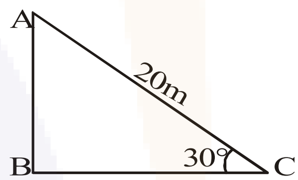
Class X : MATH
Chapter - 9 : Some Applications of Trignometry
Questions & Answers - Ex : 9.1 - NCERT Book

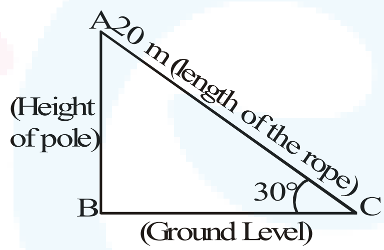
Q1. A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole if the angle made by the rope with the ground level is 30° (see fig.).



Sol. AC = 20 m is the length of the rope.

Let AB = h metres be the height of the pole

$\angle ACB = 30°$ (Given)

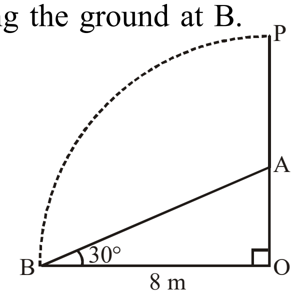


Now, $\frac{AB}{AC} = \sin 30° = \frac{1}{2} \quad \Rightarrow \quad \frac{h}{20} = \frac{1}{2} \Rightarrow h = 10$ m

Q2. A tree breaks due to storm and the broken part bends so that the top of the trees touches the ground making an angle of 30° with the ground. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

Sol. Let tree is broken at A and its top is touching the ground at B.

Now, in right ΔAOB, we have



$\frac{AO}{OB} = \tan 30°$

$\Rightarrow \frac{AO}{8} = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{AO}{8} = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad AO = \frac{8}{\sqrt{3}}$ m

Also, $\frac{AB}{OB} = \sec 30°$

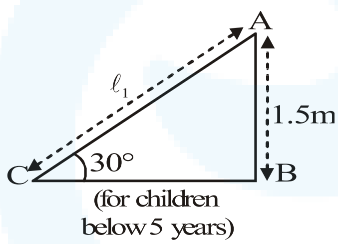
$\Rightarrow \frac{AB}{8} = \frac{2}{\sqrt{3}} \quad \Rightarrow \quad AB = \frac{2 \times 8}{\sqrt{3}} = \frac{16}{\sqrt{3}}$ m

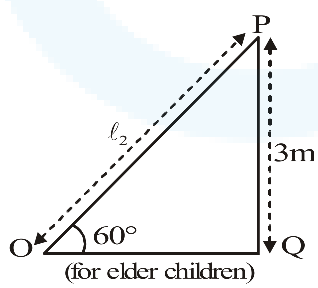
Now, height of the tree OP = OA + AB

$= \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}} = \frac{24}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \text{m} = 8\sqrt{3}\text{m}$

Q3. A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she perfers to have a slide whose top is at a height of 1.5 m and is inclinded at an angle of 30° to the ground, whereas for elder children, she wants to have a steep slide at a height of 3 m, and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?

Sol. In figure, ℓ_1 is the length of the slide made for children below the age of 5 years and ℓ_2 is the length of the slide made for elder children.





In figure, AB = 1.5 m, AC = ℓ_1 m and ∠ACB = 30°; PQ = 3 m, OP = ℓ_2 m and ∠POQ = 60°

$\frac{AB}{AC} = \sin 30°$ and $\frac{PQ}{OP} = \sin 60°$

$\Rightarrow \frac{1.5}{\ell_1} = \frac{1}{2}$ and $\frac{3}{\ell_2} = \frac{\sqrt{3}}{2}$

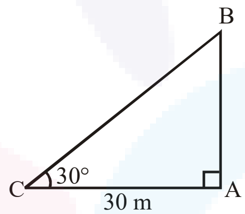
$\Rightarrow \ell_1 = 2 \times 1.5$ m and $\ell_2 = \frac{3 \times 2}{\sqrt{3}}$ m

$\Rightarrow \ell_1 = 3$ m and $\ell_2 = 2\sqrt{3}$ m

Q4. The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower.

