## CLASS IX: MATHS

## Chapter 7: Triangles

## Questions and Solutions | Exercise 7.3-NCERT Books

Q1. $\triangle \mathrm{ABC}$ and $\triangle \mathrm{DBC}$ are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see figure). If AD is extended to intersect BC at P , show that

(i) $\triangle \mathrm{ABD} \cong \triangle \mathrm{ACD}$
(ii) $\triangle \mathrm{ABP} \cong \triangle \mathrm{ACP}$
(iii) AP bisects $\angle \mathrm{A}$ as well as $\angle \mathrm{D}$
(iv) AP is the perpendicular bisector of BC .

Sol. In $\triangle \mathrm{ABD}$ and $\triangle \mathrm{ACD}$,
$\mathrm{AB}=\mathrm{AC}$
$\mathrm{DB}=\mathrm{DC}$
( $\because \triangle \mathrm{ABC}$ is isosceles)
$\mathrm{AD}=\mathrm{AD}$
( $\because \triangle \mathrm{DBC}$ is isosceles)
(Common side)
$\therefore \triangle \mathrm{ABD} \cong \triangle \mathrm{ACD}$
(By SSS congruence rule)
(ii) Now, $\triangle \mathrm{ABD} \cong \triangle \mathrm{ACD}$

$$
\begin{equation*}
\Rightarrow \angle \mathrm{BAD}=\angle \mathrm{CAD} \tag{ВуСРСТ}
\end{equation*}
$$

In $\triangle \mathrm{ABP}$ and $\triangle \mathrm{ACP}$,

$$
\begin{array}{ll}
\mathrm{AB}=\mathrm{AC} & (\because \triangle \mathrm{ABC} \text { is isosceles) } \\
\Rightarrow \angle \mathrm{BAP}=\angle \mathrm{CAP} & (\text { By } 1) \\
\mathrm{AP}=\mathrm{AP} & (\text { common side }) \tag{By1}
\end{array}
$$

$\therefore \triangle \mathrm{ABP} \cong \triangle \mathrm{ACP} \quad$ (By SAS congruence rule)
(iii) $\triangle \mathrm{ABD} \cong \triangle \mathrm{ADC}$ (Proved above)
$\angle \mathrm{BAD}=\angle \mathrm{CAD}$
(by CPCT)
$\angle \mathrm{ADB}=\angle \mathrm{ADC}$
(by CPCT)
$180-\angle \mathrm{ADB}=180-\angle \mathrm{ADC}$
$\Rightarrow \angle \mathrm{BDP}=\angle \mathrm{CDP}$
AP bisects $\angle \mathrm{A}$ as well as $\angle \mathrm{D}$
(iv) $\triangle \mathrm{ABP} \cong \triangle \mathrm{ACP}$
$\Rightarrow \mathrm{BP}=\mathrm{CP}$
(By CPCT)
$\Rightarrow \mathrm{AP}$ bisects BC
$\angle \mathrm{APB}=\angle \mathrm{APC}(\mathrm{By} \mathrm{CPCT})$
$\angle \mathrm{APB}+\angle \mathrm{APC}=180^{\circ}$
$2 \angle \mathrm{APB}=180^{\circ}$
$\angle \mathrm{APB}=90^{\circ}$
AP is perpendicular bisector of BC

Q2. $A D$ is an altitude of an isosceles triangle $A B C$ in which $A B=A C$. Show that
(i) AD bisects BC
(ii) AD bisects $\angle \mathrm{A}$

Sol. Given : AD is an altitude of an isosceles triangle ABC in which $\mathrm{AB}=\mathrm{AC}$.


To Prove: (i) AD bisect BC . (ii) AD bisect $\angle \mathrm{A}$.
Proof : (i) In right $\triangle \mathrm{ADB}$ and right $\triangle \mathrm{ADC}$.
Hyp. $\mathrm{AB}=$ Hyp. AC
$\angle \mathrm{ADB}=\angle \mathrm{ADC}$
(Each $90^{\circ}$ )
Side $\mathrm{AD}=$ side AD
(Common)
$\Delta \mathrm{ADB} \cong \triangle \mathrm{ADC}$
(RHS rule)
$\Rightarrow \mathrm{BD}=\mathrm{CD}$
(By CPCT)
$\Rightarrow \mathrm{AD}$ bisect BC
(ii) $\triangle \mathrm{ADB} \cong \triangle \mathrm{ADC}$
$\angle \mathrm{BAD}=\angle \mathrm{CAD}$
(By CPCT)
$\Rightarrow \mathrm{AD}$ bisect $\angle \mathrm{A}$

