Class XI : Physics
Chapter 3 :Motion In A Plane

Questions and Solutions | Exercises - NCERT Books

Question 1:

State, for each of the following physical quantities, if it is a scalar or a vector:

volume, mass, speed, acceleration, density, number of moles, velocity, angular frequency, displacement, angular velocity.

Answer

Scalar: Volume, mass, speed, density, number of moles, angular frequency

Vector: Acceleration, velocity, displacement, angular velocity

A scalar quantity is specified by its magnitude only. It does not have any direction associated with it. Volume, mass, speed, density, number of moles, and angular frequency are some of the scalar physical quantities.

A vector quantity is specified by its magnitude as well as the direction associated with it. Acceleration, velocity, displacement, and angular velocity belong to this category.

Question 2:

Pick out the two scalar quantities in the following list:

force, angular momentum, work, current, linear momentum, electric field, average velocity, magnetic moment, relative velocity.

Answer

Work and **current** are scalar quantities.

Work done is given by the dot product of force and displacement. Since the dot product of two quantities is always a scalar, work is a scalar physical quantity.

Current is described only by its magnitude. Its direction is not taken into account. Hence, it is a scalar quantity.

Question 3:

Pick out the only vector quantity in the following list:

Temperature, pressure, impulse, time, power, total path length, energy, gravitational potential, coefficient of friction, charge.

Answer

Impulse

Impulse is given by the product of force and time. Since force is a vector quantity, its product with time (a scalar quantity) gives a vector quantity.

Question 4:

State with reasons, whether the following algebraic operations with scalar and vector physical quantities are meaningful:

adding any two scalars, (b) adding a scalar to a vector of the same dimensions, (c) multiplying any vector by any scalar, (d) multiplying any two scalars, (e) adding any two vectors, (f) adding a component of a vector to the same vector.

Answer

Answer:

Meaningful

Not Meaningful

Meaningful

Meaningful

Meaningful

Meaningful

Explanation:

(a) The addition of two scalar quantities is meaningful only if they both represent the same physical quantity.

(b) The addition of a vector quantity with a scalar quantity is not meaningful.

A scalar can be multiplied with a vector. For example, force is multiplied with time to give impulse.

A scalar, irrespective of the physical quantity it represents, can be multiplied with another scalar having the same or different dimensions.

The addition of two vector quantities is meaningful only if they both represent the same physical quantity.

A component of a vector can be added to the same vector as they both have the same dimensions.

Question 5:

Read each statement below carefully and state with reasons, if it is true or false:

The magnitude of a vector is always a scalar, (b) each component of a vector is always a scalar, (c) the total path length is always equal to the magnitude of the displacement vector of a particle. (d) the average speed of a particle (defined as total path length divided by the time taken to cover the path) is either greater or equal to the magnitude of average velocity of the particle over the same interval of time, (e) Three vectors not lying in a plane can never add up to give a null vector.

Answer

Answer:

True

False

False

True

True

Explanation:

The magnitude of a vector is a number. Hence, it is a scalar.

Each component of a vector is also a vector.

Total path length is a scalar quantity, whereas displacement is a vector quantity. Hence, the total path length is always greater than the magnitude of displacement. It becomes equal to the magnitude of displacement only when a particle is moving in a straight line.

It is because of the fact that the total path length is always greater than or equal to the magnitude of displacement of a particle.

Three vectors, which do not lie in a plane, cannot be represented by the sides of a triangle taken in the same order.

Question 6:

Establish the following vector inequalities geometrically or otherwise:

$|\mathbf{a} + \mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}|$

$|\mathbf{a} + \mathbf{b}| \geq ||\mathbf{a}| - |\mathbf{b}||$

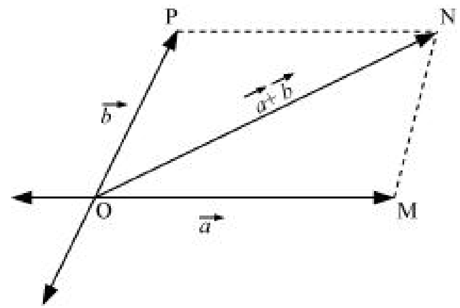
$|\mathbf{a} - \mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}|$

$|\mathbf{a} - \mathbf{b}| \geq ||\mathbf{a}| - |\mathbf{b}||$

When does the equality sign above apply?

Answer

Let two vectors $\vec{a}$ and $\vec{b}$ be represented by the adjacent sides of a parallelogram OMNP, as shown in the given figure.



Here, we can write:

$$\left|\overrightarrow{OM}\right| = \left|\vec{a}\right| \qquad \ldots (i)$$

$$\left|\overrightarrow{MN}\right| = \left|\overrightarrow{OP}\right| = \left|\vec{b}\right| \qquad \ldots (ii)$$

$$\left|\overrightarrow{ON}\right| = \left|\vec{a} + \vec{b}\right| \qquad \ldots (iii)$$

In a triangle, each side is smaller than the sum of the other two sides.

