

Class X : MATH
Chapter - 8 : Introduction To Trigonometry
Questions and Answers | Exercise - 8.2 | NCERT Book

Q1. Evaluate :

(i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

(ii) $2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

(iii) $\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

(iv) $\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

(v) $\frac{5 \cos^2 60^\circ + 4 \sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

Sol. (i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$= \frac{3}{4} + \frac{1}{4} = 1$$

(ii) $2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

$$= 2 \times (1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 2 + \frac{3}{4} - \frac{3}{4} = 2$$

(iii) $\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

$$= \frac{1/\sqrt{2}}{\frac{2}{\sqrt{3}} + 2} = \frac{1/\sqrt{2}}{2\left(\frac{1+\sqrt{3}}{\sqrt{3}}\right)} = \frac{1(\sqrt{3})}{2\sqrt{2}(1+\sqrt{3})}$$

$$= \frac{\sqrt{3}}{2(\sqrt{2})} \times \frac{(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{\sqrt{3}(\sqrt{3}-1)}{2\sqrt{2} \times 2}$$

$$= \frac{(3-\sqrt{3})}{4\sqrt{2}}$$

$$(iv) \frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ - \cos 60^\circ + \cot 45^\circ}$$

$$\frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1} = \frac{\frac{\sqrt{3} + 2\sqrt{3} - 4}{2\sqrt{3}}}{\frac{4 + \sqrt{3} + 2\sqrt{3}}{2\sqrt{3}}}$$

$$= \frac{(3\sqrt{3} - 4)(4 - 3\sqrt{3})}{(4 + 3\sqrt{3})(4 - 3\sqrt{3})}$$

$$= \frac{12\sqrt{3} - 27 - 16 + 12\sqrt{3}}{16 - 9 \times 3} = \frac{24\sqrt{3} - 43}{-11} = \frac{43 - 24\sqrt{3}}{11}$$

$$(v) \frac{5 \cos^2 60^\circ + 4 \sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$$

$$= \frac{5(\cos 60^\circ)^2 + 4(\sec 30^\circ)^2 - (\tan 45^\circ)^2}{(\sin 30^\circ)^2 + (\cos 30^\circ)^2}$$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{\frac{5}{4} + 4 \times \frac{4}{3} - 1}{\frac{1}{4} + \frac{3}{4}}$$

$$= \frac{\frac{5}{4} + \frac{16}{3} - 1}{\frac{1}{4} + \frac{3}{4}} = \frac{5}{4} + \frac{16}{3} - 1$$

$$= \frac{15 + 64 - 12}{12} = \frac{67}{12}$$

Q2. Choose the correct option and justify your choice:

- (i) $\frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} =$
 (A) $\sin 60^\circ$ (B) $\cos 60^\circ$ (C) $\tan 60^\circ$ (D) $\sin 30^\circ$
- (ii) $\frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} =$
 (A) $\tan 90^\circ$ (B) 1 (C) $\sin 45^\circ$ (D) 0
- (iii) $\sin 2A = 2 \sin A$ is true when $A =$
 (A) 0° (B) 30° (C) 45° (D) 60°
- (iv) $\frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} =$
 (A) $\cos 60^\circ$ (B) $\sin 60^\circ$ (C) $\tan 60^\circ$ (D) $\sin 30^\circ$

Sol. (i) Option (A) is correct.

$$\frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} = \frac{2 \left(\frac{1}{\sqrt{3}} \right)}{1 + \left(\frac{1}{\sqrt{3}} \right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{\sqrt{3}}{2} = \sin 60^\circ$$

(ii) Option (D) is correct.

$$\frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} = \frac{1 - 1}{1 + 1} = 0$$

(iii) Option (A) is correct.

$$\begin{aligned} \sin 2A &= 2 \sin A \\ \Rightarrow 2 \sin A \cdot \cos A &= 2 \sin A \\ \Rightarrow \cos A &= 1 \\ \Rightarrow A &= 0^\circ \end{aligned}$$

(iv) Option (C) is correct.

$$\frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$$

$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$$= \tan 60^\circ$$

Q3. If $\tan (A + B) = \sqrt{3}$ and $\tan (A - B) = \frac{1}{\sqrt{3}}$;

$0^\circ < A + B \leq 90^\circ$; $A > B$, find A and B.

Sol. $\tan (A + B) = \sqrt{3} \Rightarrow A + B = 60^\circ \dots(1)$

$\tan (A - B) = \frac{1}{\sqrt{3}} \Rightarrow A - B = 30^\circ \dots(2)$

Adding (1) and (2),

$$2A = 90^\circ \Rightarrow A = 45^\circ$$

Then from (1), $45^\circ + B = 60^\circ \Rightarrow B = 15^\circ$

Q4. State whether the following are true or false. Justify your answer.

- (i) $\sin (A + B) = \sin A + \sin B$
- (ii) The value of $\sin \theta$ increases as θ increases.
- (iii) The value of $\cos \theta$ increases as θ increases.
- (iv) $\sin \theta = \cos \theta$ for all values of θ .
- (v) $\cot A$ is not defined for $A = 0^\circ$.

Sol. (i) False.

When $A = 60^\circ$, $B = 30^\circ$

$$\begin{aligned} \text{LHS} &= \sin (A + B) = \sin (60^\circ + 30^\circ) \\ &= \sin 90^\circ = 1 \end{aligned}$$

$$\begin{aligned} \text{RHS} &= \sin A + \sin B \\ &= \sin 60^\circ + \sin 30^\circ \\ &= \frac{\sqrt{3}}{2} + \frac{1}{2} \neq 1 \end{aligned}$$

i.e., LHS \neq RHS

(ii) True.

Note that $\sin 0^\circ = 0$, $\sin 30^\circ = \frac{1}{2} = 0.5$,

$\sin 45^\circ = \frac{1}{\sqrt{2}} = 0.7$ (approx.),

$\sin 60^\circ = \frac{\sqrt{3}}{2} = 0.87$ (approx.)

and $\sin 90^\circ = 1$

i.e., value of $\sin \theta$ increases as θ increases from 0° to 90° .

(iii) False.

Note that $\cos 0^\circ = 1$,

$\cos 30^\circ = \frac{\sqrt{3}}{2} = 0.87$ (approx.)

$\cos 45^\circ = \frac{1}{\sqrt{2}} = 0.7$ (approx.),

$\cos 60^\circ = \frac{1}{2} = 0.5$ and $\cos 90^\circ = 0$

i.e., value of $\cos \theta$ decreases as θ increases from 0° to 90° .

(iv) False, it is true for only $\theta = 45^\circ$

(v) True, $\cot A = \frac{1}{0} = \text{not defined}$.