

CLASS IX: MATHS

Chapter 7: Triangles

Questions and Solutions | Exercise 7.2 - NCERT Books

Q1. In an isosceles triangle ABC, with $AB = AC$, the bisectors of $\angle B$ and $\angle C$ intersect each other at O. Join A to O. Show that : (i) $OB = OC$ (ii) AO bisects $\angle A$.

Sol. (i) In $\triangle ABC$, OB and OC are bisectors of $\angle B$ and $\angle C$.

$$\therefore \angle OBC = \frac{1}{2} \angle B \quad \dots(1)$$

$$\angle OCB = \frac{1}{2} \angle C \quad \dots(2)$$

Also, $AB = AC$ (Given)

$$\Rightarrow \angle B = \angle C \quad \dots(3)$$

From (1), (2), (3), we have

$$\angle OBC = \angle OCB$$

Now, in $\triangle OBC$, we have

$$\angle OBC = \angle OCB$$

$$\Rightarrow OB = OC$$

(Sides opposite to equal angles are equal)

$$(ii) \angle OBA = \frac{1}{2} \angle B \text{ and } \angle OCA = \frac{1}{2} \angle C$$

$$\Rightarrow \angle OBA = \angle OCA \quad (\because \angle B = \angle C)$$

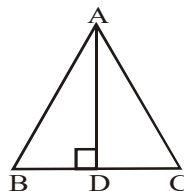
$AB = AC$ and $OB = OC$

$\therefore \triangle OAB \cong \triangle OAC$ (SAS congruence criteria)

$$\Rightarrow \angle OAB = \angle OAC$$

\Rightarrow AO bisects $\angle A$.

Q2. In $\triangle ABC$, AD is the perpendicular bisector of BC. Show that $\triangle ABC$ is an isosceles triangle in which $AB = AC$.



Sol. Given : In $\triangle ABC$, AD is perpendicular bisector of BC.

To Prove : $\triangle ABC$ is isosceles \triangle with $AB = AC$

Proof : In $\triangle ADB$ and $\triangle ADC$

$$\angle ADB = \angle ADC \quad (\text{Each } 90^\circ)$$

$$DB = DC \quad (\text{AD is } \perp \text{ bisector of BC})$$

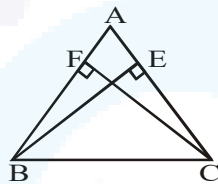
$$AD = AD \quad (\text{Common})$$

$$\triangle ADB \cong \triangle ADC \quad (\text{By SAS rule})$$

$$AB = AC \quad (\text{By CPCT})$$

$\therefore \triangle ABC$ is an isosceles \triangle with $AB = AC$

Q3. ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively. Show that these altitudes are equal.



Sol. In $\triangle ABE$ and $\triangle ACF$, we have

$$\angle BEA = \angle CFA \quad (\text{Each} = 90^\circ)$$

$$\angle A = \angle A \quad (\text{Common angle})$$

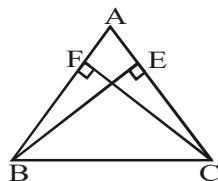
$$AB = AC \quad (\text{Given})$$

$$\therefore \triangle ABE \cong \triangle ACF \quad (\text{By AAS congruence criteria})$$

$$\Rightarrow BE = CF \quad (\text{By CPCT})$$

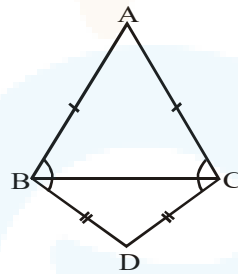
Q4. ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (See figure). Show that (i) $\triangle ABE \cong \triangle ACF$

(ii) $AB = AC$, i.e., ABC is an isosceles triangle.



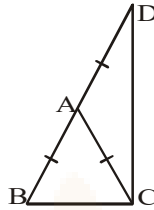
Sol. (i) In $\triangle ABE$ and $\triangle ACF$, we have
 $\angle A = \angle A$ (Common)
 $\angle AEB = \angle AFC$ (Each = 90°)
 $BE = CF$ (Given)
 $\therefore \triangle ABE \cong \triangle ACF$ (By ASA congruence)
 (ii) $\triangle ABE \cong \triangle ACF$
 $\Rightarrow AB = AC$ (By CPCT)

Q5. ABC and DBC are two isosceles triangles on the same base BC (see figure). Show that $\angle ABD = \angle ACD$.



Sol. Given : ABC and BCD are two isosceles triangle on common base BC .
 To prove : $\angle ABC = \angle ACD$
 Proof : ABC is an isosceles Triangle on base BC
 $\therefore \angle ABC = \angle ACB$... (1)
 $\because DBC$ is an isosceles Δ on base BC .
 $\angle DBC = \angle DCB$... (2)
 Adding (1) and (2)
 $\angle ABC + \angle DBC = \angle ACB + \angle DCB$
 $\Rightarrow \angle ABD = \angle ACD$

Q6. $\triangle ABC$ is an isosceles triangle in which $AB = AC$. Side BA is produced to D such that $AD = AB$ (see figure). Show that $\angle BCD$ is a right angle.



Sol. In $\triangle ABC$, $AB = AC$
 $\Rightarrow \angle ACB = \angle ABC$... (1)

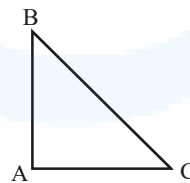
In $\triangle ACD$,
 $AD = AB$ (By construction)
 $\Rightarrow AD = AC$
 $\Rightarrow \angle ACD = \angle ADC$... (2)

Adding (1) and (2),
 $\angle ACB + \angle ACD = \angle ABC + \angle ADC$
 $\Rightarrow \angle BCD = \angle ABC + \angle ADC$

In $\triangle BCD$, $\angle DBC + \angle ABC + \angle BCD + \angle CDB = 180^\circ$
 $\Rightarrow 2 \angle BCD = 180^\circ$
 $\Rightarrow \angle BCD = 90^\circ$

Q7. ABC is a right angled triangle in which $\angle A = 90^\circ$ and $AB = AC$. Find $\angle B$ and $\angle C$.

Sol. In $\triangle ABC$
 $AB = AC$
 $\angle B = \angle C$... (1)
 (angles opposite to equal sides are equal)



In $\triangle ABC$
 $\angle A + \angle B + \angle C = 180^\circ$
 $90^\circ + \angle B + \angle C = 180^\circ$
 $\angle B + \angle C = 90^\circ$... (2)
 from (1) and (2)
 $\angle B = \angle C = 45^\circ$



Q8. Show that the angles of an equilateral triangle are 60° each.

Sol. $\triangle ABC$ is equilateral triangle.

$$\Rightarrow AB = BC = CA$$

Now, $AB = BC$

$$\Rightarrow \angle C = \angle A \dots(1)$$

$$\Rightarrow \angle C = \angle A \dots(1)$$

Similarly, $\angle A = \angle B \dots(2)$

From (1) and (2),

$$\angle A = \angle B = \angle C \dots(3)$$

$$\text{Also, } \angle A + \angle B + \angle C = 180^\circ \dots(4)$$

$$\Rightarrow \angle A = \angle B = \angle C = \frac{1}{3} \times 180^\circ = 60^\circ$$