Therefore, in ΔOMN, we have:

ON < (OM + MN)

$$\left|\vec{a} + \vec{b}\right| < \left|\vec{a}\right| + \left|\vec{b}\right| \qquad \ldots (iv)$$

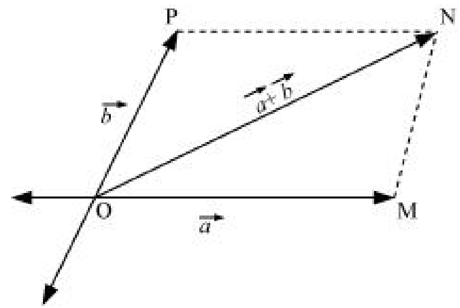
If the two vectors $\vec{a}$ and $\vec{b}$ act along a straight line in the same direction, then we can write:

$$\left|\vec{a} + \vec{b}\right| = \left|\vec{a}\right| + \left|\vec{b}\right| \qquad \ldots (v)$$

Combining equations (*iv*) and (*v*), we get:

$$\left|\vec{a} + \vec{b}\right| \le \left|\vec{a}\right| + \left|\vec{b}\right|$$

Let two vectors $\vec{a}$ and $\vec{b}$ be represented by the adjacent sides of a parallelogram OMNP, as shown in the given figure.



Here, we have:

$$\left|\overrightarrow{OM}\right| = \left|\vec{a}\right| \quad \text{... }(i)$$

$$\left|\overrightarrow{MN}\right| = \left|\overrightarrow{OP}\right| = \left|\vec{b}\right| \quad \text{... }(ii)$$

$$\left|\overrightarrow{ON}\right| = \left|\vec{a} + \vec{b}\right| \quad \text{... }(iii)$$

In a triangle, each side is smaller than the sum of the other two sides.

Therefore, in ΔOMN, we have:

$$ON + MN > OM$$

$$ON + OM > MN$$

$$\left|\overrightarrow{ON}\right| > \left|\overrightarrow{OM} - \overrightarrow{OP}\right| \quad (\because OP = MN)$$

$$\left|\vec{a}+\vec{b}\right| > \left|\left|\vec{a}\right| - \left|\vec{b}\right|\right| \quad \ldots (iv)$$

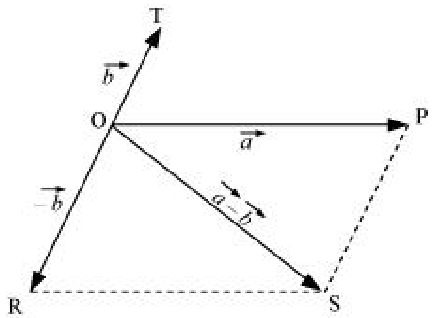
If the two vectors $\vec{a}$ and $\vec{b}$ act along a straight line in the same direction, then we can write:

$$\left|\vec{a}+\vec{b}\right| = \left|\left|\vec{a}\right| - \left|\vec{b}\right|\right| \quad \ldots (v)$$

Combining equations (*iv*) and (*v*), we get:

$$\left|\vec{a}+\vec{b}\right| \geq \left|\left|\vec{a}\right| - \left|\vec{b}\right|\right|$$

Let two vectors $\vec{a}$ and $\vec{b}$ be represented by the adjacent sides of a parallelogram PORS, as shown in the given figure.



Here we have:

$$\left|\overrightarrow{OR}\right| = \left|\overrightarrow{PS}\right| = \left|\vec{b}\right| \quad \ldots (i)$$

$$\left|\overrightarrow{OP}\right| = \left|\vec{a}\right| \quad \ldots (ii)$$

In a triangle, each side is smaller than the sum of the other two sides. Therefore, in ΔOPS, we have:

$$OS < OP + PS$$

$$\left|\vec{a}-\vec{b}\right| < \left|\vec{a}\right| + \left|-\vec{b}\right|$$

$$\left|\vec{a}-\vec{b}\right| < \left|\vec{a}\right| + \left|\vec{b}\right| \quad \ldots (iii)$$

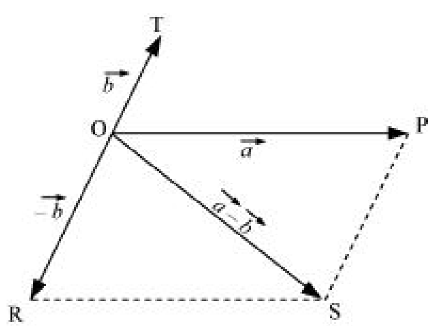
If the two vectors act in a straight line but in opposite directions, then we can write:

$$\left|\vec{a}-\vec{b}\right| = \left|\vec{a}\right| + \left|\vec{b}\right| \quad \ldots (iv)$$

Combining equations (*iii*) and (*iv*), we get:

$$\left|\vec{a}-\vec{b}\right| \le \left|\vec{a}\right| + \left|\vec{b}\right|$$

Let two vectors $\vec{a}$ and $\vec{b}$ be represented by the adjacent sides of a parallelogram PORS, as shown in the given figure.



