous Exercise 8 - NCERT Books

Question 1:

If *f* is a function satisfying $f(x+y)=f(x)f(y)$ for all $x, y \in N$ such that $f(1)=3 \text{ and } \sum_{x=1}^{n} f(x)=120$, find the value of *n*.

Answer

It is given that,

f (*x* + *y*) = *f* (*x*) × *f* (*y*) for all *x*, *y* ∈ N … (1)

f (1) = 3

Taking *x* = *y* = 1 in (1), we obtain

f (1 + 1) = *f* (2) = *f* (1) *f* (1) = 3 × 3 = 9

Similarly,

f (1 + 1 + 1) = *f* (3) = *f* (1 + 2) = *f* (1) *f* (2) = 3 × 9 = 27

f (4) = *f* (1 + 3) = *f* (1) *f* (3) = 3 × 27 = 81

∴ *f* (1), *f* (2), *f* (3), …, that is 3, 9, 27, …, forms a G.P. with both the first term and common ratio equal to 3.

It is known that, $S_n=\frac{a(r^n-1)}{r-1}$

It is given that, $\sum_{x=1}^{n} f(x)=120$

$$\therefore 120=\frac{3(3^n-1)}{3-1}$$

$$\Rightarrow 120=\frac{3}{2}(3^n-1)$$

$$\Rightarrow 3^n-1=80$$

$$\Rightarrow 3^n=81=3^4$$

$$\therefore n=4$$

Thus, the value of *n* is 4.

Question 2:

The sum of some terms of G.P. is 315 whose first term and the common ratio are 5 and 2, respectively. Find the last term and the number of terms.

Answer

Let the sum of *n* terms of the G.P. be 315.

It is known that, $S_n = \frac{a(r^n - 1)}{r - 1}$

It is given that the first term *a* is 5 and common ratio *r* is 2.

$$\therefore 315 = \frac{5(2^n - 1)}{2 - 1}$$

$$\Rightarrow 2^n - 1 = 63$$

$$\Rightarrow 2^n = 64 = (2)^6$$

$$\Rightarrow n = 6$$

∴Last term of the G.P = 6^{th} term = ar^{6-1} = $(5)(2)^5$ = (5)(32) = 160

Thus, the last term of the G.P. is 160.

Question 3:

The first term of a G.P. is 1. The sum of the third term and fifth term is 90. Find the common ratio of G.P.

Answer

Let *a* and *r* be the first term and the common ratio of the G.P. respectively.

$\therefore a = 1$

$a_3 = ar^2 = r^2$

$a_5 = ar^4 = r^4$

$\therefore r^2 + r^4 = 90$

$\Rightarrow r^4 + r^2 - 90 = 0$

$$\Rightarrow r^2 = \frac{-1 + \sqrt{1 + 360}}{2} = \frac{-1 \pm \sqrt{361}}{2} = \frac{-1 \pm 19}{2} = -10 \text{ or } 9$$

$$\therefore r = \pm 3 \qquad \text{(Taking real roots)}$$

Thus, the common ratio of the G.P. is ±3.

Question 4:

The sum of three numbers in G.P. is 56. If we subtract 1, 7, 21 from these numbers in that order, we obtain an arithmetic progression. Find the numbers.

Answer

Let the three numbers in G.P. be a, ar, and ar^2.

From the given condition, $a + ar + ar^2 = 56$

$\Rightarrow a(1 + r + r^2) = 56$

… (1)

$a - 1$, $ar - 7$, $ar^2 - 21$ forms an A.P.

$\therefore (ar - 7) - (a - 1) = (ar^2 - 21) - (ar - 7)$

$\Rightarrow ar - a - 6 = ar^2 - ar - 14$

$\Rightarrow ar^2 - 2ar + a = 8$

$\Rightarrow ar^2 - ar - ar + a = 8$

$\Rightarrow a(r^2 + 1 - 2r) = 8$

$\Rightarrow a(r - 1)^2 = 8$ … (2)

$\Rightarrow 7(r^2 - 2r + 1) = 1 + r + r^2$

$\Rightarrow 7r^2 - 14r + 7 - 1 - r - r^2 = 0$

$\Rightarrow 6r^2 - 15r + 6 = 0$

$\Rightarrow 6r^2 - 12r - 3r + 6 = 0$

$\Rightarrow 6r(r - 2) - 3(r - 2) = 0$

$\Rightarrow (6r - 3)(r - 2) = 0$

When $r = 2$, $a = 8$

Therefore, when $r = 2$, the three numbers in G.P. are 8, 16, and 32.