Sol. In right ABC, AB = height of the tower and point C is 30m away from the foot of the tower,

$\therefore$ AC = 30 m



Now $\frac{AB}{AC} = \tan 30°$

$\Rightarrow \frac{h}{30} = \frac{1}{\sqrt{3}}$

[$\because \tan 30° = \frac{1}{\sqrt{3}}$]

$\Rightarrow h = \frac{30}{\sqrt{3}} = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 10\sqrt{3}$

Thus, the required height of the tower is $10\sqrt{3}$ m

Q5. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. find the length of the string, assuming that there is no slack in the string.

Sol. P is the position of the kite. Its height from the point Q (on the ground) = PQ = 60 m

Let OP = ℓ be the length of the string.

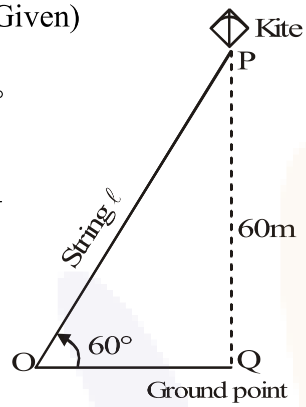
$\angle POQ = 60°$ (Given)

Now, $\frac{PQ}{OP} = \sin 60°$

$\Rightarrow \frac{60}{\ell} = \sin 60° = \frac{\sqrt{3}}{2}$

$\Rightarrow \frac{60}{\ell} = \frac{\sqrt{3}}{2}$

$\Rightarrow \ell = 40\sqrt{3}$m



Q6. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

Sol. PQ = 30 m is the height of the building. OA = 1.5 m is the height of the boy. Its first position is at OA OR is horizontal line through the position of the eye at O.

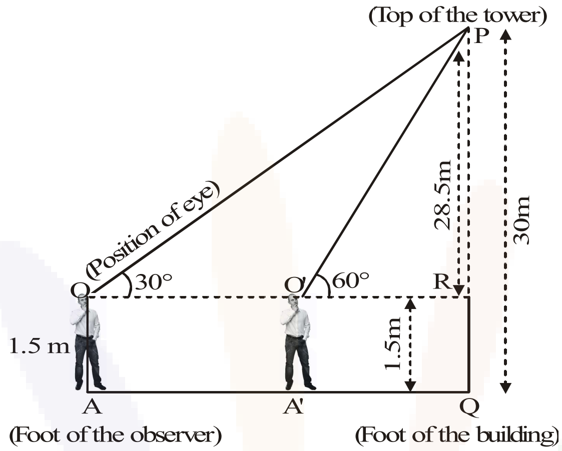
$\angle POR = 30°$ (Given)

The second position of the boy is at O'A' and

$\angle PO'R = 60°$.

Here, RQ = OA = 1.5 m

and PR = 30 m – 1.5 m = 28.5 m



From ΔPOR, From ΔPO'R,

$\frac{PR}{OR} = \tan 30°$ $\qquad$ $\frac{PR}{O'R} = \tan 60°$

$\Rightarrow \frac{28.5}{OR} = \frac{1}{\sqrt{3}}$ $\qquad$ $\frac{28.5}{O'R} = \sqrt{3}$

$\Rightarrow$ OR = $28.5 \times \sqrt{3}$ m ..(1) $\Rightarrow$ O'R$=\frac{28.5}{\sqrt{3}}$ m ...(2)

The distance walked by the boy towards the building.