The following relations can be written for the given parallelogram.

$$OS + PS > OP \quad \text{... }(i)$$

$$OS > OP - PS \quad \text{... }(ii)$$

$$\left|\vec{a} - \vec{b}\right| > \left|\vec{a}\right| - \left|\vec{b}\right| \quad \text{... }(iii)$$

The quantity on the LHS is always positive and that on the RHS can be positive or negative. To make both quantities positive, we take modulus on both sides as:

$$\left|\left|\vec{a} - \vec{b}\right|\right| > \left|\left|\vec{a}\right| - \left|\vec{b}\right|\right|$$

$$\left|\vec{a} - \vec{b}\right| > \left|\left|\vec{a}\right| - \left|\vec{b}\right|\right| \quad \text{... }(iv)$$

If the two vectors act in a straight line but in the opposite directions, then we can write:

$$\left|\vec{a} - \vec{b}\right| = \left|\left|\vec{a}\right| - \left|\vec{b}\right|\right| \quad \text{... }(v)$$

Combining equations (*iv*) and (*v*), we get:

$$\left|\vec{a} - \vec{b}\right| \geq \left|\left|\vec{a}\right| - \left|\vec{b}\right|\right|$$

Question 7:

Given **a** + **b** + **c** + **d** = **0**, which of the following statements are correct:

a, b, c, and **d** must each be a null vector,

The magnitude of (**a** + **c**) equals the magnitude of (**b**+ **d**),

The magnitude of *a* can never be greater than the sum of the magnitudes of **b***,* **c**, and **d***,*

b + **c** must lie in the plane of **a** and **d** if **a** and **d** are not collinear, and in the line of **a** and **d**, if they are collinear?

Answer

Answer: (a) Incorrect

In order to make **a** + **b** + **c** + **d** = **0**, it is not necessary to have all the four given vectors to be null vectors. There are many other combinations which can give the sum zero.

Answer: (b) Correct

a + b + c + d = 0

a + c = – (b + d)

Taking modulus on both the sides, we get:

| a + c | = | –(b + d)| = | b + d |

Hence, the magnitude of (**a** + **c**) is the same as the magnitude of (**b** + **d**).

Answer: (c) Correct

a + b + c + d = 0

a = (b + c + d)

Taking modulus both sides, we get:

| a | = | b + c + d |

$|\mathbf{a}| \leq |\mathbf{a}| + |\mathbf{b}| + |\mathbf{c}|$ … (*i*)

Equation (*i*) shows that the magnitude of **a** is equal to or less than the sum of the magnitudes of **b**, **c**, and **d**.

Hence, the magnitude of vector *a* can never be greater than the sum of the magnitudes of **b**, **c**, and **d**.

Answer: (d) Correct

For **a** + **b** + **c** + **d** = 0

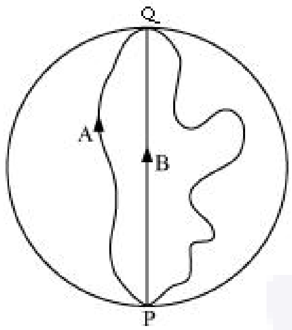
a + (**b** + **c**) + **d** = 0

The resultant sum of the three vectors **a**, (**b** + **c**), and **d** can be zero only if (**b** + **c**) lie in a plane containing **a** and **d**, assuming that these three vectors are represented by the three sides of a triangle.

If **a** and **d** are collinear, then it implies that the vector (**b** + **c**) is in the line of **a** and **d**. This implication holds only then the vector sum of all the vectors will be zero.

Question 8:

Three girls skating on a circular ice ground of radius 200 m start from a point P on the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in Fig. 4.20. What is the magnitude of the displacement vector for each? For which girl is this equal to the actual length of the path skated?



Answer

Displacement is given by the minimum distance between the initial and final positions of a particle. In the given case, all the girls start from point P and reach point Q. The magnitudes of their displacements will be equal to the diameter of the ground.

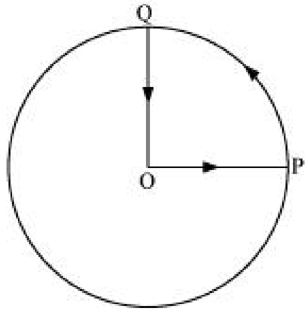
Radius of the ground = 200 m

Diameter of the ground = 2 × 200 = 400 m

Hence, the magnitude of the displacement for each girl is 400 m. This is equal to the actual length of the path skated by girl B.

