Class X : MATH

Chapter 11 : Areas Related To Circles

Questions & Ansswers - Exercise : 11.1 - NCERT Book

Q1. Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°.

Sol. Radius, r = 6 cm; sector angle, θ = 60 degrees

Area of the sector

$$= \frac{\theta}{360} \times \pi r^2 = \frac{60}{360} \times \frac{22}{7} \times (6)^2 \text{ cm}^2$$

$$= \frac{1}{6} \times \frac{22}{7} \times (6)^2 \text{ cm}^2 = \frac{132}{7} \text{cm}^2$$

Q2. Find the area of a quadrant of a circle whose circumference is 22 cm.

Sol. Let radius of the circle = r

$\therefore \quad 2\pi r = 22$

$$\Rightarrow 2 \times \frac{22}{7} \times r = 22$$

$$\Rightarrow r = 22 \times \frac{7}{22} \times \frac{1}{2} = \frac{7}{2} \text{ cm}$$

Here, θ = 90°

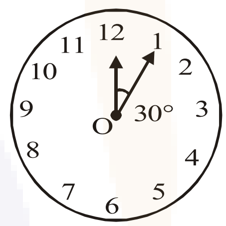
$\therefore$ Area of the $\left(\frac{1}{4}\right)^{th}$ quadrant of the circle,

$$= \frac{\theta}{360} \times \pi r^2 = \frac{90°}{360°} \times \frac{22}{7}\left(\frac{7}{2}\right)^2 \text{ cm}^2$$

$=\frac{1}{4}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\,cm^2=\frac{77}{8}\,cm^2$

Q3. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

Sol. We know that in 1 hour (i.e., 60 minutes), the minute hand rotates 360°.



In 5 minutes, minute hand will rotate

$=\frac{360°}{60}\times 5 = 30°$

Therefore, the area swept by the minute hand in 5 minutes will be the area of a sector of 30° in a circle of 14 cm radius.

Area of sector of angle $\theta = \frac{\theta}{360°}\times \pi r^2$

Area of sector of 30° $=\frac{30°}{360°}\times\frac{22}{7}\times 14\times 14$

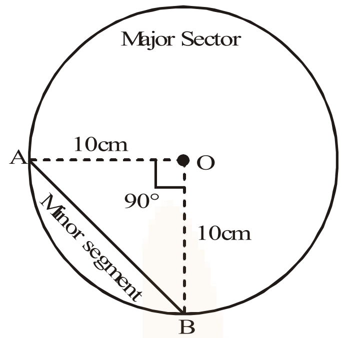
$=\frac{22}{12}\times 2\times 14=\frac{11\times 14}{3}=\frac{154}{3}\,cm^2$

Therefore, the area swept by the minute hand in 5 minutes is $\frac{154}{3}\,cm^2$

Q4. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding: (i) minor segment (ii) major sector. (Use $\pi = 3.14$)

Sol. Here, the radius of the circle is r = 10 cm.

Sector angle of the minor sector made corresponding to the chord AB is 90°



Now, the area of the minor sector $= \frac{90}{360} \times \pi r^2$

$= \frac{1}{4} \times \pi \times (10)^2\ cm^2 = \frac{1}{4} \times 3.14 \times 100\ cm^2$

$= \frac{314}{4} cm^2 = 78.5\ cm^2$

Then, the area of the minor segment

= The area of the minor sector

– The area of the ΔOAB

$= 78.5\ cm^2 - \frac{1}{2} \times OA \times OB\ (\because\ \angle AOB = 90°)$

$= 78.5\ cm^2 - \frac{1}{2} \times 10 \times 10\ cm^2$

$= (78.5 - 50)\ cm^2 = 28.5\ cm^2$

The area of the major sector

$= \left(\frac{360-90}{360}\right) \times \pi r^2 = \frac{270}{360} \times 3.14 \times (10)^2\ cm^2$

$= \frac{3}{4} \times 314\ cm^2 = \frac{3 \times 157}{2} cm^2 = 235.5\ cm^2$

Q5. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:

(i) the length of the arc

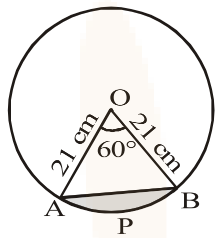
(ii) area of the sector formed by the arc

