Class X : MATH
Chapter 5 : Arithmetic Progressions
Questions & Answers - Exercise : 5.2 - NCERT Book

Q1. Fill in the blanks in the following table, given that a is the first term, d the common difference and a_n, the n^{th} term of the AP.

	a	d	n	a_n
(i)	7	3	8	...
(ii)	–18	...	10	0
(iii)	...	–3	18	–5
(iv)	–18.9	2.5	...	3.6
(v)	3.5	0	105	...

Sol. (i) $a = 7, d = 3, n = 8$

$a_8 = a + 7d = 7 + 7 \times 3 = 28.$

Hence, $a_8 = 28.$

(ii) $a = -18, n = 10, a_n = 0, d = ?$

$a_n = a + (n - 1)d$

$0 = -18 + (10 - 1)d$

$18 = 9d \quad \Rightarrow \quad d = \frac{18}{9} = 2$

Hence, $d = 2$

(iii) $d = -3, n = 18, a_n = -5$

$a_n = a + (n - 1)d$

$-5 = a + (18 - 1)(-3)$

$-5 = a + (17)(-3)$

$-5 = a - 51$

$a = 51 - 5 = 46$

Hence, $a = 46$

(iv) $a = -18.9, d = 2.5$

$t_n = 3.6$

$\Rightarrow$ $a + (n-1)\,d = 3.6$

$\Rightarrow$ $-18.9 + (n-1) \times (2.5) = 3.6$

$\Rightarrow$ $(n-1) \times (2.5) = 3.6 + 18.9 = 22.5$

$\Rightarrow$ $n - 1 = \frac{22.5}{2.5} = \frac{225}{25} = 9$

$\Rightarrow$ $n = 10$

(v) $a = 3.5$, $d = 0$, $n = 105$

Then $a_{105} = a + 104\,d = 3.5 + 0 = 3.5$

Q2. Choose the correct choice in the following and justify

(i) 30th term of the AP : 10, 7, 4,.... is

(A) 97 (B) 77 (C) – 77 (D) – 87

(ii) 11th term of the AP : $-3, -\frac{1}{2}, 2, \ldots$ is

(A) 28 (B) 22 (C) – 38 (D) $-48\frac{1}{2}$

Sol. (i) $a = 10$, $d = -3$

$t_{30} = a + 29d = 10 + 29 \times (-3)$

$= 10 - 87 = -77$

Hence, the correct option is (C)

(ii) $a = -3$, $d = 5/2$

$t_{11} = a + 10d = -3 + 10 \times 5/2 = 22$

Hence, the correct option is (B)

Q3. In the following APs, find the missing terms in the boxes :

(i) 2, $\square$, 26

(ii) $\square$, 13, $\square$, 3

(iii) 5, $\square$, $\square$, $9\frac{1}{2}$

(iv) – 4, $\square$, $\square$, $\square$, $\square$, 6

(v) $\square$, 38, $\square$, $\square$, $\square$, – 22

Sol. (i) $a = 2, a + 2d = 26 \quad \Rightarrow 2 + 2d = 26$

$\Rightarrow \quad 2d = 26 - 2 = 24 \quad \Rightarrow d = 12$

Then the missing term

$t_2 = a + d = 2 + 12 = 14$

(ii) $a + d = 13 \quad ...(1)$

$a + 3d = 3 \quad ...(2)$

Subtracting (1) from (2), we get

$(a + 3d) - (a + d) = 3 - 13$

$\Rightarrow \quad 2d = -10 \quad \Rightarrow \quad d = -5$

from (1), $a - 5 = 13$

$\Rightarrow \quad a = 18$

Therefore, the first missing term is 18

The next missing term

$t_3 = t_2 + d = 13 + (-5) = 8$

(iii) $a = 5$

$a_4 = 9\frac{1}{2} = \frac{19}{2} \qquad a + 3d = \frac{19}{2}$

$\frac{19}{2} = 5 + 3d$

$\frac{19}{2} - 5 = 3d$

$\frac{9}{2} = 3d \qquad d = \frac{3}{2}$

$a_2 = a + d = 5 + \frac{3}{2} = \frac{13}{2}$

$a_3 = a + 2d = 5 + 2\left(\frac{3}{2}\right) = 8$

Therefore, the missing terms are $\frac{13}{2}$ and 8 respectively.