Question 9:

A cyclist starts from the centre O of a circular park of radius 1 km, reaches the edge P of the park, then cycles along the circumference, and returns to the centre along QO as shown in Fig. 4.21. If the round trip takes 10 min, what is the (a) net displacement, (b) average velocity, and (c) average speed of the cyclist?



Answer

Displacement is given by the minimum distance between the initial and final positions of a body. In the given case, the cyclist comes to the starting point after cycling for 10 minutes. Hence, his net displacement is zero.

Average velocity is given by the relation:

Average velocity $= \dfrac{\text{Net displacement}}{\text{Total time}}$

Since the net displacement of the cyclist is zero, his average velocity will also be zero.

Average speed of the cyclist is given by the relation:

Average speed $= \dfrac{\text{Total path length}}{\text{Total time}}$

Total path length = OP + PQ + QO

$$= 1 + \frac{1}{4}(2\pi \times 1) + 1$$

$$= 2 + \frac{1}{2}\pi = 3.570 \text{ km}$$

Time taken = 10 min $= \dfrac{10}{60} = \dfrac{1}{6}$ h

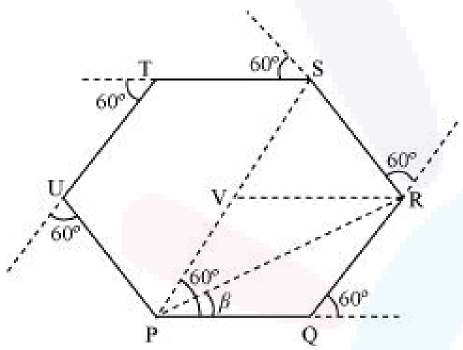
∴Average speed $= \dfrac{3.570}{\frac{1}{6}} = 21.42$ km/h

Question 10:

On an open ground, a motorist follows a track that turns to his left by an angle of 60° after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of the displacement with the total path length covered by the motorist in each case.

Answer

The path followed by the motorist is a regular hexagon with side 500 m, as shown in the given figure



Let the motorist start from point P.

The motorist takes the third turn at S.

∴Magnitude of displacement = PS = PV + VS = 500 + 500 = 1000 m

Total path length = PQ + QR + RS = 500 + 500 +500 = 1500 m

The motorist takes the sixth turn at point P, which is the starting point.

∴Magnitude of displacement = 0

Total path length = PQ + QR + RS + ST + TU + UP

= 500 + 500 + 500 + 500 + 500 + 500 = 3000 m

The motorist takes the eight turn at point R

∴Magnitude of displacement = PR

$$= \sqrt{PQ^2 + QR^2 + 2(PQ)\cdot(QR)\cos 60^\circ}$$

$$= \sqrt{500^2 + 500^2 + (2 \times 500 \times 500 \times \cos 60^\circ)}$$

$$= \sqrt{250000 + 250000 + \left(500000 \times \frac{1}{2}\right)}$$

$$= 866.03 \text{ m}$$

$$\beta = \tan^{-1}\left(\frac{500\sin 60^\circ}{500 + 500\cos 60^\circ}\right) = 30^\circ$$

Therefore, the magnitude of displacement is 866.03 m at an angle of 30° with PR.

Total path length = Circumference of the hexagon + PQ + QR

= 6 × 500 + 500 + 500 = 4000 m

The magnitude of displacement and the total path length corresponding to the required turns is shown in the given table

Turn	**Magnitude of displacement (m)**	**Total path length (m)**
Third	1000	1500
Sixth	0	3000
Eighth	866.03; 30°	4000

Question 11:

A passenger arriving in a new town wishes to go from the station to a hotel located 10 km away on a straight road from the station. A dishonest cabman takes him along a circuitous path 23 km long and reaches the hotel in 28 min. What is (a) the average speed of the taxi, (b) the magnitude of average velocity? Are the two equal?

Answer

Total distance travelled = 23 km

Total time taken = 28 min $= \frac{28}{60}$ h

∴Average speed of the taxi $= \frac{\text{Total distance travelled}}{\text{Total time taken}}$

$$= \frac{23}{\left(\frac{28}{60}\right)} = 49.29 \text{ km/h}$$

Distance between the hotel and the station = 10 km = Displacement of the car

∴Average velocity $= \frac{10}{\frac{28}{60}} = 21.43 \text{ km/h}$

Therefore, the two physical quantities (averge speed and average velocity) are not equal.

Question 12:

The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 m s^{-1} can go without hitting the ceiling of the hall?