(iii) area of the segment formed by the corresponding chord

Sol. Here, radius = 21 cm and $\theta = 60°$

(i) Circumference of the circle = $2\pi r$

$$= 2 \times \frac{22}{7} \times 21 \text{ cm} = 2 \times 22 \times 3 \text{ cm} = 132 \text{ cm}$$



$\therefore$ Length of arc APB

$$= \frac{\theta}{360°} \times 2\pi r = \frac{60°}{360°} \times 132 \text{ cm}$$

$$= \frac{1}{6} \times 132 \text{ cm} = 22 \text{ cm}$$

(ii) Area of the sector with sector angle 60°

$$= \frac{60°}{360°} \times \pi r^2 = \frac{60°}{360°} \times \frac{22}{7} \times 21 \times 21 \text{ cm}^2$$

$$= 11 \times 21 \text{ cm}^2 = 231 \text{ cm}^2$$

(iii) Area of the segment APB = [Area of the sector AOB] – [Area of ΔAOB] .....(1)

In ΔAOB, OA = OB = 21 cm

$\therefore$ $\angle A = \angle B = 60°$ $\quad$ [$\because \angle O = 60°$]

$\Rightarrow$ AOB is an equilateral Δ.

$\therefore$ AB = 21 cm

$\therefore$ area of $\Delta AOB = \frac{\sqrt{3}}{4}(\text{side})^2$

$= \frac{\sqrt{3}}{4} \times 21 \times 21 \text{ cm}^2 = \frac{441\sqrt{3}}{4} \text{ cm}^2$ ...(2)

From (1) and (2), we have

Area of segment $= [231 \text{ cm}^2] - \left[\frac{441\sqrt{3}}{4} \text{cm}^2\right] = \left(231 - \frac{441\sqrt{3}}{4}\right) \text{cm}^2$

Q6. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

Sol. Here, radius (r) = 15 cm and

Sector angle (θ) = 60°

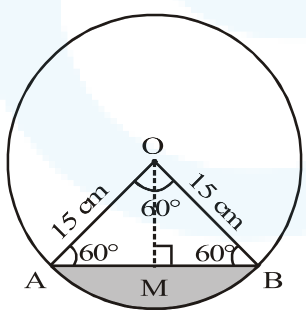
$\therefore$ Area of the sector

$= \frac{\theta}{360°} \times \pi r^2 = \frac{60°}{360°} \times \frac{314}{100} \times 15 \times 15 \text{ cm}^2$

$= \frac{11775}{100} \text{ cm}^2 = 117.75 \text{ cm}^2$

Since $\angle O = 60°$ and OA = OB = 15 cm

$\therefore$ AOB is an equilateral triangle.



$\Rightarrow$ AB = 15 cm and $\angle A = 60°$

Draw OM $\perp$ AB, in ΔAMO

$\therefore \frac{OM}{OA} = \sin 60° = \frac{\sqrt{3}}{2}$

$\Rightarrow$ OM = OA $\times \frac{\sqrt{3}}{2} = \frac{15\sqrt{3}}{2}$ cm

Now, ar(ΔAOB) = $\frac{1}{2} \times$ AB $\times$ OM

$= \frac{1}{2} \times 15 \times 15\frac{\sqrt{3}}{2}$ cm^2 = $\frac{225\sqrt{3}}{4}$ cm^2

$= \frac{225 \times 1.73}{4}$ cm^2 = 97.3125

Now area of the minor segment
= (Area of minor sector) – (ar ΔAOB)
= (117.75 – 97.3125) cm^2 = 20.4375 cm^2
Area of the major segment
= [Area of the circle] – [Area of the minor segment]

$= \pi r^2 - 20.4375$ cm^2 $= \left[\frac{314}{100} \times 15^2\right] - 20.4375$ cm^2 = 706.5 – 20.4375 cm^2 = 686.0625 cm^2

Q7. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. (Use π = 3.14 and $\sqrt{3}$ = 1.73)

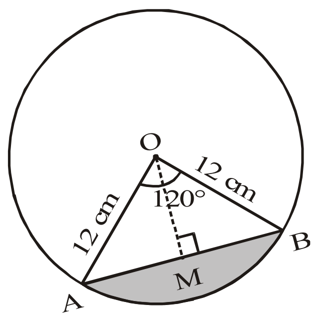
Sol. Here θ = 120° and r = 12 cm

$\therefore$ Area of the sector $= \frac{\theta}{360°} \times \pi r^2$

$= \frac{120}{360} \times \frac{314}{100} \times 12 \times 12$ cm^2

$= \frac{314 \times 4 \times 12}{100}$ cm^2 $= \frac{15072}{100}$ cm^2

= 150.72 cm^2 .....(1)