(iv) $a = -4$

$a_6 = 6$

$a + 5d = 6$

$6 = -4 + 5d$

$10 = 5d$

$d = 2$

$a_2 = a + d = -4 + 2 = -2$

$a_3 = a + 2d = -4 + 2(2) = 0$

$a_4 = a + 3d = -4 + 3(2) = 2$

$a_5 = a + 4d = -4 + 4(2) = 4$

Therefore, the missing terms are –2, 0, 2, and 4 respectively.

(v) $a_2 = 38$

$a_6 = -22$

$38 = a + d$ ....(1)

$-22 = a + 5d$ ....(2)

On subtracting equation (1) from (2), we obtain

$-22 - 38 = 4d$

$-60 = 4d$

$d = -15$

$a = a_2 - d = 38 - (-15) = 53$

$a_3 = a + 2d = 53 + 2(-15) = 23$

$a_4 = a + 3d = 53 + 3(-15) = 8$

$a_5 = a + 4d = 53 + 4(-15) = -7$

Therefore, the missing terms are 53, 23, 8, and –7 respectively.

Q4. Which term of the AP : 3, 8, 13, 18, ... is 78?

Sol. $a = 3, d = 5$

Let $t_n = 78$

$\Rightarrow$ $a + (n - 1)\, d = 78$

$\Rightarrow$ $3 + (n - 1) \times 5 = 78$ $\Rightarrow$ $5n - 2 = 78$

$\Rightarrow$ $5n = 80$ $\Rightarrow$ $n = 16$

Hence, $t_{16} = 78$

Q5. Find the number of terms in each of the following AP's :

(i) 7, 13, 19. ....., 205

(ii) 18, $15\frac{1}{2}$, 13, ..., – 47

Sol. (i) $a = 7, d = 6,$

$t_n = 205$

$\Rightarrow$ $a + (n - 1)\, d = 205$

$\Rightarrow$ $7 + (n - 1) \times 6 = 205$ $\Rightarrow$ $6n + 1 = 205$

$\Rightarrow$ $6n = 204$ $\Rightarrow$ $n = 34$

Hence, 34 terms

(ii) $a = 18$

$d = a_2 - a_1 = 15\frac{1}{2} - 18$

$d = \frac{31 - 36}{2} = -\frac{5}{2}$

Let there are n terms in this A.P.

Therefore, $a_n = -47$ and we know that

$a_n = a + (n - 1)d$

$-47 = 18 + (n - 1)\left(-\frac{5}{2}\right)$

$-47 = 18 + (n-1)\left(-\frac{5}{2}\right)$

$-65 = (n-1)\left(-\frac{5}{2}\right)$

$(n-1) = \frac{-130}{-5}$

$(n-1) = 26$

$n = 27$

Therefore, this given A.P. has 27 terms in it.

Q6. Check whether – 150 is a term of the AP : 11, 8, 5, 2, ...

Sol. $a = 11, d = -3$

Let if possible $t_n = -150$

$\Rightarrow a + (n-1)d = -150$

$\Rightarrow 11 + (n-1) \times (-3) = -150$

$\Rightarrow 11 - 3n + 3 = -150$

$\Rightarrow 14 - 3n = -150$

$\Rightarrow 3n = 14 + 150 = 164$

$\Rightarrow n = \frac{164}{3} = 54\frac{2}{3}$

It is not possible because n is to be natural number.

Hence, – 150 cannot be a term of the AP.

Q7. Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.

Sol. Given that,

$a_{11} = 38$

$a_{16} = 73$

We know that,

$a_n = a + (n - 1)\,d$

$a_{11} = a + (11 - 1)\,d$

$38 = a + 10d \qquad ......(1)$

Similarly,

$a_{16} = a + (16 - 1)\,d$

$73 = a + 15d \qquad ......(2)$

On subtracting (1) from (2), we obtain

$35 = 5d$

$d = 7$

From equation (1),

$38 = a + 10 \times (7)$

$38 - 70 = a$

$a = -32$

$a_{31} = a + (31 - 1)\,d$

$= -32 + 30\,(7)$

$= -32 + 210$

$= 178$

Hence, 31^{st} term is 178.