Answer

Speed of the ball, $u = 40$ m/s

Maximum height, $h = 25$ m

In projectile motion, the maximum height reached by a body projected at an angle θ, is given by the relation:

$$h = \frac{u^2 \sin^2 \theta}{2g}$$

$$25 = \frac{(40)^2 \sin^2 \theta}{2 \times 9.8}$$

$\sin^2 \theta = 0.30625$

$\sin \theta = 0.5534$

$\therefore \theta = \sin^{-1}(0.5534) = 33.60°$

Horizontal range, $R = \frac{u^2 \sin 2\theta}{g}$

$$= \frac{(40)^2 \times \sin 2 \times 33.60}{9.8}$$

$$= \frac{1600 \times \sin 67.2}{9.8}$$

$$= \frac{1600 \times 0.922}{9.8} = 150.53 \text{ m}$$

Question 13:

A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball?

Answer

Maximum horizontal distance, $R = 100$ m

The cricketer will only be able to throw the ball to the maximum horizontal distance when the angle of projection is 45°, i.e., $\theta = 45°$.

The horizontal range for a projection velocity v, is given by the relation:

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$100 = \frac{u^2}{g} \sin 90°$$

$$\frac{u^2}{g} = 100 \qquad \ldots (i)$$

The ball will achieve the maximum height when it is thrown vertically upward. For such motion, the final velocity v is zero at the maximum height H.

Acceleration, $a = -g$

Using the third equation of motion:

$$v^2 - u^2 = -2gH$$

$$H = \frac{1}{2} \times \frac{u^2}{g} = \frac{1}{2} \times 100 = 50 \text{ m}$$

Question 14:

A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude and direction of acceleration of the stone?

Answer

Length of the string, l = 80 cm = 0.8 m

Number of revolutions = 14

Time taken = 25 s

Frequency, $\nu = \frac{\text{Number of revolutions}}{\text{Time taken}} = \frac{14}{25}$ Hz

Angular frequency, $\omega = 2\pi\nu$

$$= 2 \times \frac{22}{7} \times \frac{14}{25} = \frac{88}{25} \text{ rad s}^{-1}$$

Centripetal acceleration, $a_c = \omega^2 r$

$$= \left(\frac{88}{25}\right)^2 \times 0.8$$

$$= 9.91 \text{ m/s}^2$$

The direction of centripetal acceleration is always directed along the string, toward the centre, at all points.

Question 15:

An aircraft executes a horizontal loop of radius 1.00 km with a steady speed of 900 km/h. Compare its centripetal acceleration with the acceleration due to gravity.

Answer

Radius of the loop, $r = 1$ km $= 1000$ m

Speed of the aircraft, $v = 900$ km/h $= 900 \times \frac{5}{18} = 250 \text{ m/s}$

Centripetal acceleration, $a_c = \frac{v^2}{r}$

$$= \frac{(250)^2}{1000} = 62.5 \text{ m/s}^2$$

Acceleration due to gravity, g = 9.8 m/s^2

$$\frac{a_c}{g} = \frac{62.5}{9.8} = 6.38$$

$$a_c = 6.38\,g$$

Question 16::

Read each statement below carefully and state, with reasons, if it is true or false:

The net acceleration of a particle in circular motion is *always* along the radius of the circle towards the centre

The velocity vector of a particle at a point is *always* along the tangent to the path of the particle at that point

The acceleration vector of a particle in *uniform* circular motion averaged over one cycle is a null vector

Answer

False

The net acceleration of a particle in circular motion is not always directed along the radius of the circle toward the centre. It happens only in the case of uniform circular motion.

True

At a point on a circular path, a particle appears to move tangentially to the circular path. Hence, the velocity vector of the particle is always along the tangent at a point.

True

In uniform circular motion (UCM), the direction of the acceleration vector points toward the centre of the circle. However, it constantly changes with time. The average of these vectors over one cycle is a null vector.

Question 17:

The position of a particle is given by

$\mathbf{r} = 3.0t\ \hat{\mathbf{i}} - 2.0t^2\ \hat{\mathbf{j}} + 4.0\ \hat{\mathbf{k}}$ m

Where t is in seconds and the coefficients have the proper units for **r** to be in metres.

Find the **v** and **a** of the particle?

What is the magnitude and direction of velocity of the particle at $t = 2.0$ s?