Now, area of $\Delta AOB = \frac{1}{2} \times AB \times OM$ .....(2) $[\because OM \perp AB]$

In ΔOAB, $\angle O = 120°$

$\Rightarrow \angle A + \angle B = 180° - 120° = 60°$

$\because$ $OB = OA = 12$ cm

$\Rightarrow$ $\angle A = \angle B = 30°$

So, $\frac{OM}{OA} = \sin 30° = \frac{1}{2}$ $\Rightarrow OM = OA \times \frac{1}{2}$

$\Rightarrow$ $OM = 12 \times \frac{1}{2} = 6$ cm

and $\frac{AM}{OA} = \cos 30° = \frac{\sqrt{3}}{2}$

$\Rightarrow$ $AM = \frac{\sqrt{3}}{2} OA = \frac{\sqrt{3}}{2} \times 12 = 6\sqrt{3}$ cm

$\therefore$ $AB = 2(AM) = 12\sqrt{3}$ cm.

Now, from (2),

Area of $\Delta AOB = \frac{1}{2} \times AB \times OM$

$= \frac{1}{2} \times 12\sqrt{3} \times 6 \text{ cm}^2 = 36\sqrt{3} \text{ cm}^2$

$= 36 \times 1.73 \text{ cm}^2 = 62.28 \text{ cm}^2$ .....(3)

From (1) and (3)

Area of the minor segment

= [Area of sector] – [Area of ΔAOB]

$= [150.72 \text{ cm}^2] - [62.28 \text{ cm}^2] = 88.44 \text{ cm}^2$

Q8. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope. Find

(i) the area of that part of the field in which the horse can graze.

(ii) the increase in the grazing area if the rope were 10 m long instead of 5 m. (Use $\pi = 3.14$)

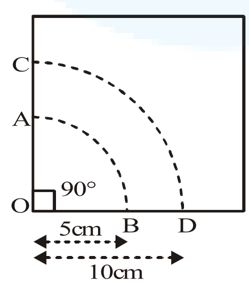
Sol. (i) $r = 5$ m, $\theta = 90°$

The required area (Grazing area for horse)

= The sector area of the sector OAB

$$= \frac{90}{360} \times \pi r^2 = \frac{1}{4} \times 3.14 \times (5)^2 \text{ m}^2$$

$$= \frac{1}{4} \times 78.50 \text{ m}^2 = 19.625 \text{ m}^2$$



(ii) Now, the radius for the sector OCD = 10 m

and sector angle = 90°

The area of the sector OCD

$= \frac{90}{360} \times \pi \times (10)^2 \text{ m}^2 = \frac{1}{4} \times 3.14 \times 100 \text{ m}^2 = 78.5 \text{ m}^2$

Therefore, the increase of grazing area

= The area of sector OCD

– The area of sector OAB

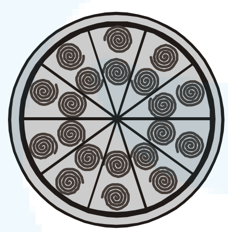
$= 78.5 \text{ m}^2 - 19.625 \text{ m}^2$

$= 58.875 \text{ m}^2$

Q9. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in fig. Find:

(i) the total length of the silver wire required.

(ii) the area of each sector of the brooch.



Sol. Diameter of the circle = 35 mm

$\therefore$ Radius (r) = $\frac{35}{2}$ mm

(i) Circumference = $2\pi r$

$= 2 \times \frac{22}{7} \times \frac{35}{2}$ mm $= 22 \times 5 = 110$ mm

Length of 1 piece of wire used to make diameter to divide the circle into 10 equal sectors = 35 mm

$\therefore$ Length 5 pieces = 5 × 35 = 175 mm

$\therefore$ Total length of the silver wire

= 110 + 175 mm = 285 mm

(ii) Since the circle is divided into 10 equal sectors,

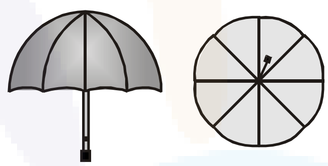
$\therefore$ Sector angle $\theta = \frac{360°}{10} = 36°$

$\Rightarrow$ Area of each sector

$= \frac{\theta}{360°} \times \pi r^2 = \frac{36°}{360°} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} \text{mm}^2$

$= \frac{11 \times 35}{4} \text{ mm}^2 = \frac{385}{4} \text{ mm}^2$

Q10. An umbrella has 8 ribs which are equally spaced. Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.