= OO' = OR – O'R

$= 28.5 \times \sqrt{3}\text{ m} - \frac{28.5}{\sqrt{3}}\text{ m}$ [From (1) and (2)]

$= 28.5 \times \left\{\sqrt{3} - \frac{1}{\sqrt{3}}\right\}\text{ m}$

$= 28.5 \times \frac{(3-1)}{\sqrt{3}} m = 28.5 \times \frac{2}{\sqrt{3}} m$

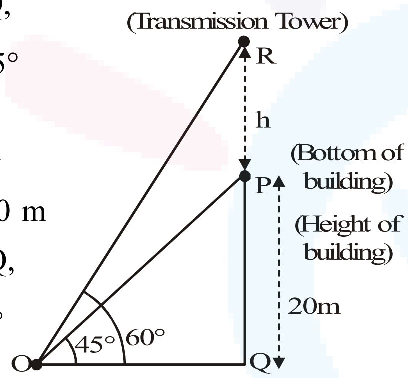
$= \frac{57}{\sqrt{3}} m = 19\sqrt{3}\ m$

Q7. From a point on the ground the angles of elevation of the bottom and top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

Sol. PQ = 20 m is the height of the building.

Let PR = h metres be the height of the transmission tower. P is the bottom and R is the top of the transmission tower.

$\angle POQ = 45°$ and $\angle ROQ = 60°$



From ΔOPQ,

$\frac{PQ}{OQ} = \tan 45°$

$\Rightarrow \frac{20}{OQ} = 1$

$\Rightarrow OQ = 20$ m

From ΔORQ,

$\frac{RQ}{OQ} = \tan 60°$

$\Rightarrow \frac{20+h}{20} = \sqrt{3}$

($\because$ RQ = PQ + PR = 20 + h metres and OQ = 20 metres)

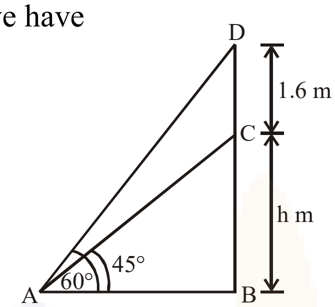
$\Rightarrow 1 + \frac{h}{20} = \sqrt{3} \Rightarrow \frac{h}{20} = (\sqrt{3}-1)$

$\Rightarrow h = 20(\sqrt{3}-1)$ m

Q8. A statue, 1.6 m tall, stands on the top of pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

Sol. In the figure, DC represents the statue and BC represents the pedestal.

Now, in right ΔABC, we have



$$\frac{AB}{BC} = \cot 45° = 1$$

$$\Rightarrow \frac{AB}{h} = 1$$

$\Rightarrow$ AB = h metres.

Now in right ΔABD, we have

$$\frac{BD}{AB} = \tan 60° = \sqrt{3}$$

$$\Rightarrow BD = \sqrt{3} \times AB = \sqrt{3} \times h$$

$$\Rightarrow h + 1.6 = \sqrt{3}\, h$$

$$\Rightarrow h(\sqrt{3} - 1) = 1.6$$

$$\Rightarrow h = \frac{1.6}{\sqrt{3}-1} = \frac{1.6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

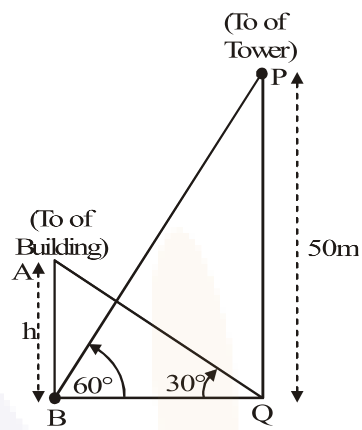
$$\Rightarrow h = \frac{1.6}{3-1} \times (\sqrt{3}+1) = \frac{1.6}{2} \times \left(\sqrt{3}+1\right)$$

$$= 0.8(\sqrt{3} + 1)\text{m}$$

Thus, the height of the pedestal is $0.8(\sqrt{3} + 1)$m.

Q9. The angle of elevation of the top of the building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

Sol. PQ = 50 metres is the height of the tower. Let AB = h metres be the height of the building. Angle of elevation of the top of the building from the foot of the tower = 30°, i.e., ∠AQB = 30°.