Answer

$\vec{v}(t) = (3.0\ \hat{\mathbf{i}} - 4.0t\ \hat{\mathbf{j}});\ \vec{a} = -4.0\ \hat{\mathbf{j}}$

The position of the particle is given by:

$\vec{r} = 3.0t\ \hat{\mathbf{i}} - 2.0t^2\ \hat{\mathbf{j}} + 4.0\ \hat{\mathbf{k}}$

Velocity $\vec{v}$, of the particle is given as:

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left(3.0t\ \hat{\mathbf{i}} - 2.0t^2\ \hat{\mathbf{j}} + 4.0\ \hat{\mathbf{k}}\right)$$
$$\therefore \vec{v} = 3.0\ \hat{\mathbf{i}} - 4.0t\ \hat{\mathbf{j}}$$

Acceleration $\vec{a}$, of the particle is given as:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(3.0\ \hat{\mathbf{i}} - 4.0t\ \hat{\mathbf{j}}\right)$$
$$\therefore \vec{a} = -4.0\ \hat{\mathbf{j}}$$

8.54 m/s, 69.45° below the x-axis

We have velocity vector, $\vec{v} = 3.0\ \hat{\mathbf{i}} - 4.0t\ \hat{\mathbf{j}}$

At $t = 2.0$ s:

$\vec{v} = 3.0\ \hat{\mathbf{i}} - 8.0\ \hat{\mathbf{j}}$

The magnitude of velocity is given by:

$$|\vec{v}| = \sqrt{3^2 + (-8)^2} = \sqrt{73} = 8.54 \text{ m/s}$$

$$\text{Direction, } \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$
$$= \tan^{-1}\left(\frac{-8}{3}\right) = -\tan^{-1}(2.667)$$
$$= -69.45°$$

The negative sign indicates that the direction of velocity is below the *x*-axis.

Question 18:

A particle starts from the origin at $t = 0$ s with a velocity of $10.0\ \hat{\mathbf{j}}$ m/s and moves in the *x-y* plane with a constant acceleration of $(8.0\ \hat{\mathbf{i}} + 2.0\ \hat{\mathbf{j}})\ \text{m s}^{-2}$.

At what time is the *x*-coordinate of the particle 16 m? What is the *y*-coordinate of the particle at that time?

What is the speed of the particle at the time?

Answer

Velocity of the particle, $\vec{v} = 10.0\ \hat{\mathbf{j}}$ m/s

Acceleration of the particle $\vec{a} = (8.0\ \hat{\mathbf{i}} + 2.0\ \hat{\mathbf{j}})$

Also,

But, $\vec{a} = \frac{d\vec{v}}{dt} = 8.0\ \hat{\mathbf{i}} + 2.0\ \hat{\mathbf{j}}$

$$d\vec{v} = (8.0\ \hat{\mathbf{i}} + 2.0\hat{\mathbf{j}})dt$$

Integrating both sides:

$$\vec{v}(t) = 8.0t\ \hat{\mathbf{i}} + 2.0t\ \hat{\mathbf{j}} + \vec{u}$$

Where,

$\vec{u}$ = Velocity vector of the particle at $t = 0$

$\vec{v}$ = Velocity vector of the particle at time t

But, $\vec{v} = \dfrac{d\vec{r}}{dt}$

$$d\vec{r} = \vec{v}dt = \left(8.0t\ \hat{\mathbf{i}} + 2.0t\ \hat{\mathbf{j}} + \vec{u}\right)dt$$

Integrating the equations with the conditions: at $t = 0$; $r = 0$ and at $t = t$; $r = r$

$$\vec{r} = \vec{u}t + \frac{1}{2}8.0t^2\ \hat{\mathbf{i}} + \frac{1}{2} \times 2.0t^2\ \hat{\mathbf{j}}$$

$$= \vec{u}t + 4.0t^2\ \hat{\mathbf{i}} + t^2\ \hat{\mathbf{j}}$$

$$= \left(10.0\hat{\mathbf{j}}\right)t + 4.0t^2\ \hat{\mathbf{i}} + t^2\ \hat{\mathbf{j}}$$

$$x\ \hat{\mathbf{i}} + y\ \hat{\mathbf{j}} = 4.0t^2\ \hat{\mathbf{i}} + \left(10t + t^2\right)\ \hat{\mathbf{j}}$$

Since the motion of the particle is confined to the x-y plane, on equating the coefficients of $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$, we get:

$$x = 4t^2$$

$$t = \left(\frac{x}{4}\right)^{\frac{1}{2}}$$

And $y = 10t + t^2$

When $x = 16$ m:

$$t = \left(\frac{16}{4}\right)^{\frac{1}{2}} = 2\ \text{s}$$

$\therefore y = 10 \times 2 + (2)^2 = 24$ m

Velocity of the particle is given by:

$\vec{v}(t) = 8.0t\ \hat{\mathbf{i}} + 2.0t\ \hat{\mathbf{j}} + \vec{u}$

at $t = 2$ s

$\vec{v}(t) = 8.0 \times 2\ \hat{\mathbf{i}} + 2.0 \times 2\ \hat{\mathbf{j}} + 10\ \hat{\mathbf{j}}$