Sol. Here, radius (r) = 45 cm

Since circle is divided in 8 equal parts,

$\therefore$ Sector angle corresponding to each part

$\theta = \frac{360°}{8} = 45°$

$\Rightarrow$ Area of a sector (part)

$= \frac{\theta}{360°} \times \pi r^2 = \frac{45°}{360°} \times \frac{22}{7} \times 45 \times 45 \text{ cm}^2$

$= \frac{11 \times 45 \times 45}{4 \times 7} \text{cm}^2 = \frac{22275}{28} \text{ cm}^2$

$\therefore$ The required area between the two ribs

$= \frac{22275}{28} \text{ cm}^2$

Q11. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

Sol. Here, one blade of a wipe sweeps a sector area of a circle of radius 25 cm.

The sector angle = 115°

i.e., r = 25 cm

and $\theta = 115°$

The area covered by one blade

$$= \frac{115}{360} \times \pi \times (25)^2 \text{ cm}^2$$

Then, the area covered by two blades

$$= 2 \times \frac{115}{360} \times \frac{22}{7} \times 625 \text{ cm}^2$$

$$= \frac{23}{18} \times \frac{11}{7} \times 625 \text{ cm}^2$$

$$= \frac{158125}{126} \text{cm}^2$$

Q12. To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. (Use $\pi = 3.14$)

Sol. Here, Radius (r) = 16.5 km and

Sector angle (θ) = 80°

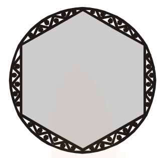
$\therefore$ Area of the sea surface over which the ships are warned

$$= \frac{\theta}{360°} \times \pi r^2 = \frac{80°}{360°} \times \frac{314}{100} \times \frac{165}{10} \times \frac{165}{10} \text{km}^2$$

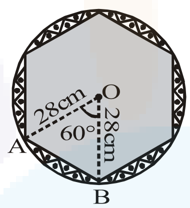
$$= \frac{157 \times 11 \times 11}{100} \text{km}^2 = \frac{18997}{100} \text{km}^2$$

$= 189.97 \text{ km}^2$

Q13. A round table cover has six equal designs as shown in fig. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs 0.35 per cm^2. (Use $\sqrt{3} = 1.7$)



Sol. Here, r = 28 cm. $\theta = \frac{360°}{6} = 60°$



In the figure ΔOAB is equilateral having side 28 cm.

The area of one shaded designed portion

= The area of the sector OAB

– The area of the ΔOAB

$$= \left\{\frac{60}{360} \times \pi \times (28)^2 - \frac{\sqrt{3}}{4} \times (28)^2\right\} cm^2$$

$$= \left\{\frac{1}{6} \times \frac{22}{7} \times 28 \times 28 - \frac{1.7}{4} \times 28 \times 28\right\} cm^2$$

$$= \left\{\frac{11}{3} \times 112 - 1.7 \times 196\right\} cm^2$$

$$= \left\{\frac{1232}{3} - 333.2\right\} cm^2$$

The total area of six designed portions

$= 6 \times \left\{\frac{1232}{3} - 333.2\right\} cm^2$

$= 2464 - 1999.2\ cm^2 \ \ = 464.8\ cm^2$

The total cost of making the designs at the rate of Rs. 0.35 per cm^2

= Rs. 0.35 × 464.8 = Rs. 162.68.

Q14. Tick the correct answer in the following :

Area of a sector of angle p (in degree) of a circle with radius R is.

(A) $\frac{p}{180} \times 2\pi R$ (B) $\frac{p}{180} \times \pi R^2$ (C) $\frac{p}{360} \times 2\pi R$ (D) $\frac{p}{720} \times 2\pi R^2$

Sol. (D) Here, radius (r) = R

Angle of sector (θ) = p°

∴ Area of the sector

$= \frac{\theta}{360} \times \pi r^2 = \frac{p}{360°} \times \pi R^2$

$= \frac{2}{2} \times \left(\frac{p}{360°} \times \pi r^2\right) = \frac{p}{720°} \times 2\pi R^2$