When $r = \frac{1}{2}$, the three numbers in G.P. are 32, 16, and 8. Thus, in either case, the three required numbers are 8, 16, and 32.

Question 5:

A G.P. consists of an even number of terms. If the sum of all the terms is 5 times the sum of terms occupying odd places, then find its common ratio.

Answer

Let the G.P. be T_1, T_2, T_3, T_4, … T_{2n}.

Number of terms = $2n$

According to the given condition,

$T_1 + T_2 + T_3 + \ldots + T_{2n} = 5 [T_1 + T_3 + \ldots + T_{2n-1}]$

$\Rightarrow T_1 + T_2 + T_3 + \ldots + T_{2n} - 5 [T_1 + T_3 + \ldots + T_{2n-1}] = 0$

$\Rightarrow T_2 + T_4 + \ldots + T_{2n} = 4 [T_1 + T_3 + \ldots + T_{2n-1}]$

Let the G.P. be a, ar, ar^2, ar^3, …

$$\therefore \frac{ar\left(r^n - 1\right)}{r-1} = \frac{4 \times a\left(r^n - 1\right)}{r-1}$$

$$\Rightarrow ar = 4a$$

$$\Rightarrow r = 4$$

Thus, the common ratio of the G.P. is 4.

Questions 6.

If $\frac{a+bx}{a-bx}=\frac{b+cx}{b-cx}=\frac{c+dx}{c-dx}(x\neq 0)$, then show that *a*, *b*, *c* and *d* are in G.P.

Answer

It is given that,

$$\frac{a+bx}{a-bx}=\frac{b+cx}{b-cx}$$

$$\Rightarrow (a+bx)(b-cx)=(b+cx)(a-bx)$$

$$\Rightarrow ab-acx+b^2x-bcx^2=ab-b^2x+acx-bcx^2$$

$$\Rightarrow 2b^2x=2acx$$

$$\Rightarrow b^2=ac$$

$$\Rightarrow \frac{b}{a}=\frac{c}{b} \qquad ...(1)$$

Also, $\frac{b+cx}{b-cx}=\frac{c+dx}{c-dx}$

$$\Rightarrow (b+cx)(c-dx)=(b-cx)(c+dx)$$

$$\Rightarrow bc-bdx+c^2x-cdx^2=bc+bdx-c^2x-cdx^2$$

$$\Rightarrow 2c^2x=2bdx$$

$$\Rightarrow c^2=bd$$

$$\Rightarrow \frac{c}{d}=\frac{d}{c} \qquad ...(2)$$

From (1) and (2), we obtain

$$\frac{b}{a}=\frac{c}{b}=\frac{d}{c}$$

Thus, *a*, *b*, *c*, and *d* are in G.P.

Question 7:

Let S be the sum, P the product and R the sum of reciprocals of n terms in a G.P. Prove that $P^2R^n = S^n$

Answer

Let the G.P. be $a, ar, ar^2, ar^3, \ldots ar^{n-1} \ldots$

According to the given information,

$$S = \frac{a(r^n - 1)}{r - 1}$$

$$P = a^n \times r^{1+2+\ldots+n-1}$$

$$= a^n \; r^{\frac{n(n-1)}{2}} \qquad \left[\because \text{ Sum of first } n \text{ natural numbers is } n\frac{(n+1)}{2}\right]$$

$$R = \frac{1}{a} + \frac{1}{ar} + \ldots + \frac{1}{ar^{n-1}}$$

$$= \frac{r^{n-1} + r^{n-2} + \ldots r + 1}{ar^{n-1}}$$

$$= \frac{1(r^n - 1)}{(r-1)} \times \frac{1}{ar^{n-1}} \qquad [\because 1, r, \ldots r^{n-1} \text{ forms a G.P}]$$

$$= \frac{r^n - 1}{ar^{n-1}(r-1)}$$

$$\therefore P^2R^n = a^{2n} r^{n(n-1)} \frac{(r^n - 1)^n}{a^n r^{n(n-1)} (r-1)^n}$$

$$= \frac{a^n (r^n - 1)^n}{(r-1)^n}$$

$$= \left[\frac{a(r^n - 1)}{(r-1)}\right]^n$$

$$= S^n$$

Hence, $P^2 R^n = S^n$

Question 8:

If a, b, c, d are in G.P, prove that $(a^n+b^n),(b^n+c^n),(c^n+d^n)$ are in G.P.