Angle of elevation of the top of the tower from the foot of the building

= 60,i.e.,∠PBQ = 60

From ΔAQB

$$\frac{h}{BQ} = \tan 30° = \frac{1}{\sqrt{3}}$$

$\Rightarrow$ BQ = $h\sqrt{3}$ ...(1)

From ΔPBQ

$$\frac{50}{BQ} = \tan 60° = \sqrt{3}$$

$\Rightarrow$ BQ = $\frac{50}{\sqrt{3}}$ ...(2)

From (1) and (2), we have $h\sqrt{3} = \frac{50}{\sqrt{3}}$

$\Rightarrow$ h = $\frac{50}{3}$m, i.e., h = $16\frac{2}{3}$m

Q10. Two poles of equal heights are standing opposite to each other on either side of the road which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles and the distances of the point from the poles.

Sol. Let AB and CD be the towels & P is the point between them.

AB = h metres

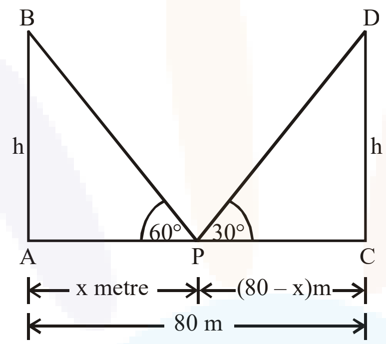
CD = h metres

AP = x m

CP = (80 – x) m

Now, in right ΔAPB, We have

$$\frac{AB}{AP} = \tan 60° \quad \Rightarrow \quad \frac{h}{x} = \sqrt{3}$$



$$\Rightarrow \quad h = x\sqrt{3} \qquad ....(1)$$

Again in right ΔCPD, we have

$$\frac{CD}{CP} = \tan 30°$$

$$\Rightarrow \quad \frac{h}{(80-x)} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad h = \frac{80-x}{\sqrt{3}} \qquad ....(2)$$

From (1) and (2), we get

$$\sqrt{3}x = \frac{80-x}{\sqrt{3}}$$

$$\Rightarrow \quad \sqrt{3} \times \sqrt{3} \times x = 80 - x \Rightarrow 3x = 80 - x$$

$$\Rightarrow \quad 3x + x = 80 \Rightarrow 4x = 80 \quad \Rightarrow x = \frac{80}{4} = 20$$

$\therefore$ CP = 80 – x = 80 – 20 = 60 m

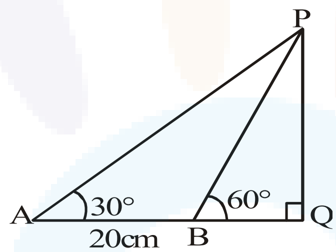
Now, from (1), we have

$h = \sqrt{3} \times 20 = 1.732 \times 20 = 34.64$

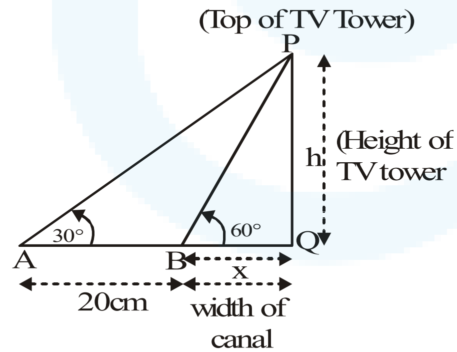
Thus, the required point is 20 m away from the first pole and 60 m away from the second pole.

Height of each pole = 34.64 m.

Q11. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30° (see fig.). Find the height of the tower and the width of the canal.



Sol. Let PQ = h metres be the height of the tower and BQ = x metres be the width of the canal $\angle PBQ = 60°$



Now, the angle of elevation of the top of the tower from the point A = 30°,

i.e., $\angle PAQ = 30°$ where AB = 20 metres.