Q8. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.

Sol. $t_3 = 12$, t_{50} (last term) $= 106$

$\Rightarrow \ a + 2d = 12 \qquad ...(1)$

and $a + 49d = 106 \qquad ...(2)$

Subtracting (1) from (2), we get

$47d = 106 - 12 = 94 \Rightarrow d = 2$

From (1), $a + 2 \times 2 = 12 \qquad \Rightarrow a = 8$

$t_{29} = a + 28d = 8 + 28 \times 2 = 64$

Q9. If the 3rd and 9th terms of an AP are 4 and – 8 respectively, which term of this AP is zero?

Sol. Given that,

$a_3 = 4$

$a_9 = -8$

We know that,

$a_n = a + (n - 1)\,d$

$a_3 = a + (3 - 1)\,d$

$4 = a + 2d \qquad \text{......(I)}$

$a_9 = a + (9 - 1)\,d$

$-8 = a + 8d \qquad \text{......(II)}$

On subtracting equation (I) from (II), we obtain

$-12 = 6d$

$d = -2$

From equation (I), we obtain

$4 = a + 2\,(-2)$

$4 = a - 4$

$a = 8$

Let n^{th} term of this A.P. be zero.

$a_n = a + (n - 1)\,d$

$0 = 8 + (n - 1)\,(-2)$

$0 = 8 - 2n + 2$

$2n = 10$

$n = 5$

Hence, 5^{th} term of this A.P. is 0.

Q10. The 17th term of an AP exceeds its 10th term by 7. Find the common difference.

Sol. $a_{17} - a_{10} = 7$

$(a + 16d) - (a + 9d) = 7$

$7d = 7$

$d = 1$

Therefore, the common difference is 1.

Q11. Which term of the AP : 3, 15, 27, 39, ... will be 132 more than its 54th term?

Sol. $a = 3,\ d = 12$

Let us suppose $t_n = t_{54} + 132$

$\Rightarrow\ a + (n - 1)\,d = a + 53\,d + 132$

$\Rightarrow\ (n - 1)\,d - 53\,d = 132$

$\Rightarrow\ \{n - 1 - 53)\,d = 132$

$\Rightarrow\ (n - 54) \times 12 = 132$

$\Rightarrow\ n - 54 = 11$

$\Rightarrow\ n = 65$

Hence, t_{65} is 132 more than t_{54}.

Q12. Two APs have the same common difference. The difference between their 100 th terms is 100, what is the difference between their 1000th terms?

Sol. Let the two APs with same common difference d be

$a,\ a + d,\ a + 2\,d,\ ....$

$b,\ b + d,\ b + 2d,\ ....\ (a > b)$

We are given that

{100th term of the first AP}

– {100th term of the second AP} = 100

$\Rightarrow$ {a + 99d} – {b + 99d} = 100

$\Rightarrow$ a – b = 100 ...(1)

Now, {1000th term of the first AP)

– {1000th term of the second AP}

= {a + 999 d} – {b + 999 d} = a – b = 100

{By (1)}

Q13. How many three-digit numbers are divisible by 7?

Sol. First three-digit number that is divisible by 7 = 105

Next number = 105 + 7 = 112

Therefore, 105, 112, 119, …

All are three digit numbers which are divisible by 7 and thus, all these are terms of an A.P. having first term as 105 and common difference as 7.

The maximum possible three-digit number is 999. When we divide it by 7, the remainder will be 5. Clearly, 999 - 5 = 994 is the maximum possible three-digit number that is divisible by 7.

The series is as follows.

105, 112, 119, ..., 994

Let 994 be the n^{th} term of this A.P.

a = 105

d = 7

$a_n = 994$

n = ?

$a_n = a + (n - 1)\, d$

994 = 105 + (n – 1) 7

$889 = (n - 1)\ 7$

$(n - 1) = 127$

$n = 128$

Therefore, 128 three-digit numbers are divisible by 7.

Q14. How many multiples of 4 lie between 10 and 250?

Sol. The multiples of 4 between 10 and 250 are 12, 16, 20, 24...., 248.

Let these numbers be n.

