Class X : MATH
Chapter 5 : Arithmetic Progressions
Questions & Answers - Exercise : 5.1 - NCERT Book

Q1. In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

(i) The taxi fare after each km when the fare is Rs. 15 for the first km and Rs. 8 for each additional km.

(ii) The amount of air present in a cylinder when a vacuum pump removes 1/4 of the air remaining in the cylinder at a time.

(iii) The cost of digging a well after every metre of digging, when it costs Rs. 150 for the first metre and rises by Rs. 50 for each subsequent metre.

(iv) The amount of money in the account every year, when Rs. 10000 is deposited at compound interest at 8% per annum.

Sol. (i) t_n denotes the taxi fare (in Rs.) for the first n km.

Now, $t_1 = 15$,

$t_2 = 15 + 8 = 23$,

$t_3 = 23 + 8 = 31$,

$t_4 = 31 + 8 = 39$,....

List of fares after 1 km, 2 km, 3 km, 4 km, ... respectively is 15, 23, 31, 39, ... (in Rs.).

Here, $t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = ... = 8$.

Thus, the list forms an AP.

(ii) $t_1 = x$ units ; $t_2 = x - \frac{1}{4}x = \frac{3}{4}x$ units ;

$t_3 = \frac{3}{4}x - \frac{1}{4}\left(\frac{3}{4}x\right) = \frac{3}{4}x - \frac{3}{16}x = \frac{9}{16}x$ units

$t_4 = \frac{9}{16}x - \frac{1}{4}\left(\frac{9}{16}x\right) = \frac{27}{64}x$ units

The list of numbers is x, $\frac{3}{4}x, \frac{9}{16}x, \frac{27}{64}x, \ldots.$

It is not an AP because $t_2 - t_1 \neq t_3 - t_2$.

(iii) Cost of digging for first metre = 150

Cost of digging for first 2 metres

= 150 + 50 = 200

Cost of digging for first 3 metres

= 200 + 50 = 250

Cost of digging for first 4 metres

= 250 + 50 = 300

Clearly, 150, 200, 250, 300.... forms an A.P.

Here, $t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = \ldots. = 50$.

Thus, the list forms an AP.

(iv) We know that if Rs P is deposited at r% compound interest per annum for n years, our money will be $P\left(1+\frac{r}{100}\right)^n$ after n years.

Therefore, after every year, our money will be

$10000\left(1+\frac{8}{100}\right), 10000\left(1+\frac{8}{100}\right)^2,$

$10000\left(1+\frac{8}{100}\right)^3, 10000\left(1+\frac{8}{100}\right)^4, \ldots.$

Clearly, adjacent terms of this series do not have the same difference between them. Therefore, this is not an A.P.

Q2. Write first four terms of the AP, when the first term a and the common difference d are given as follows:

(i) $a = 10, d = 10$ (ii) $a = -2 \quad d = 0$

(iii) $a = 4, d = -3$ (iv) $a = -1, d = 1/2$

(v) $a = -1.25, \quad d = -0.25$

Sol. (i) $t_1 = a = 10,$

$t_2 = 10 + d = 10 + 10 = 20,$

$t_3 = 20 + d = 20 + 10 = 30,$

$t_4 = 30 + d = 30 + 10 = 40, ...$

Thus, the AP is 10, 20, 30, 40, ...

(ii) Given $a = -2$ and $d = 0$

$t_1 = -2, t_2 = -2 + 0 = -2,$

$t_3 = -2 + 0 = -2, t_4 = -2 + 0 = -2, ...$

Thus, the AP is –2, – 2, – 2, – 2, ...

(iii) $a = 4, d = -3$

$t_1 = a = 4$

$t_2 = a_1 + d = 4 - 3 = 1$

$t_3 = a_2 + d = 1 - 3 = -2$

$t_4 = a_3 + d = -2 - 3 = -5$

Therefore, the series will be 4, 1, –2 –5 …

First four terms of this A.P. will be 4, 1, –2 and –5.

(iv) $a = -1, d = \frac{1}{2}$

$t_1 = a = -1$

$t_2 = a_1 + d = -1 + \frac{1}{2} = -\frac{1}{2}$

$t_3 = a_2 + d = -\frac{1}{2} + \frac{1}{2} = 0$

$t_4 = a_3 + d = 0 + \frac{1}{2} = \frac{1}{2}$

Clearly, the series will be

$-1, -\frac{1}{2}, 0, \frac{1}{2}$ … … … … .