Q3. Two sides AB and BC and median AM of one triangle ABC are respectively equal to side PQ and QR and median PN of $\triangle \mathrm{PQR}$ (see figure). Show that :
(i) $\triangle \mathrm{ABM} \cong \triangle \mathrm{PQN}$
(ii) $\triangle \mathrm{ABC} \cong \triangle \mathrm{PQR}$


Sol. (i) $\mathrm{BM}=\frac{1}{2} \mathrm{BC} \quad(\because \mathrm{M}$ is mid-point of BC$)$

$$
\begin{array}{ll}
\mathrm{QN}=\frac{1}{2} \mathrm{QR} & (\because \mathrm{~N} \text { is mid-point of } \mathrm{QR}) \\
\Rightarrow \mathrm{BM}=\mathrm{QN} & (\because \mathrm{BC}=\mathrm{QR} \text { is given })
\end{array}
$$

Now, in $\triangle \mathrm{ABM}$ and $\triangle \mathrm{PQN}$, we have
$\mathrm{AB}=\mathrm{PQ}$ (Given)
$\mathrm{BM}=\mathrm{QN}$
(Proved)
$\mathrm{AM}=\mathrm{PN}$
(Given)
$\therefore \triangle \mathrm{ABM} \cong \triangle \mathrm{PQN} \quad(\mathrm{SSS}$ congruence criteria)
(ii) $\triangle \mathrm{ABM} \cong \triangle \mathrm{PQN}$
$\Rightarrow \angle \mathrm{ABM}=\angle \mathrm{PQN}$
$\angle \mathrm{B}=\angle \mathrm{Q}$
Now, in $\triangle \mathrm{ABC}$ and $\triangle \mathrm{PQN}$,
$\mathrm{AB}=\mathrm{PQ}, \angle \mathrm{B}=\angle \mathrm{Q}$ and $\mathrm{BC}=\mathrm{QN}$
$\Rightarrow \triangle \mathrm{ABC} \cong \triangle \mathrm{PQN} \quad[$ by SAS cong. $]$

Q4. BE and CF are two equal altitudes of a triangle ABC . Using RHS congruence rule, prove that the triangle ABC is isosceles.

Sol. Given $B E$ and $C F$ are two altitude of $\triangle \mathrm{ABC}$.


To prove: $\triangle \mathrm{ABC}$ is isosceles.
Proof : In right $\triangle \mathrm{BEC}$ and right $\triangle \mathrm{CFB}$ side
$\mathrm{BE}=$ side CF
(Given)
Hyp. $\mathrm{BC}=$ Hyp CB
(Common)
$\angle \mathrm{BEC}=\angle \mathrm{BFC}$
(Each 90 ${ }^{\circ}$ )
$\triangle \mathrm{BEC} \cong \triangle \mathrm{CFB}$
(RHS Rule)
$\therefore \angle \mathrm{BCE}=\angle \mathrm{CBF}$
(By CPCT)
$\mathrm{AB}=\mathrm{AC}$
(Side opp. to equal angles are equal)
$\triangle \mathrm{ABC}$ is isosceles.

Q5. ABC is an isosceles triangle with $\mathrm{AB}=\mathrm{AC}$. Draw $\mathrm{AP} \perp \mathrm{BC}$ to show that $\angle \mathrm{B}=\angle \mathrm{C}$

Sol. In $\triangle \mathrm{APB}$ and $\triangle \mathrm{APC}$
$\mathrm{AB}=\mathrm{AC}$
$\angle \mathrm{APB}=\angle \mathrm{APC}$
(Each $=90^{\circ}$ )
$\mathrm{AP}=\mathrm{AP}$
(common side)


Therefore, by RHS congruence criteria, we have
$\Delta \mathrm{APB} \cong \triangle \mathrm{APC}$
$\Rightarrow \angle \mathrm{ABP}=\angle \mathrm{ACP}$
(By CPCT)
$\Rightarrow \angle \mathrm{B}=\angle \mathrm{C}$