From ΔPBQ,

$\frac{h}{x} = \tan 60°$

$\Rightarrow \frac{h}{x} = \sqrt{3}$ or $h = x\sqrt{3}$ ...(1)

From ΔPAQ,

$\frac{h}{20+x} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow h = \frac{20+x}{\sqrt{3}}$ ...(2)

From (1) and (2), we have $x\sqrt{3} = \frac{20+x}{\sqrt{3}}$

$\Rightarrow$ 3x = 20 + x or 2x = 20 $\Rightarrow$ x = 10

From (1), h = $10\sqrt{3}$ m

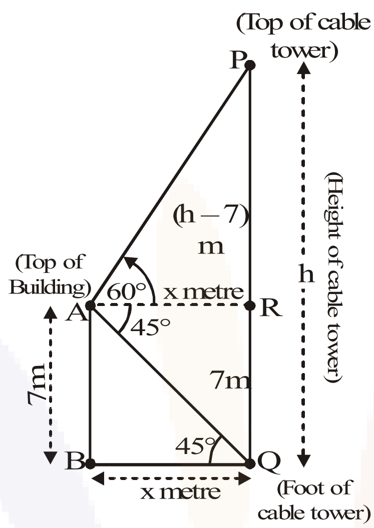
Q12. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Sol. Let PQ = h metres be the height of the cable tower.

AB = 7 metres is the height of the bulding

$\angle$PAR = 60° is the angle of elevation of the top of the cable tower from the top of the building.

$\angle$RAQ = 45° is the angle of depression of the foot of the cable tower from the top of the building. Then $\angle$AQB = 45°.



Now, BQ = AR = x metres (say)

From ΔAQB, $\frac{AB}{BQ} = \tan 45° \Rightarrow \frac{7}{x} = 1 \Rightarrow x = 7$ m

Now, from ΔPAR, $\frac{PR}{AR} = \tan 60° \Rightarrow \frac{PQ - QR}{x} = \sqrt{3}$

$\Rightarrow \frac{h-7}{x} = \sqrt{3} \Rightarrow \frac{h-7}{7} = \sqrt{3} \Rightarrow h = 7(\sqrt{3}+1)$

Hence, the height of the cable tower is $7(\sqrt{3}+1)$ metre.

Q13. As observed from the top of a 75m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

Sol. In the figure, let AB represent the light house.

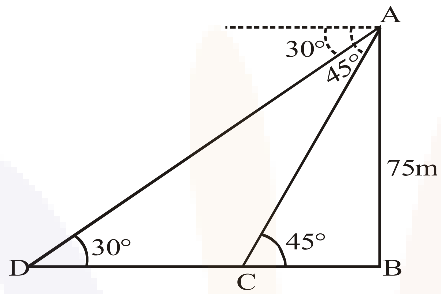
∴ AB = 75 m

Let the two ships be C and D such that angle of depression from A are 45° and 30°

respectively.

Now, in right ΔABC, we have

$\frac{AB}{BC} = \tan 45°$



$\Rightarrow \frac{75}{BC} = 1 \Rightarrow BC = 75$

Again, in right ΔABD, We have

$\frac{AB}{BD} = \tan 30° \quad \Rightarrow \frac{75}{BD} = \frac{1}{\sqrt{3}}$

$\Rightarrow BD = 75\sqrt{3}$

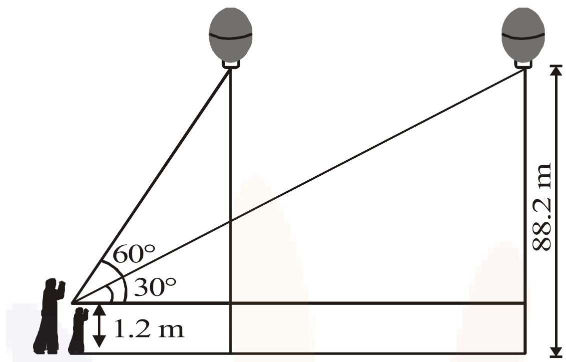
Since the distance between the two ships