First four terms of this A.P. will be

$-1, -\frac{1}{2}$, 0 and $-\frac{1}{2}$

(v) $a = -1.25, d = -0.25$

$t_1 = a = -1.25$

$t_2 = a_1 + d = -1.25 - 0.25 = -1.50$

$t_3 = a_2 + d = -1.50 - 0.25 = -1.75$

$t_4 = a_3 + d = -1.75 - 0.25 = -2.00$

Clearly, the series will be –1.25, –1.50, –1.75, –2.00 … … ....

First four terms of this A.P. will be –1.25, –1.50, –1.75 and –2.00.

Q3. For the following APs, write the first term and the common difference :

(i) 3, 1, – 1, –3, ...

(ii) –5, – 1, 3, 7, ...

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, ...$

(iv) 0.6, 1.7, 2.8, 3.9, ...

Sol. (i) $a = 3, d = t_2 - t_1 = 1 - 3 = -2,$

i.e., $d = -2$

(ii) $a = -5, d = 4$

(iii) $a = \frac{1}{3}$

$d = t_2 - t_1 = \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$

(iv) 0.6, 1.7, 2.8, 3.9 …

$a = 0.6$

$d = t_2 - t_1$

$= 1.7 - 0.6$

$= 1.1$

Q4. Which of the following are APs ? If they form an AP, find the common difference d and write three more terms.

(i) 2, 4, 8, 16,....

(ii) $2, \frac{5}{2}, 3, \frac{7}{2}, ...$

(iii) – 1.2, – 3.2, – 5.2, – 7.2, ...

(iv) – 10, – 6, – 2, 2, ...

(v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, ...$

(vi) 0.2, 0.22, 0.222, 0.2222, ...

(vii) 0, – 4, – 8, – 12, ...

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, ...$

(ix) 1, 3, 9, 27, ...

(x) a, 2a, 3a, 4a, ...

(xi) a, a^2, a^3, a^4, ...

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, ...$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, ...$

(xiv) 1^2, 3^2, 5^2, 7^2, ...

(xv) 1^2, 5^2, 7^2, 73, ...

Sol. (i) Not an AP because $t_2 - t_1 = 2$
and $t_3 - t_2 = 8 - 4 = 4$,
i.e., $t_2 - t_1 \neq t_3 - t_2$.

(ii) It is an AP. a = 2, d = 1/2
[$\because$ $t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = 1/2$]

$t_5 = \frac{7}{2} + \frac{1}{2} = 4$, $t_6 = 4 + \frac{1}{2} = \frac{9}{2}$,

$t_7 = \frac{9}{2} + \frac{1}{2} = 5$.

(iii) We have : –1.2, –3.2, –5.2, –7.2, ...
$\therefore$ $t_1 = -1.2$, $t_2 = -3.2$, $t_3 = -5.2$, $t_4 = -7.2$

$t_2 - t_1 = -3.2 + 1.2 = -2$

$t_3 - t_2 = -5.2 + 3.2 = -2$

$t_4 - t_3 = -7.2 + 5.2 = -2$

$\because\ t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = -2$

$\Rightarrow d = -2$

$\therefore$ The given numbers from an A.P. such that $d = -2$.

Now, $t_5 = t_4 + (-2) = -7.2 + (-2) = -9.2$,

$t_6 = t_5 + (-2) = -9.2 + (-2) = -11.2$

and $t_7 = t_6 + (-2)$

$= -11.2 + (-2) = -13.2$

Thus, $d = -2$ and $t_5 = -9.2$, $t_6 = -11.2$ and $t_7 = -13.2$

(iv) It is an AP.

$a = -10$, $d = 4$, $t_5 = 6$, $t_6 = 10$, $t_7 = 14$.

(v) It is an AP.

$a = 3$, $d = \sqrt{2}$

$t_5 = 3 + 3\sqrt{2} + \sqrt{2} = 3 + 4\sqrt{2}$,

$t_6 = 3 + 5\sqrt{2}$, $t_7 = 3 + 6\sqrt{2}$.

(vi) It is not AP.

$t_2 - t_1 = 0.22 - 0.2 = 0.02$,

$t_3 - t_2 = 0.222 - 0.22 = 0.002$, ...

i.e., $t_2 - t_1 \neq t_3 - t_2$.

(vii) We have : 0, –4, –8, –12, ...

