Class X : MATH
Chapter 5 : Arithmetic Progressions
Questions & Answers - Exercise : 5.4 - NCERT Book

Q1. Which term of the AP: is its first negative term? [Hint : Find n for an < 0]

Sol. Given AP is

121, 117, 113, ...,

Here a = 121 and d = -4

let suppose nth term of the AP is first negative team

Then,

$a_n = a + (n - 1)d$

If nth term is negative then $a_n < 0$

$\Rightarrow 121 + (n-1)(-4) < 0$

$\Rightarrow 125 < 4n$

$\Rightarrow n > \frac{125}{4} = 31.25$

Therefore, first negative term must be 32nd term

Q2. The sum of the third and the seventh terms of an AP is 6 and their product is 8. find the sum of first sixteen terms of the AP.

Sol. It is given that sum of third and seventh terms of an AP are and their product is 8

$a_3 = a + 2d$

$a_7 = a + 6d$

Now,

$a_3 + a_7 = a + 2d + a + 6d = 6$

$\Rightarrow 2a + 8d = 6$

$\Rightarrow a + 4d = 3 \Rightarrow a = 3 - 4d \quad ...(i)$

And

$a_3 \cdot a_7 = (a+2d)\cdot(a+6d) = a^2 + 8ad + 12d^2 = 8$ ......(ii)

put value from equation (i) in (ii) we will get

$\Rightarrow (3-4d)^2 + 8(3-4d)d + 12d^2 = 8$

$\Rightarrow 9 + 16d^2 - 24d + 24d - 32d^2 + 12d^2 = 8$

$\Rightarrow 4d^2 = 1$

$\Rightarrow d = \pm\frac{1}{2}$

Now,

Case (i) $d = \frac{1}{2}$

$a = 3 - 4\times\frac{1}{2} = 1$

Then,

$S_{16} = \frac{10}{2}\left\{2\times1 + (16-1)\frac{1}{2}\right\}$

$S_{16} = 76$

Case (ii) $d = -\frac{1}{2}$

$a = 3 - 4\times\left(-\frac{1}{2}\right) = 5$

Then,

$S_{16} = \frac{16}{2}\left\{2\times1 + (16-1)\left(-\frac{1}{2}\right)\right\}$

$s_{16} = 20$

Q3. A ladder has rungs \small 25 cm apart. (see Fig. 5.7). The rungs decrease uniformly in length from 45 cm at the bottom to \small 25 cm at the top. If the top and the bottom rungs are $2\frac{1}{2}$ m apart, what is the length of the wood required for the rungs? [Hint: Number of rungs $=\frac{250}{25}+1$]

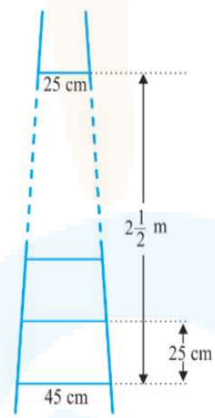

Fig. 5.7

It is given that
The total distance between the top and bottom rung

$$=2\frac{1}{2}\text{m}=250\,\text{cm}$$

Distance between any two rungs = 25 cm

Total number of rungs $=\frac{200}{25}+1=11$

And it is also given that bottom-most rungs is of 45 cm length and topmost is of 25 cm length. As it is given that the length of rungs decrease uniformly, it will form an AP with

$a=25, a_{11}=45$ and $n=11$

Now, we know that

$a_{11} = a + 10d$

$\Rightarrow 45 = 25 + 10d$

$\Rightarrow d = 2$

Now, total length of the wood required for the rungs is equal to

$$S_{11} = \frac{11}{2}\{2 \times 25 + (11-1)2\}$$

$$S_{11} = \frac{11}{2}\{50 + 20\}$$

$$S_{11} = \frac{11}{2} \times 70$$

$S_{11} = 385\,\text{cm}$

Therefore, the total length of the wood required for the rungs is equal to 385 cm

Q4. The houses of a row are numbered consecutively from 1 to 49 . Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x.

[**Hint :** $S_{x-1} = S_{49} - S_x$]

Sol. It is given that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it

And 1, 2, 3, ..., 49 form an AP with **a = 1 and d = 1**

Now, we know that

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

Suppose their exist an n term such that (n < 49)

Now, according to given conditions

Sum of first n - 1 terms of AP = Sum of terms following the nth term

Sum of first n - 1 term of AP = Sum of whole AP - Sum of first m terms of AP

i.e.

$S_{n-1} = S_{49} - S_n$

$$\frac{n-1}{2}\{2a+((n-1)-1)d\} = \frac{49}{2}\{2a+(49-1)d\} - \frac{n}{2}\{2a+(n-1)d\}$$

$$\frac{n-1}{2}\{n\} = \frac{49}{2}\{50\} - \frac{n}{2}\{n+1\}$$

$$\frac{n^2}{2} - \frac{n}{2} = 1225 - \frac{n^2}{2} - \frac{n}{2}$$

$$n^2 = 1225$$

$$n = \pm 35$$

Given House number are not negative so we reject n = -35
Therefore, the sum of no of houses preceding the house no 35 is equal to the sum of no of houses following the house no 35

Q5. A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete. Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m. (see figh. 5.8). Calculate the total volume of concrete required to build the terrace.
[**Hint :** Volume of concrete required to build the first step

$$= \frac{1}{4} \times \frac{1}{2} \times 50 m^3]$$

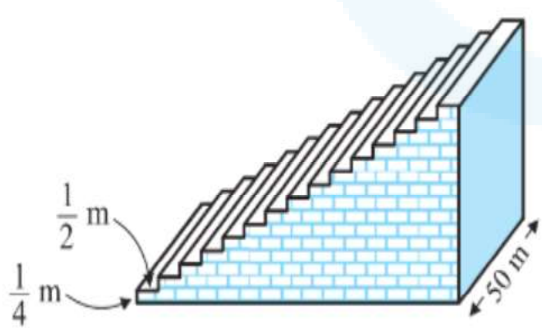

Fig. 5.8

Sol. It is given that

football ground comprises of \small 15 steps each of which is 50 m long

and Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m

Now,

The volume required to make the first step = $\frac{1}{4}\times\frac{1}{2}\times 50 = 6.25\text{m}^3$

similarly,

The volume required to make 2nd step =

$$\left(\frac{1}{4}+\frac{1}{4}\right)\times\frac{1}{2}\times 50 = \frac{1}{2}\times\frac{1}{2}\times 50 = 12.5\text{m}^3$$

And

the volume required to make 3rd step =

$$\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}\right)\times\frac{1}{2}\times 50 = \frac{3}{4}\times\frac{1}{2}\times 50 = 18.75\,\text{m}^3$$

And so on

We can clearly see that this is an AP with a = 6.25 and d = 6.25

Now, the total volume of concrete required to build the terrace of 15 such step is

$$S_{15} = \frac{15}{2}\{2\times 6.25 + (15-1)6.25\}$$

$$S_{15} = \frac{10}{2}\{12.5 + 87.5\}$$

$$S_{15} = \frac{10}{2}\times 100$$

$$S_{15} = 15\times 50 = 750$$

Therefore, the total volume of concrete required to build the terrace of 15 such steps is 750m^3