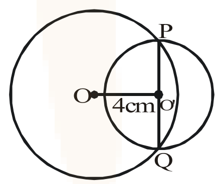
CLASS IX: MATHS
Chapter 9: Circles

Questions and Solutions | Exercise 9.2 - NCERT Books

Q1. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of common chord.

Sol. We know that if two circles intersect each other at two points, then the line joining their centres is the perpendicular bisector of their common chord.



$\therefore$ Length of the common chord

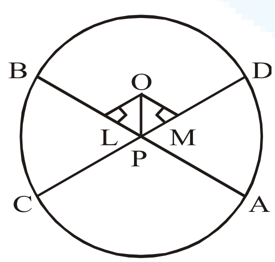
$\Rightarrow$ $PQ = 2O'P = 2 \times 3 = 6$ cm

Q2. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

Sol. O is the centre of the circle. Chords AB and CD of the circle are equal. P is the point of intersection of AB and CD. Join OP, draw $OL \perp AB$ and $OM \perp OD$.

Here, we find $OL = OM$ $(\because AB = CD)$ ...(1)

In ΔOLP and ΔOMP,



$OL = OM$ (By 1)

$OP = OP$ (Common hypotenuse)

$\angle OLP = \angle OMP$ (Each $= 90°$)

Then we have $\Delta OLP \cong \Delta OMP$ (RHS congruence)

By CPCT, or $PL = PM$ ...(2)

Now, AL = BL = ½ AB; CM = DM = ½ CD

$\Rightarrow$ AL = CM ($\because$ AB = CD) ...(3)

and BL = DM ...(4)

Subtracting (1) from (3),

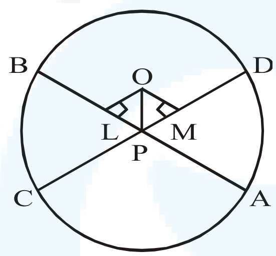
AL – PL = CM – PM $\Rightarrow$ AP = CP

Adding (2) from (4),

PL + BL = PM + DM $\Rightarrow$ PB = PD

Q3. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.

Sol. O is the centre of the circle. Chords AB and CD of the circle are equal. P is the point of intersection of AB and CD. Join OP, draw OL ⊥ AB and OM ⊥ OD.



Here, we find OL = OM ($\because$ AB = CD) ...(1)

In ΔOLP and ΔOMP,

OL = OM (By 1)

OP = OP (Common hypotenuse)

∠OLP = ∠OMP (Each = 90°)

Then we have ΔOLP ≅ ΔOMP (RHS congruence)

By CPCT, or PL = PM ...(2)

Now, AL = BL = ½ AB;

CM = DM = ½ CD

$\Rightarrow$ AL = CM ($\because$ AB = CD) ...(3)

and BL = DM ...(4)

Subtracting (1) from (3),

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Adding (2) from (4),

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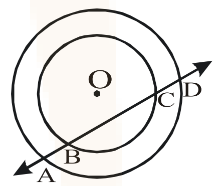
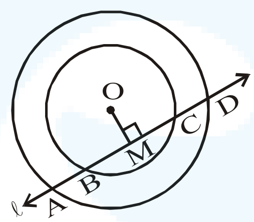
Q4. If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD (see fig).



Sol. **Given :** Two circles with the common centre O. A line "ℓ" intersects the outer circle at A and D and the inner circle at B and C.



To prove : AB = CD

Construction : Draw OM $\perp$ ℓ.

Proof : OM $\perp$ ℓ [Construction]

For the outer circle,

$\therefore$ AM = MD [Perpendicular from the centre bisects the chord] .... (1)

For the inner circle,

OM $\perp$ ℓ [Construction]

$\therefore$ BM = MC [Perpendicular from the centre to the chord bisects the chord] ...(2)

Subtracting (2) from (1), we have

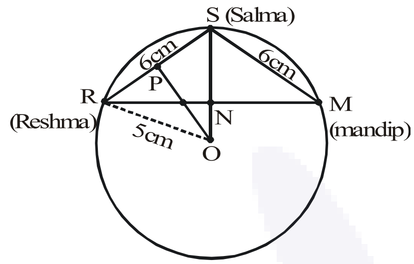
$\Rightarrow$ AM – BM = MD – MC

$\Rightarrow$ AB = CD

Q5. Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5 m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6 m each, what is the distance between Reshma and Mandip?

Sol. We draw SN $\perp$ RS.

Now, SN bisects RM and also SN (produced) passes through the centre O.



Put RN = x

The ar(ΔORS) = $\frac{1}{2}$ × OS × RN

$= \frac{1}{2} \times 5 \times x$ ($\because$ OS = OR = 5 m)

i.e., ar(ΔORS) = $\frac{5}{2}$x ...(1)

Now, draw OP $\perp$ RS, P is mid-point of RS.

$\Rightarrow$ PR = PS = 3 m $\Rightarrow$ $OP^2 = (5)^2 - (3)^2 = 16 \Rightarrow OP = 4$ m

Here, ar(ΔORS) = $\frac{1}{2} \times RS \times OP = \frac{1}{2} \times 6 \times 4$

i.e., ar(ΔORS) = 12 m^2

From (1) and (2),

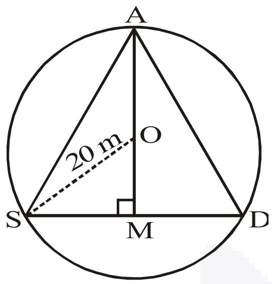
$\frac{5}{2}x = 12 \Rightarrow x = 4.8 \text{ m} \Rightarrow RM = 2x = 2 \times 4.8 \text{ m} \Rightarrow RM = 9.6 \text{ m}$

Thus, distance between Reshma and Mandip is 9.6 m.

Q6. A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Sol. Let Ankur, Syed and David are sitting at A, S and D respectively such that AS = SD = AD i.e., ΔASD is an equilateral triangle.

Let the length of each side of the equilateral triangle is 2x metres.



Draw AM $\perp$ SD.

Since, ΔASD is an equilateral triangle,

$\therefore$ AM passes through O.

$\Rightarrow$ $SM = \frac{1}{2}SD = \frac{1}{2}(2x) = x$

Now, in ΔASM, we have $AM^2 + SM^2 = AS^2$

$\Rightarrow$ $AM^2 = AS^2 - SM^2 = (2x)^2 - x^2 = 4x^2 - x^2 = 3x^2$

$\Rightarrow$ $AM = \sqrt{3}x$.

Now, $OM = AM - OA = \left(\sqrt{3}x - 20\right)$ m

$\Rightarrow$ (OS = OA = 20 cm)

$\Rightarrow$ $(20)^2 = x^2 + \left(\sqrt{3}x - 20\right)^2$

$\Rightarrow$ $400 = x^2 + 3x^2 - 40\sqrt{3}x + 400$

$\Rightarrow$ $4x^2 = 40\sqrt{3}x \Rightarrow 4x = 40\sqrt{3} \Rightarrow x = 10\sqrt{3}$ m

Now, $SD = 2x = 2 \times 10\sqrt{3}$ m $= 20\sqrt{3}$ m

Thus, the length of the string of each phone = $20\sqrt{3}$ m