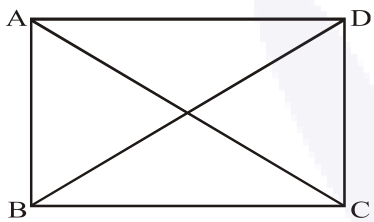
CLASS IX: MATHS
Chapter 8: Quadrilaterals

Questions and Solutions | Exercise 8.1 - NCERT Books

Q1. If the diagonals of a parallelogram are equal, then show that it is a rectangle.

Sol. **Given :** ABCD is a parallelogram with diagonal
AC = diagonal BD



To prove : ABCD is a rectangle.

Proof : In triangle ABC and ABD,

	AB = AB	[Common]	
	AC = BD	[Given]	
	AD = BC	[Opp. Sides of a ||gm]	
∴	ΔABC ≅ BAD	[By SSS congruency]	
⇒	∠DAB = ∠CBA	[By C.P.C.T.]	....(i)

[∵ AD||BC and AB cuts them, the sum of the interior angle of the same side of transversal is 180°]

$\angle DAB + \angle CBA = 180°$ .....(ii)

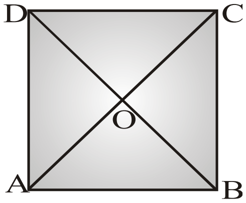
From eq. (i) and (ii), $\angle DAB = \angle CBA = 90°$

Hence, ABCD is a rectangle

Q2. Show that the diagonals of a square are equal and bisect each other at right angles.

Sol. **Given:** ABCD is a square.

To Prove : (i) AC = BD (ii) AC and BD bisect each other at right angles.



Proof: In ΔABC and ΔBAD,

AB = BA [Common]

BC = AD [Opp. sides of square ABCD]

$\angle ABC = \angle BAD$ [Each = 90° ($\because$ ABCD is a square]

$\therefore$ $\Delta ABC \cong \Delta BAD$ [SAS Rule]

$\therefore$ AC = BD ... (i) [C.P.C.T.]

In ΔAOD and ΔBOC

AD = CB [Opp. sides of square ABCD]

$\angle OAD = \angle OCB$

[Alternate angles as $AD \parallel BC$ and transversal AC intersects them]

$\angle ODA = \angle OBC$

[Alternate angles as $AD \parallel BC$ and transversal BD intersects them]

$\Delta AOD \cong \Delta BOC$ [ASA Rule]

$\therefore$ OA = OC and OB = OD ...(ii) [C.P.C.T.]

So, O is the mid point of AC and BD.

Now, In ΔAOB and ΔCOB

AB = BC [Given]

OA = OC [from (ii)]

OB = OB [Common]

$\therefore$ $\Delta AOB \cong \Delta COB$ [By SSS Rule]

$\therefore$ $\angle AOB = \angle BOC$ [C.P.C.T]

But $\angle AOB + \angle BOC = 180°$ [Linear pair]

$\angle AOB + \angle AOB = 180°$

[AOB = BOC proved earlier]

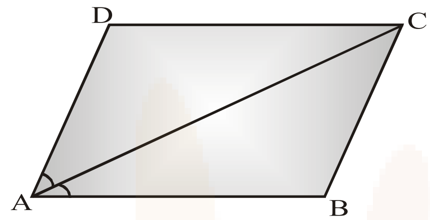
$\Rightarrow$ $2\angle AOB = 180°$

$\Rightarrow$ $\angle AOB = \frac{180°}{2} = 90°$

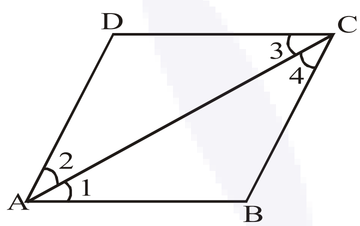
∴ $\angle AOB = \angle BOC = 90°$

∴ AC and BD bisect each other at right angles.

Q3. In figure, ABCD is a parallelogram. Diagonal AC bisects $\angle A$. Show that
(i) it bisects $\angle C$ also (ii) ABCD is a rhombus.



Sol. **Given :**



Diagonal AC bisects $\angle A$ of the parallelogram ABCD.

To prove :

(i) AC bisects $\angle C$

(ii) ABCD is a rhombus

Proof :

(i) Since AB||DC and AC intersects them.

∴ $\angle 1 = \angle 3$ [Alternate angles] ...(i)

Similarly $\angle 2 = \angle 4$ ...(ii)

But $\angle 1 = \angle 2$ [Given] ...(iii)

∴ $\angle 3 = \angle 4$ [Using eq. (i), (ii) and (iii)]

Thus AC bisects $\angle C$.

(ii) $\angle 2 = \angle 3 = \angle 4 = \angle 1$

$\Rightarrow$ AD = CD [Sides opposite to equal angles]

∴ AB = CD = AD = BC

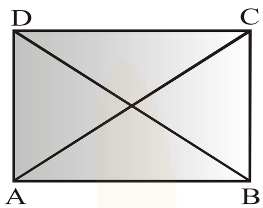
Hence, ABCD is a rhombus.

Q4. ABCD is a rectangle in which diagonal AC bisects ∠A as well as ∠C. Show that

(i) ABCD is a square

(ii) diagonal BD bisects ∠B as well as ∠D.

Sol. **Given :** ABCD is a rectangle in which diagonal AC bisects ∠A as well as ∠C.



To prove : (i) ABCD is a square

(ii) diagonal BD bisects ∠B as well as ∠D.

