CLASS IX: MATHS
Chapter 8: Quadrilaterals

Questions and Solutions | Exercise 8.2 - NCERT Books

Q1. ABCD is a quadrilateral in which P, Q, R and S are mid points of the sides AB, BC, CD and DA (fig.) AC is a diagonal. Show that

(i) SR||AC and SR = 1/2 C

(ii) PQ = SR

(iii) PQRS is a parallelogram.

Sol. **Given :** ABCD is a quadrilateral in which P, Q, R and S are mid-points of AB, BC, CD and DA. AC is a diagonal.

To prove : (i) SR || AC and SR = $\frac{1}{2}$ AC

(ii) PQ = SR

(iii) PQRS is a parallelogram.

Proof : (i) In ΔDAC,

∵ S is the mid-point of DA and R is the mid-point of DC

∴ SR || AC and SR = $\frac{1}{2}$ AC [By Mid-point theorem]

(ii) In ΔBAC,

∵ P is the mid-point of AB and Q is the mid-point of BC

∴ PQ || AC and PQ = $\frac{1}{2}$ AC [By Mid-point theorem]

But from (i) SR = $\frac{1}{2}$ AC & (ii) PQ = $\frac{1}{2}$ AC

⇒ PQ = SR

(iii) PQ || AC [From (ii)]

SR || AC [From (i)]

∴ PQ || SR

[Two lines parallel to the same line are parallel to each other]

Also, PQ = SR [From (ii)]

∴ PQRS is a parallelogram.

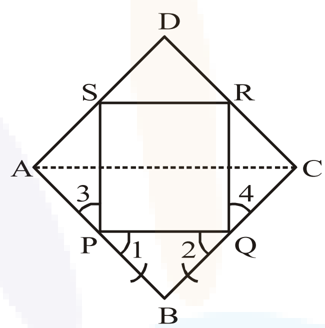
[A quadrilateral is a parallelogram if a pair of opposite sides is parallel and is of equal length]

Q2. ABCD is a rhombus and P, Q, R and S are the mid points of sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.

Sol. **Given :** P, Q, R and S are the mid-points of respective sides AB, BC, CD and DA of rhombus. PQ, QR, RS and SP are joined.

To prove : PQRS is a rectangle.

Construction : Join A and C.



Proof : In ΔABC, P is the mid-point of AB and Q is the mid-point of BC.

∴ PQ || AC and PQ = $\frac{1}{2}$ AC .....(i)

In ΔADC, R is the mid-point of CD and S is the mid-point of AD.

∴ SR || AC and SR = $\frac{1}{2}$ AC ....(ii)

From eq. (i) and (ii), PQ || SR and PQ = SR

∴ PQRS is a parallelogram.

Now ABCD is a rhombus [Given]

∴ AB = BC

⇒ $\frac{1}{2}$ AB = $\frac{1}{2}$ BC ⇒ PB = BQ

∴ ∠1 = ∠2 [Angles opposite to equal sides are equal]

Now in triangles APS and CQR, we have,

AP = CQ [P and Q are the mid-points of AB and BC and AB = BC]

Similarly, AS = CR and PS = QR

[Opposite sides of a parallelogram]

∴ ΔAPS ≅ ΔCQR [By SSS congruency]

⇒ ∠3 = ∠4 [By C.P.C.T.]

Now, we have ∠1 + ∠SPQ + ∠3 = 180°

and $\angle 2 + \angle PQR + \angle 4 = 180°$

$\therefore$ $\angle 1 + \angle SPQ + \angle 3 = \angle 2 + \angle PQR + \angle 4$

Since $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$ [Proved above]

$\therefore$ $\angle SPQ = \angle PQR$ .....(iii)

Now PQRS is a parallelogram [Proved above]

$\therefore$ $\angle SPQ + \angle PQR = 180°$ .....(iv)

[Interior angles]

Using eq. (iii) and (iv),

$\angle SPQ + \angle SPQ = 180°$

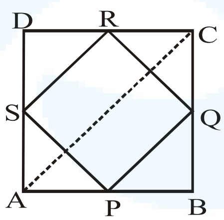
$\Rightarrow$ $2\angle SPQ = 180° \Rightarrow \angle SPQ = 90°$

Hence, PQRS is a rectangle.

Q3. ABCD is a rectangle and P,Q,R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

Sol. **Given :** A rectangle ABCD in which P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. PQ, QR, RS and SP are joinned.

To prove : PQRS is a rhombus.



Construction : Join AC.

Proof : In Δ ABC, P and Q are the mid-points of sides AB, BC respectively.

$\therefore$ $PQ \parallel AC$ and $PQ = \frac{1}{2}AC$ ....(i)

In Δ ADC, R and S are the mid-points of sides CD, AD respectively.

$\therefore$ $SR \parallel AC$ and $SR = \frac{1}{2}AC$ ...(ii)

From eq.(i) and (ii), $PQ \parallel SR$ and $PQ=SR$ ...(iii)

$\therefore$ PQRS is a parallelogram.