$= CD = BD - BC = 75\sqrt{3} - 75 = 75[\sqrt{3} - 1]$

$= 75[1.732 - 1] = 75 \times 0.732 = 54.9$

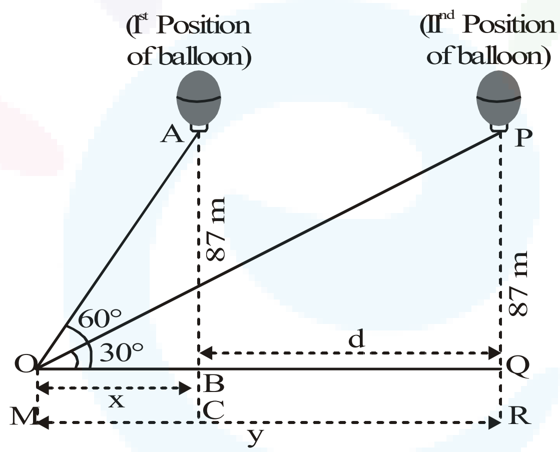
Thus, the required distance between the ships is 54.9 m.

Q14. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30°. Find the distance travelled by the balloon during the interval.



Sol. From figure, we have ∠POQ = 30° is the angle of elevation for the first position of the balloon. Let

OQ = y m.



We are given that

AC = 88.2 m., AB = 88.2 – 1.2 = 87 m

For the second position of the balloon, we have

∠POQ = 30°. Let OB = x m.

We have to find d = BQ = (y – x)

$$\frac{AB}{OB} = \tan 60° \text{ and } \frac{PQ}{OQ} = \tan 30°$$

$\Rightarrow \frac{87}{x} = \sqrt{3}$ and $\frac{87}{y} = \frac{1}{\sqrt{3}}$

$\Rightarrow x = \frac{87}{\sqrt{3}} m$ and $y = 87\sqrt{3}$ m

Then $d = y - x = \left\{87\sqrt{3} - \frac{87}{\sqrt{3}}\right\} m$

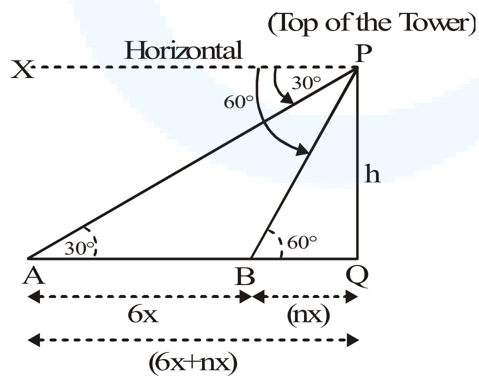
$= 87 \times \left\{\sqrt{3} - \frac{1}{\sqrt{3}}\right\} m$

$= 87 \times \frac{2}{\sqrt{3}} m = 87 \times \frac{2}{3} \times \sqrt{3}$ m

$= \frac{174}{3}\sqrt{3} m = 58\sqrt{3}$ m

Q15. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower.

Sol. Let PQ = h metres be the height of the tower. P is the top of the tower. PX is horizontal line through P. The first and second positions of the car are at A and B respectively.



$\angle APX = 30°$
(Angle of depression of the car when observed at A)
and $\angle BPX = 60°$
(Angle of depression of the car when observed at B)
Then $\angle PAQ = 30°$ and $\angle PBQ = 60°$
Let the speed of the car be x m/second
Then distance AB = 6 x metres.
Let the time taken from B to Q be n second.
Then distance BQ = nx metres.
In right ΔPAQ,

In right ΔPAQ	In right ΔPBQ
$\frac{h}{6x+nx} = \tan 30° = \frac{1}{\sqrt{3}}$	$\frac{h}{nx} = \tan 60° = \sqrt{3}$
$\Rightarrow h = \frac{(n+6)x}{\sqrt{3}} \quad \ldots(1)$	$\Rightarrow h = nx\sqrt{3} \quad \ldots(2)$

From (1) and (2), we have

$$\frac{(n+6)x}{\sqrt{3}} = nx\sqrt{3}$$

$\Rightarrow$ $n + 6 = n\sqrt{3} \times \sqrt{3}$
$\Rightarrow$ $3n = n + 6$
$\Rightarrow$ $2n = 6$
$\Rightarrow$ $n = 3$
Hence, the time from B to Q = 3 seconds.