$= 16\ \hat{\mathbf{i}} + 14\ \hat{\mathbf{j}}$

$\therefore$ Speed of the particle:

$|\vec{v}| = \sqrt{(16)^2 + (14)^2}$

$= \sqrt{256 + 196} = \sqrt{452}$

$= 21.26$ m/s

Question 19:

$\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are unit vectors along x- and y-axis respectively. What is the magnitude and direction of the vectors $\hat{\mathbf{i}} + \hat{\mathbf{j}}$, and $\hat{\mathbf{i}} - \hat{\mathbf{j}}$? What are the components of a vector $\mathbf{A} = 2\ \hat{\mathbf{i}} + 3\ \hat{\mathbf{j}}$ along the directions of $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\hat{\mathbf{i}} - \hat{\mathbf{j}}$? [You may use graphical method]

Answer

Consider a vector $\vec{P}$, given as:

$\vec{P} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$

$P_x\ \hat{\mathbf{i}} + P_y\ \hat{\mathbf{j}} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$

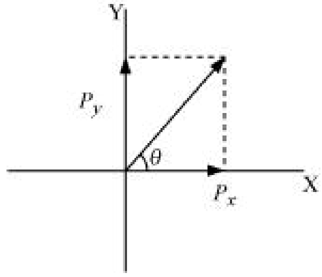
On comparing the components on both sides, we get:

$P_x = P_y = 1$

$|\vec{P}| = \sqrt{P_x^2 + P_y^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$ ... (i)

Hence, the magnitude of the vector $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ is $\sqrt{2}$.

Let θ be the angle made by the vector $\vec{P}$, with the x-axis, as shown in the following figure.



$\therefore \tan\theta = \left(\frac{P_y}{P_x}\right)$

$\theta = \tan^{-1}\left(\frac{1}{1}\right) = 45°$ ... (*ii*)

Hence, the vector $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ makes an angle of 45° with the *x*-axis.

Let $\vec{Q} = \hat{\mathbf{i}} - \hat{\mathbf{j}}$

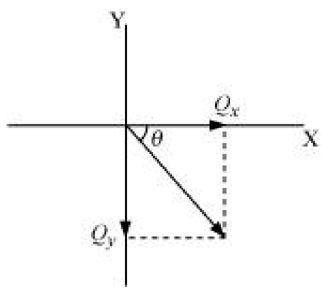
$Q_x\ \hat{\mathbf{i}} - Q_y\ \hat{\mathbf{j}} = \hat{\mathbf{i}} - \hat{\mathbf{j}}$

$Q_x = Q_y = 1$

$\left|\vec{Q}\right| = \sqrt{Q_x^2 + Q_y^2} = \sqrt{2}$ ... (*iii*)

Hence, the magnitude of the vector $\hat{\mathbf{i}} - \hat{\mathbf{j}}$ is $\sqrt{2}$.

Let θ be the angle made by the vector $\vec{Q}$, with the *x*- axis, as shown in the following figure.



$\therefore \tan\theta = \left(\frac{Q_y}{Q_x}\right)$

$\theta = -\tan^{-1}\left(-\frac{1}{1}\right) = -45°$ ... (*iv*)

Hence, the vector $\hat{\mathbf{i}} - \hat{\mathbf{j}}$ makes an angle of $-45°$ with the *x*-axis.

It is given that:

$\vec{A} = 2\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}}$

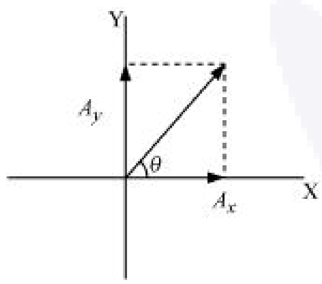
$A_x\,\hat{\mathbf{i}} + A_y\,\hat{\mathbf{j}} = 2\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}}$

On comparing the coefficients of $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$, we have:

$A_x = 2$ and $A_y = 3$

$|\vec{A}| = \sqrt{2^2 + 3^2} = \sqrt{13}$

Let $\vec{A}_x$ make an angle θ with the *x*-axis, as shown in the following figure.