$a = 12, d = 4$

$t_n = 248$

$\Rightarrow a + (n - 1)\ d = 248$

$\Rightarrow 12 + (n - 1) \times 4 = 248$

$\Rightarrow 4n + 8 = 248 \Rightarrow n = 60.$

Q15. For what value of n, are the nth terms of two APs 63, 65, 67, ... and 3, 10, 17, ... equal?

Sol. Two APs are 63, 65, 67, ..., 3, 10, 17, ...

From (1), First term = 63 and common difference = 2.

Its nth term $= 63 + (n - 1) \times 2 = 2n + 61.$

From (2), First term = 3 and common difference = 7

Its nth term $= 3 + (n - 1) \times 7 = 7n - 4$

Putting $7n - 4 = 2n + 61$

$\Rightarrow 7n - 2n = 61 + 4 \Rightarrow 5n = 65 \Rightarrow n = 13$

Q16. Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12.

Sol. $a_3 = 16$

$a + (3 - 1)\ d = 16$

$a + 2d = 16$ .... (1)

$a_7 - a_5 = 12$

$[a + (7-1)d] - [a + (5-1)d] = 12$

$(a + 6d) - (a + 4d) = 12$

$2d = 12$

$d = 6$

From equation (1), we obtain

$a + 2(6) = 16$

$a + 12 = 16$

$a = 4$

Therefore, A.P. will be

4, 10, 16, 22, …

Q17. Find the 20th term from the last term of the AP 3, 8, 13, ..., 253.

Sol. The AP is 3, 8, 13, ..., 253

Its first term = 3 and the common difference = 5.

Now, the AP in the reverse order will have the first term = 253 and the common difference = – 5.

The 20th term from the end of the AP (1)

= The 20 term of the AP in the reverse order

$= a + 19d$

$= 253 + 19 \times (-5) = 253 - 95 = 158.$

Q18. The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.

Sol. $t_4 + t_8 = 24$; $t_6 + t_{10} = 44$

$\Rightarrow$ $(a + 3d) + (a + 7d) = 24$;

$(a + 5d) + (a + 9d) = 44$

$\Rightarrow$ $2a + 10d = 24$; $2a + 14d = 44$

We have $a + 5d = 12$ ...(1)
and $a + 7d = 22$ ...(2)
Subtracting (1) from (2), we get
$2d = 10 \Rightarrow d = 5$
From (i) $a + 5 \times 5 = 12$, $a = -13$
$t_1 = -13$, $t_2 = -8$, $t_3 = -3$

Q19. Subba Rao started work in 1995 at an annual salary of Rs. 5000 and received an increment of Rs. 200 each year. In which year did his income reach Rs. 7000 ?

Sol. It can be observed that the incomes that Subba Rao obtained in various years are in A.P. as every year, his salary is increased by Rs. 200.
Therefore, the salaries of each year after 1995 are
5000, 5200, 5400, ...
Here, $a = 5000$
$d = 200$
Let after n^{th} year, his salary be Rs 7000.
Therefore, $a_n = a + (n - 1) d$
$7000 = 5000 + (n - 1) 200$
$200(n - 1) = 2000$
$(n - 1) = 10$
$n = 11$
Therefore, in 11th year, his salary will be Rs 7000.

Q20. Ramkali saved Rs. 5 in the first week of a year and then increased her weekly savings by Rs. 1.75. If in the nth week, her weekly savings become Rs. 20.75, find n.

Sol. t_1 = Rs. 5 (savings in the Ist week)

t_2 = Rs. 5 + Rs. 1.75 = Rs. 6.75

(savings in the 2nd week)

t_3 = Rs. 6.75 + Rs. 1.75 = Rs. 8.50

(savings in the 3rd week)

....

t_n = Rs. 20.75

$\Rightarrow$ $a + (n - 1)\, d = 20.75$

$\Rightarrow$ $5 + (n - 1) \times 1.75 = 20.75$

$\Rightarrow$ $(n - 1) \times 1.75 = 15.75$

$\Rightarrow$ $n - 1 = \frac{15.75}{1.75} = \frac{1575}{175} = 9 \Rightarrow n = 10$

Hence, in the 10th week, Ramkali's savings will be Rs. 20.75