Answer

It is given that a, b, c,and d are in G.P.

$\therefore b^2 = ac$ … (1)

$c^2 = bd$ … (2)

$ad = bc$ … (3)

It has to be proved that $(a^n + b^n)$, $(b^n + c^n)$, $(c^n + d^n)$ are in G.P. i.e.,

$(b^n + c^n)^2 = (a^n + b^n)(c^n + d^n)$

Consider L.H.S.

$(b^n + c^n)^2 = b^{2n} + 2b^nc^n + c^{2n}$

$= (b^2)^n + 2b^nc^n + (c^2)^n$

$= (ac)^n + 2b^nc^n + (bd)^n$ [Using (1) and (2)]

$= a^n c^n + b^nc^n + b^n c^n + b^n d^n$

$= a^n c^n + b^nc^n + a^n d^n + b^n d^n$ [Using (3)]

$= c^n (a^n + b^n) + d^n (a^n + b^n)$

$= (a^n + b^n)(c^n + d^n)$

= R.H.S.

$\therefore (b^n + c^n)^2 = (a^n + b^n)(c^n + d^n)$

Thus, $(a^n + b^n)$, $(b^n + c^n)$, and $(c^n + d^n)$ are in G.P.

Questions 9.

If *a* and *b* are the roots of $x^2-3x+p=0$ and c, d are roots of $x^2-12x+q=0$, where *a*, *b*, *c*, *d*, form a G.P. Prove that (*q* + *p*): (*q* – *p*) = 17:15.

Answer

It is given that *a* and *b* are the roots of $x^2 - 3x + p = 0$

∴ $a + b = 3$ and $ab = p$ … (1)

Also, *c* and *d* are the roots of $x^2-12x+q=0$

∴$c + d = 12$ and $cd = q$ … (2)

It is given that *a*, *b*, *c*, *d* are in G.P.

Let $a = x$, $b = xr$, $c = xr^2$, $d = xr^3$

From (1) and (2), we obtain

$x + xr = 3$

$\Rightarrow x(1 + r) = 3$

$xr^2 + xr^3 = 12$

$\Rightarrow xr^2(1 + r) = 12$

On dividing, we obtain

$$\frac{xr^2(1+r)}{x(1+r)}=\frac{12}{3}$$

$$\Rightarrow r^2=4$$

$$\Rightarrow r=\pm 2$$

When $r=2, x=\frac{3}{1+2}=\frac{3}{3}=1$

When $r=-2, x=\frac{3}{1-2}=\frac{3}{-1}=-3$

Case I:

When $r = 2$ and $x = 1$,

$ab = x^2r = 2$

$cd = x^2r^5 = 32$

$$\therefore \frac{q+p}{q-p}=\frac{32+2}{32-2}=\frac{34}{30}=\frac{17}{15}$$

i.e., $(q+p):(q-p)=17:15$

Case II:

When $r = -2$, $x = -3$,

$ab = x^2 r = -18$

$cd = x^2 r^5 = -288$

$$\therefore \frac{q+p}{q-p} = \frac{-288-18}{-288+18} = \frac{-306}{-270} = \frac{17}{15}$$

$$\text{i.e., } (q+p):(q-p) = 17:15$$

Thus, in both the cases, we obtain $(q + p)$: $(q - p) = 17:15$

Question 10:

The ratio of the A.M and G.M. of two positive numbers a and b, is m: n. Show that

$$a:b = \left(m+\sqrt{m^2-n^2}\right):\left(m-\sqrt{m^2-n^2}\right).$$

Answer

Let the two numbers be a and b.