$\therefore\ t_1 = 0$, $t_2 = -4$, $t_3 = -8$, $t_4 = -12$

$t_2 - t_1 = -4 - 0 = -4$

$t_3 - t_2 = -8 + 4 = -4$

$t_4 - t_3 = -12 + 8 = -4$

$\because\ t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = -4 \Rightarrow d = -4$

$\therefore$ The given numbers from an A.P.

Now, $t_5 = t_4 + (-4) = -12 + (-4) = -16$

$t_6 = t_5 + (-4) = -16 + (-4) = -20$

$t_7 = t_6 + (-4) = -20 + (-4) = -24$

Thus, $d = -4$ and $t_5 = -16,\ t_6 = -20,\quad t_7 = -24$.

(viii) We have : $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \ldots\ldots$

$\therefore\ t_1 = t_2 = t_3 = t_4 = -\frac{1}{2}$

$t_2 - t_1 = 0,\ t_3 - t_2 = 0,\ t_4 - t_3 = 0 \Rightarrow d = 0$

$\therefore$ The given numbers from an A.P.

Now, $t_5 = -\frac{1}{2} + 0 = -\frac{1}{2}$

$t_6 = -\frac{1}{2} + 0 = -\frac{1}{2},\ t_7 = -\frac{1}{2} + 0 = -\frac{1}{2}$

Thus, $d = 0$ and $t_5 = -\frac{1}{2},\ t_6 = -\frac{1}{2},\ t_7 = -\frac{1}{2}$

(ix) Not an A.P. Here, $t_2 - t_1 \neq t_3 - t_2$.

(x) We have : a, 2a, 3a, 4a, ...

$\therefore$ $t_1 = a, t_2 = 2a, t_3 = 3a, t_4 = 4a$

$t_2 - t_1 = 2a - a = a,$

$t_3 - t_2 = 3a - 2a = a$ and

$t_4 - t_3 = 4a - 3a = a$

$\because$ $t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = a$

$\Rightarrow$ $d = a$

$\therefore$ The given numbers from an A.P.

Now, $t_5 = t_4 + a = 4a + a = 5a, t_6 = t_5 + a$

$= 5a + a = 6a$ and $t_7 = t_6 + a = 6a + a = 7a$

Thus, $d = a$ and $t_5 = 5a, t_6 = 6a, t_7 = 7a$

(xi) Not an AP if $a \neq 1$.

Here, $t_2 - t_1 = a^2 - a = a(1 - a),$

$t_3 - t_2 = a^3 - a^2 = a^2(1 - a)$

$t_3 - t_2 \neq t_2 - t_1$ when $a \neq 1$.

It will be an AP if $a = 1$.

Hence, the given sequence is an AP only when $a = 1$.

In this case, first term = 1,

common difference = 0

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots$ can be rewritten as

$\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, \ldots\ldots$

$a = \sqrt{2}, d = \sqrt{2}$

$t_5 = 5\sqrt{2},\ t_6 = 6\sqrt{2},\ t_7 = 7\sqrt{2},$

i.e., $t_5 = \sqrt{50}$, $t_6 = \sqrt{72},\ t_7 = \sqrt{98}.$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \ldots$ can be rewritten as

$\sqrt{3}, \sqrt{2} \times \sqrt{3}, 3, 2\sqrt{3}, \ldots$

Here, $t_2 - t_1 \neq t_3 - t_2$

Therefore, the given list is not an AP.

(xiv) We have $1^2, 3^2, 5^2, 7^2, \ldots\ldots\ldots$

$$\therefore \left.\begin{array}{l} t_1 = 1^2 = 1 \\ t_2 = 3^2 = 9 \end{array}\right\} \Rightarrow t_2 - t_1 = 9 - 1 = 8$$

$$\left.\begin{array}{l} t_3 = 5^2 = 25 \\ t_4 = 7^2 = 49 \end{array}\right\} \Rightarrow t_4 - t_3 = 49 - 25 = 24$$

$\because t_2 - t_1 \neq t_4 - t_3$

$\therefore$ The given numbers do not form an A.P.

(xv) $1^2, 5^2, 7^2, 73, \ldots$ can be rewritten as $1, 25, 49, 73, \ldots.$

Here, $t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = \ldots. = 24$

Hence, it is an AP.

$\therefore t_5 = 97, \quad t_6 = 121, \quad t_7 = 145$