Proof :

(i) ∵ AB || DC and transversal AC intersects them.

so, ∠BAC = ∠DCA [Alternate angles]

But ∠BAC = ∠DAC [∵ AC bisects ∠A]

∴ ∠ DCA = ∠DAC

⇒ DA = CD

[Sides opposite to equal angles of a triangle are equal]

But AB = CD and DA = BC [Opposite side of a rectangle]

∴ AB = BC = CD = DA

Also ∠A = ∠B = ∠C = ∠D = 90°

[∵ ABCD is a rectangle]

Hence, ABCD is a square

(ii) In Δ BAD and Δ BCD,

BA = BC [∵ ABCD is a square]

AD = CD [∵ ABCD is a square]

BD = BD [Common]

∴ ΔBAD ≅ ΔBCD [By SSS congruence rule]

$\therefore$ $\angle ABD = \angle CBD$ [By C.P.C.T.]

$\angle ADB = \angle CDB$ [By C.P.C.T.]

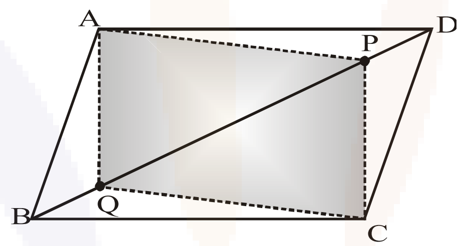
Hence, diagonal BD bisect $\angle B$ as well as $\angle D$

Q5. In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ. Show that :

(i) $\Delta APD \cong \Delta CQB$ (ii) AP = CQ

(iii) $\Delta AQB \cong \Delta CPD$ (iv) AQ = CP

(v) APCQ is a parallelogram



Sol. (i) In ΔAPD and ΔCQB, we have

DP = BQ [Given]

AD = CB

[Opposite sides of parallelogram ABCD]

$\angle ADP = \angle CBQ$ [Pair of alternate angles]

$\Rightarrow$ $\Delta APD \cong \Delta CQB$ [SAS congruence criteria]

(ii) Then, by CPCT, we have AP = CQ

(iii) We can prove

$\Delta AQB \cong \Delta CPD$ [as we have done in (i)]

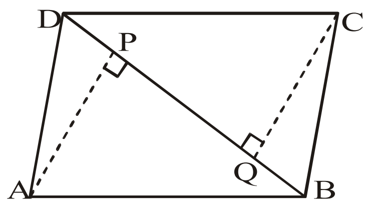
(iv) By CPCT, we have AQ = CP

(v) Now, we have AP = CQ and AQ = CP

Hence, APCQ is a parallelogram.

Q6. ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD. Show that

(i) $\Delta APB \cong \Delta CQD$ (ii) AP = CQ



Sol. **Given :** ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD respectively.

To prove : (i) $\Delta APB \cong \Delta CQD$ (ii) AP = CQ

Proof :

(i) In ΔAPB and ΔCQD,

AB = CD	[Opp. side of ‖ gm ABCD]
$\angle ABP = \angle CDQ$	[$\therefore$ AB ‖ DC and transversal BD intersect them]
$\angle APB = \angle CQD$	[Each = 90°]
$\therefore \Delta APB \cong \Delta CQD$	[AAS Rule]

(ii) $\therefore$ AP = CQ [C.P.C.T.]

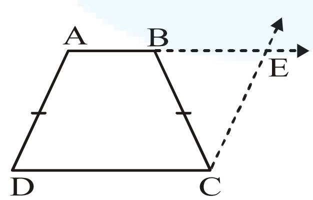
Q7. ABCD is a trapezium in which AB‖CD and AD = BC. Show that (fig)

(i) $\angle A = \angle B$

(ii) $\angle C = \angle D$

(iii) $\Delta ABC \cong \Delta BAD$

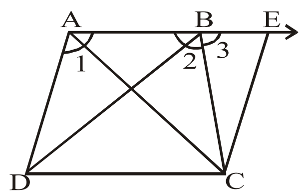
(iv) diagonal AC = diagonal BD



Sol. **Given :** ABCD is a trapezium.

AB ‖ CD and AD = BC

To Prove :



(i) $\angle A = \angle B$

(ii) $\angle C = \angle D$

(iii) $\Delta\ ABC \cong \Delta\ BAD$

(iv) Diagonal AC = Diagonal BD

Construction : Draw CE || AD and extend AB to intersect CE at E.

Proof :

(i) As AECD is a parallelogram.

[By construction]

$\therefore$ AD = EC

But AD = BC [Given]

$\therefore$ BC = EC

$\Rightarrow$ $\angle 3 = \angle 4$ [Angles opposite to equal sides are equal]

Now, $\angle 1 + \angle 4 = 180°$ [Interior angles]

and $\angle 2 + \angle 3 = 180°$ [Linear pair]

$\Rightarrow$ $\angle 1 + \angle 4 = \angle 2 + \angle 3$

$\Rightarrow$ $\angle 1 = \angle 2$ [$\because \angle 3 = \angle 4$]

$\Rightarrow$ $\angle A = \angle B$

(ii) $\angle 3 = \angle BCD$ [Alternate interior angles]

$\angle D = \angle 4$ [Opposite angles of a parallelogram]

But $\angle 3 = \angle 4$ [Δ BCE is an isosceles triangle]

$\therefore$ $\angle BCD = \angle ADC$

$\therefore$ $\angle C = \angle D$

(iii) In ΔABC and ΔBAD,

AB = AB [Common]

$\angle 1 = \angle 2$ [Proved]

AD = BC [Given]

$\therefore$ $\Delta ABC \cong \Delta BAD$ [By SAS congruency]

$\Rightarrow$ AC = BD [By C.P.C.T.]