A.M $= \frac{a+b}{2}$ and G.M. $= \sqrt{ab}$

According to the given condition,

$$\frac{a+b}{2\sqrt{ab}} = \frac{m}{n}$$

$$\Rightarrow \frac{(a+b)^2}{4(ab)} = \frac{m^2}{n^2}$$

$$\Rightarrow (a+b)^2 = \frac{4ab\, m^2}{n^2}$$

$$\Rightarrow (a+b) = \frac{2\sqrt{ab}\, m}{n} \qquad \ldots(1)$$

Using this in the identity $(a - b)^2 = (a + b)^2 - 4ab$, we obtain

$$(a-b)^2 = \frac{4ab\,m^2}{n^2} - 4ab = \frac{4ab\left(m^2-n^2\right)}{n^2}$$

$$\Rightarrow (a-b) = \frac{2\sqrt{ab}\sqrt{m^2-n^2}}{n} \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2a = \frac{2\sqrt{ab}}{n}\left(m+\sqrt{m^2-n^2}\right)$$

$$\Rightarrow a = \frac{\sqrt{ab}}{n}\left(m+\sqrt{m^2-n^2}\right)$$

Substituting the value of *a* in (1), we obtain

$$b = \frac{2\sqrt{ab}}{n}m - \frac{\sqrt{ab}}{n}\left(m+\sqrt{m^2-n^2}\right)$$

$$= \frac{\sqrt{ab}}{n}m - \frac{\sqrt{ab}}{n}\sqrt{m^2-n^2}$$

$$= \frac{\sqrt{ab}}{n}\left(m-\sqrt{m^2-n^2}\right)$$

$$\therefore a:b = \frac{a}{b} = \frac{\frac{\sqrt{ab}}{n}\left(m+\sqrt{m^2-n^2}\right)}{\frac{\sqrt{ab}}{n}\left(m-\sqrt{m^2-n^2}\right)} = \frac{\left(m+\sqrt{m^2-n^2}\right)}{\left(m-\sqrt{m^2-n^2}\right)}$$

Thus, $a : b = \left(m+\sqrt{m^2-n^2}\right):\left(m-\sqrt{m^2-n^2}\right)$

Also, $\frac{1}{c}, \frac{1}{d}, \frac{1}{c}$ are in A.P.

$$\frac{1}{d}-\frac{1}{c}=\frac{1}{e}-\frac{1}{d}$$

$$\frac{2}{d}=\frac{1}{c}+\frac{1}{e} \qquad \ldots(3)$$

It has to be proved that *a*, *c*, *e* are in G.P. i.e., $c^2 = ae$

From (1), we obtain

$$2b=a+c$$

$$\Rightarrow b=\frac{a+c}{2}$$

From (2), we obtain

$$d=\frac{c^2}{b}$$

Substituting these values in (3), we obtain

$$\frac{2b}{c^2}=\frac{1}{c}+\frac{1}{e}$$

$$\Rightarrow \frac{2(a+c)}{2c^2}=\frac{1}{c}+\frac{1}{e}$$

$$\Rightarrow \frac{a+c}{c^2}=\frac{e+c}{ce}$$

$$\Rightarrow \frac{a+c}{c}=\frac{e+c}{e}$$

$$\Rightarrow (a+c)e=(e+c)c$$

$$\Rightarrow ae+ce=ec+c^2$$

$$\Rightarrow c^2=ae$$

Thus, *a*, *c*, and *e* are in G.P.

Question 11:

Find the sum of the following series up to *n* terms:

(i) 5 + 55 + 555 + … (ii) .6 +.66 +. 666 +…

Answer

(i) 5 + 55 + 555 + …

Let S_n = 5 + 55 + 555 + ….. to n terms

$$=\frac{5}{9}[9+99+999+\ldots\text{to n terms}]$$

$$=\frac{5}{9}\left[(10-1)+\left(10^2-1\right)+\left(10^3-1\right)+\ldots\text{to n terms}\right]$$

$$=\frac{5}{9}\left[\left(10+10^2+10^3+\ldots\text{n terms}\right)-(1+1+\ldots\text{ n terms})\right]$$

$$=\frac{5}{9}\left[\frac{10\left(10^n-1\right)}{10-1}-n\right]$$

$$=\frac{5}{9}\left[\frac{10\left(10^n-1\right)}{9}-n\right]$$

$$=\frac{50}{81}\left(10^n-1\right)-\frac{5n}{9}$$

(ii) .6 +.66 +. 666 +…

Let S_n = 06. + 0.66 + 0.666 + … to n terms

$$=6[0.1+0.11+0.111+\ldots\text{to n terms}]$$

$$=\frac{6}{9}[0.9+0.99+0.999+\ldots\text{to n terms}]$$

$$=\frac{6}{9}\left[\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{10^2}\right)+\left(1-\frac{1}{10^3}\right)+\ldots\text{to n terms}\right]$$