Now ABCD is a rectangle. [Given]

$\therefore$ AD = BC

$\Rightarrow$ $\frac{1}{2}AD = \frac{1}{2}BC$ $\Rightarrow$ AS = BQ ...(iv)

In triangles APS and BPQ,

AP = BP [P is the mid-point of AB]

$\angle PAS = \angle PBQ$ [Each 90°]

and AS = BQ [From eq. (iv)]

$\therefore$ $\Delta APS \cong \Delta BPQ$ [By SAS congruency]

$\Rightarrow$ PS = PQ [By C.P.C.T.] .....(v)

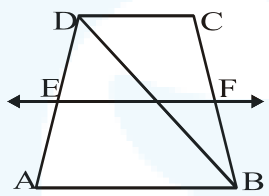
From eq.(iii) and (v), we get that PQRS is a parallelogram.

$\Rightarrow$ PS = PQ

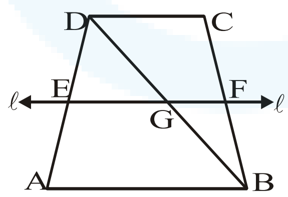
$\Rightarrow$ Two adjacent sides are equal.

Hence, PQRS is a rhombus.

Q4. ABCD is a trapezium in which AB||DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (fig.). Show that F is the mid-point of BC.



Sol. Line ℓ || AB and passes through E.



Line ℓ meets BC in F and BD in G.

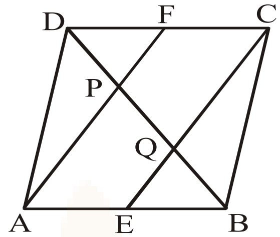
In ΔABD, E is mid-point of AD and EG || AB.

$\Rightarrow$ G is mid-point of BD.

Also, ℓ || AB and AB || CD $\Rightarrow$ ℓ || CD

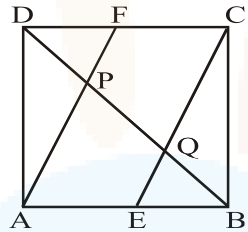
$\Rightarrow$ F is mid-point of BC. [$\because$ G is mid-point of BD]

Q5. In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (fig.). Show that the line segments AF and EC trisect the diagonal BD.



Sol. Since E and F are the mid-points of AB and CD respectively.

Given : ABCD is a parallelogram. E and F are midpoints of AB and AC respectively.



To prove : DP = PQ = QB

Proof : -

$\therefore \quad AE = \frac{1}{2}AB$ and $CF = \frac{1}{2}CD$ ....(i)

But ABCD is a parallelogram.

$\therefore \quad AB = CD$ and $AB \parallel DC$

$\Rightarrow \quad \frac{1}{2}AB = \frac{1}{2}CD$ and $AB \parallel DC$

$\Rightarrow \quad AE = FC$ and $AE \parallel FC$ [From eq. (i)]

$\therefore$ AECF is a parallelogram.

$\Rightarrow \quad FA \parallel CE \quad \Rightarrow FP \parallel CQ$

[FP is a part of FA and CQ is a part of CE] ..... (ii)

Since the line segment drawn through the mid-point of one side of a triangle and parallel to the other side bisects the third side.

In ΔDCQ, F is the mid-point of CD and

$\Rightarrow$ FP || CQ

$\therefore$ P is the is mid-point of DQ.

$\Rightarrow$ DP = PQ ....(iii)

Similarly, In ΔABP, E is the mid-point of AB and

$\Rightarrow$ EQ || AP

$\therefore$ Q is the mid-point of BP.

$\Rightarrow$ BQ = PQ ....(iv)

From eq.(iii) and (iv),

DP = PQ = BQ ....(v)

Now, BD = BQ + PQ + DP = BQ + BQ + BQ = 3BQ

$\Rightarrow$ $BQ = \frac{1}{3}BD$ .....(vi)

From eq (v) and (vi), $DP = PQ = BQ = \frac{1}{3}BD$

$\Rightarrow$ Points P and Q trisects BD. So AF and CE trisects BD.

Q6. ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that

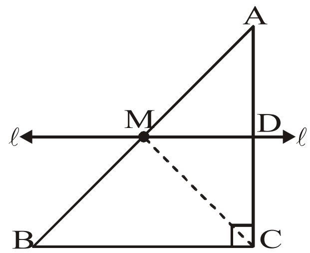
(i) D is the mid-point of AC

(ii) MD ⊥ AC

(iii) CM = MA = 1/2 AB

Sol. (i) Through M, we draw line ℓ || BC. ℓ intersects AC at D.

$\Rightarrow$ D is mid-point of AC.



(ii) $\angle ADM = \angle ACB = 90°$

[Corresponding angles]

$\Rightarrow$ $\angle ADM = 90°$ $\Rightarrow$ $MD \perp AC$.

(iii) In ΔCMD and ΔAMD;

CD = AD, MD = MD

and $\angle CDM = \angle ADM$ [Each = 90°]

Therefore, $\Delta CMD \cong \Delta AMD$

$\Rightarrow$ CM = AM; Also AM = 1/2 AB.