$\therefore \tan\theta = \left(\frac{A_y}{A_x}\right)$

$\theta = \tan^{-1}\left(\frac{3}{2}\right)$

$= \tan^{-1}(1.5) = 56.31°$

Angle between the vectors $\left(2\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}}\right)$ and $\left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right)$, $\theta' = 56.31 - 45 = 11.31°$

Component of vector $\vec{A}$, along the direction of $\vec{P}$, making an angle θ'

$$= \left(A\cos\theta'\right)\hat{P} = \left(A\cos 11.31\right)\frac{\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}\right)}{\sqrt{2}}$$

$$= \sqrt{13} \times \frac{0.9806}{\sqrt{2}}\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}\right)$$

$$= 2.5\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}\right)$$

$$= \frac{25}{10} \times \sqrt{2}$$

$$= \frac{5}{\sqrt{2}} \qquad \ldots (v)$$

Let θ'' be the angle between the vectors $\left(2\,\hat{\mathbf{i}}+3\,\hat{\mathbf{j}}\right)$ and $\left(\hat{\mathbf{i}}-\hat{\mathbf{j}}\right)$.

$$\theta'' = 45 + 56.31 = 101.31°$$

Component of vector $\vec{A}$, along the direction of $\vec{Q}$, making an angle θ''

$$= \left(A\cos\theta''\right)\vec{Q} = \left(A\cos\theta''\right)\frac{\hat{\mathbf{i}}-\hat{\mathbf{j}}}{\sqrt{2}}$$

$$= \sqrt{13}\cos\left(901.31°\right)\frac{\left(\hat{\mathbf{i}}-\hat{\mathbf{j}}\right)}{\sqrt{2}}$$

$$= -\sqrt{\frac{13}{2}}\sin 11.30°\left(\hat{\mathbf{i}}-\hat{\mathbf{j}}\right)$$

$$= -2.550 \times 0.1961\left(\hat{\mathbf{i}}-\hat{\mathbf{j}}\right)$$

$$= -0.5\left(\hat{\mathbf{i}}-\hat{\mathbf{j}}\right)$$

$$= -\frac{5}{10} \times \sqrt{2}$$

$$= -\frac{1}{\sqrt{2}} \qquad \ldots (vi)$$

Question 20:

For any arbitrary motion in space, which of the following relations are true:

$$\mathbf{v}_{\text{average}} = \left(\frac{1}{2}\right)\left(\mathbf{v}(t_1) + \mathbf{v}(t_2)\right)$$

$$\mathbf{v}_{\text{average}} = \frac{[\mathbf{r}(t_2) - \mathbf{r}(t_1)]}{(t_2 - t_1)}$$

$$\mathbf{v}(t) = \mathbf{v}(0) + \mathbf{a}t$$

$$\mathbf{r}(t) = \mathbf{r}(0) + \mathbf{v}(0)t + \left(\frac{1}{2}\right)\mathbf{a}t^2$$

$$\mathbf{a}_{\text{average}} = \frac{[\mathbf{v}(t_2) - \mathbf{v}(t_1)]}{(t_2 - t_1)}$$

(The 'average' stands for average of the quantity over the time interval t_1 to t_2)

Answer

Answer: (b) and **(e)**

(a) It is given that the motion of the particle is arbitrary. Therefore, the average velocity of the particle cannot be given by this equation.

(b) The arbitrary motion of the particle can be represented by this equation.

(c) The motion of the particle is arbitrary. The acceleration of the particle may also be non-uniform. Hence, this equation cannot represent the motion of the particle in space.

(d) The motion of the particle is arbitrary; acceleration of the particle may also be non-uniform. Hence, this equation cannot represent the motion of particle in space.

(e) The arbitrary motion of the particle can be represented by this equation.

Question 21:

Read each statement below carefully and state, with reasons and examples, if it is true or false:

A scalar quantity is one that

is conserved in a process

can never take negative values

must be dimensionless

does not vary from one point to another in space

has the same value for observers with different orientations of axes

Answer

False

Despite being a scalar quantity, energy is not conserved in inelastic collisions.

False

Despite being a scalar quantity, temperature can take negative values.

False

Total path length is a scalar quantity. Yet it has the dimension of length.

False

A scalar quantity such as gravitational potential can vary from one point to another in space.

True

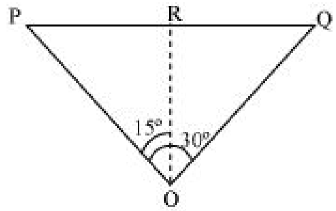
The value of a scalar does not vary for observers with different orientations of axes.

Question 22:

An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by the aircraft positions 10.0 s apart is 30°, what is the speed of the aircraft?

Answer

The positions of the observer and the aircraft are shown in the given figure.



Height of the aircraft from ground, OR = 3400 m

Angle subtended between the positions, $\angle POQ = 30°$

Time = 10 s

In ΔPRO:

$$\tan 15° = \frac{PR}{OR}$$

$$\begin{aligned} PR &= OR \tan 15° \\ &= 3400 \times \tan 15° \end{aligned}$$

ΔPRO is similar to ΔRQO.

$\therefore PR = RQ$

$PQ = PR + RQ$

$= 2PR = 2 \times 3400 \tan 15°$

$= 6800 \times 0.268 = 1822.4$ m

$\therefore$Speed of the aircraft $= \frac{1822.4}{10} = 182.24$ m/s