$$=\frac{2}{3}\left[(1+1+\ldots\text{n terms})-\frac{1}{10}\left(1+\frac{1}{10}+\frac{1}{10^2}+\ldots\text{n terms}\right)\right]$$

$$=\frac{2}{3}\left[n-\frac{1}{10}\left(\frac{1-\left(\frac{1}{10}\right)^n}{1-\frac{1}{10}}\right)\right]$$

$$=\frac{2}{3}n-\frac{2}{30}\times\frac{10}{9}\left(1-10^{-n}\right)$$

$$=\frac{2}{3}n-\frac{2}{27}\left(1-10^{-n}\right)$$

Question 12:

Find the 20^{th} term of the series $2 \times 4 + 4 \times 6 + 6 \times 8 + \ldots + n$ terms.

Answer

The given series is $2 \times 4 + 4 \times 6 + 6 \times 8 + \ldots n$ terms

$\therefore n^{th}$ term $= a_n = 2n \times (2n + 2) = 4n^2 + 4n$

$a_{20} = 4(20)^2 + 4(20) = 4(400) + 80 = 1600 + 80 = 1680$

Thus, the 20^{th} term of the series is 1680.

Question 13:

A farmer buys a used tractor for Rs 12000. He pays Rs 6000 cash and agrees to pay the balance in annual installments of Rs 500 plus 12% interest on the unpaid amount. How much will be the tractor cost him?

Answer

It is given that the farmer pays Rs 6000 in cash.

Therefore, unpaid amount = Rs 12000 – Rs 6000 = Rs 6000

According to the given condition, the interest paid annually is

12% of 6000, 12% of 5500, 12% of 5000, …, 12% of 500

Thus, total interest to be paid = 12% of 6000 + 12% of 5500 + 12% of 5000 + … + 12% of 500

= 12% of (6000 + 5500 + 5000 + … + 500)

= 12% of (500 + 1000 + 1500 + … + 6000)

Now, the series 500, 1000, 1500 … 6000 is an A.P. with both the first term and common difference equal to 500.

Let the number of terms of the A.P. be n.

$\therefore 6000 = 500 + (n - 1)\,500$

$\Rightarrow 1 + (n - 1) = 12$

$\Rightarrow n = 12$

∴Sum of the A.P $=\frac{12}{2}\left[2(500)+(12-1)(500)\right]=6[1000+5500]=6(6500)=39000$

Thus, total interest to be paid = 12% of (500 + 1000 + 1500 + … + 6000)

= 12% of 39000 = Rs 4680

Thus, cost of tractor = (Rs 12000 + Rs 4680) = Rs 16680

Question 14:

Shamshad Ali buys a scooter for Rs 22000. He pays Rs 4000 cash and agrees to pay the balance in annual installment of Rs 1000 plus 10% interest on the unpaid amount. How much will the scooter cost him?

Answer

It is given that Shamshad Ali buys a scooter for Rs 22000 and pays Rs 4000 in cash.

∴Unpaid amount = Rs 22000 – Rs 4000 = Rs 18000

According to the given condition, the interest paid annually is

10% of 18000, 10% of 17000, 10% of 16000 … 10% of 1000

Thus, total interest to be paid = 10% of 18000 + 10% of 17000 + 10% of 16000 + … + 10% of 1000

= 10% of (18000 + 17000 + 16000 + … + 1000)

= 10% of (1000 + 2000 + 3000 + … + 18000)

Here, 1000, 2000, 3000 … 18000 forms an A.P. with first term and common difference both equal to 1000.

Let the number of terms be n.

∴ 18000 = 1000 + (n – 1) (1000)

⇒ n = 18

$$\therefore 1000+2000+....+18000=\frac{18}{2}\left[2(1000)+(18-1)(1000)\right]$$
$$=9\left[2000+17000\right]$$
$$=171000$$

∴ Total interest paid = 10% of (18000 + 17000 + 16000 + … + 1000)

= 10% of Rs 171000 = Rs 17100

∴Cost of scooter = Rs 22000 + Rs 17100 = Rs 39100

Question 15:

A person writes a letter to four of his friends. He asks each one of them to copy the letter and mail to four different persons with instruction that they move the chain similarly. Assuming that the chain is not broken and that it costs 50 paise to mail one letter. Find the amount spent on the postage when 8^{th} set of letter is mailed.

Answer

The numbers of letters mailed forms a G.P.: 4, 4^2, … 4^8

First term = 4

Common ratio = 4

Number of terms = 8

It is known that the sum of *n* terms of a G.P. is given by

$$S_n=\frac{a\left(r^n-1\right)}{r-1}$$

$$\therefore S_8=\frac{4\left(4^8-1\right)}{4-1}=\frac{4(65536-1)}{3}=\frac{4(65535)}{3}=4(21845)=87380$$

It is given that the cost to mail one letter is 50 paisa.

∴Cost of mailing 87380 letters $=\text{Rs } 87380\times\frac{50}{100}$ = Rs 43690

Thus, the amount spent when 8^{th} set of letter is mailed is Rs 43690.

Question 16:

A man deposited Rs 10000 in a bank at the rate of 5% simple interest annually. Find the amount in 15^{th} year since he deposited the amount and also calculate the total amount after 20 years.

Answer

It is given that the man deposited Rs 10000 in a bank at the rate of 5% simple interest annually.

∴ Interest in first year $= \frac{5}{100} \times \text{Rs } 10000 = \text{Rs } 500$

∴Amount in 15th year = Rs $10000 + \underbrace{500 + 500 + \ldots + 500}_{14\text{times}}$

= Rs 10000 + 14 × Rs 500

= Rs 10000 + Rs 7000

= Rs 17000

Amount after 20 years = $\text{Rs } 10000 + \underbrace{500 + 500 + \ldots + 500}_{20\text{times}}$

= Rs 10000 + 20 × Rs 500

= Rs 10000 + Rs 10000

= Rs 20000

Question 17:

A manufacturer reckons that the value of a machine, which costs him Rs 15625, will depreciate each year by 20%. Find the estimated value at the end of 5 years.

Answer

Cost of machine = Rs 15625

Machine depreciates by 20% every year.

Therefore, its value after every year is 80% of the original cost i.e., $\frac{4}{5}$ of the original cost.

∴ Value at the end of 5 years = $15625 \times \underbrace{\frac{4}{5} \times \frac{4}{5} \times \ldots \times \frac{4}{5}}_{5\text{times}}$ = 5 × 1024 = 5120

Thus, the value of the machine at the end of 5 years is Rs 5120.

Question 18:

150 workers were engaged to finish a job in a certain number of days. 4 workers dropped out on second day, 4 more workers dropped out on third day and so on. It took

8 more days to finish the work. Find the number of days in which the work was completed.

Answer

Let x be the number of days in which 150 workers finish the work.

According to the given information,

$150x$ = 150 + 146 + 142 + …. (x + 8) terms

The series 150 + 146 + 142 + …. (x + 8) terms is an A.P. with first term 146, common difference –4 and number of terms as (x + 8)

$$\Rightarrow 150x = \frac{(x+8)}{2}\left[2(150)+(x+8-1)(-4)\right]$$
$$\Rightarrow 150x = (x+8)\left[150+(x+7)(-2)\right]$$
$$\Rightarrow 150x = (x+8)(150-2x-14)$$
$$\Rightarrow 150x = (x+8)(136-2x)$$
$$\Rightarrow 75x = (x+8)(68-x)$$
$$\Rightarrow 75x = 68x - x^2 + 544 - 8x$$
$$\Rightarrow x^2 + 75x - 60x - 544 = 0$$
$$\Rightarrow x^2 + 15x - 544 = 0$$
$$\Rightarrow x^2 + 32x - 17x - 544 = 0$$
$$\Rightarrow x(x+32) - 17(x+32) = 0$$
$$\Rightarrow (x-17)(x+32) = 0$$
$$\Rightarrow x = 17 \text{ or } x = -32$$

However, x cannot be negative.

$\therefore x = 17$

Therefore, originally, the number of days in which the work was completed is 17.

Thus, required number of days = (17 + 8) = 25