Class X : Math
Chapter 12 : Surface Area and Volumes
Questions & Answers - Ex : 12.1 - NCERT Book

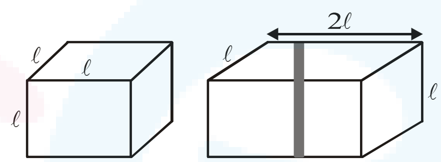
NOTE: Unless stated otherwise, take $\pi = \frac{22}{7}$

Q1. 2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

Sol. Let ℓ cm be the length of an edge of the cube having volume = 64 cm^3.

Then, $\ell^3 = 64 = (4)^3 \Rightarrow \ell = 4$ cm

Now, the dimensions of the resulting cuboid made by joining two cubes (see figure) are 8 cm × 4 cm × 4 cm (i.e., length = 8 cm, breadth = 4 cm and height = 4 cm)



Surface area of cuboid = $2(\ell b + bh + h\ell)$

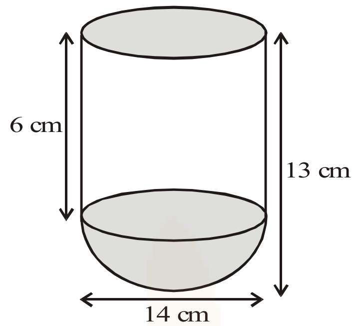
$= 2(8 \times 4 + 4 \times 4 + 4 \times 8)$

$= 2(32 + 16 + 32) = 2 \times 80 = 160 \text{ cm}^2$

Q2. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.

Sol. For hemispherical part, radius (r) = $\frac{14}{2} = 7$ cm

$\therefore$ Curved surface area = $2\pi r^2$



$= 2 \times \frac{22}{7} \times 7 \times 7 \text{ cm}^2 = 308 \text{ cm}^2$

Total height of vessel = 13 cm

$\therefore$ Height of cylinder = (13 – 7)cm = 6 cm and radius (r) = 7 cm

$\therefore$ Curved surface area of cylinder = $2\pi rh$

$= 2 \times \frac{22}{7} \times 7 \times 6 \text{ cm}^2 = 264 \text{ cm}^2$

$\therefore$ Inner surface area of vessel = Curved surface area of hemispherical part + Curved surface area of cylinder

$= (308 + 264) \text{ cm}^2 = 572 \text{ cm}^2$

Q3. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. find the total surface area of the toy.

Sol. Let r and h be the radius of cone, hemisphere and height of cone

$\therefore$ h = (15.5 – 3.5) cm

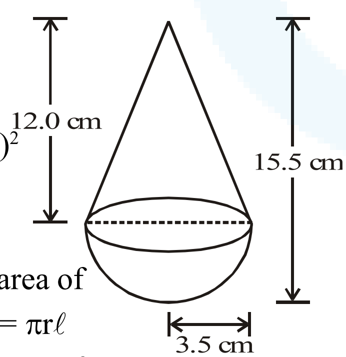
= 12.0 cm

Also $\ell^2 = h^2 + r^2$

$= 12^2 + (3.5)^2$

$= 156.25$

$\therefore \ell = 12.5$ cm



Curved surface area of the conical part = $\pi r\ell$

Curved surface area of the hemispherical part = $2\pi r^2$

Total surface area of the toy = $\pi r\ell + 2\pi r^2$

$= \pi r(\ell + 2r)$

$= \frac{22}{7} \times \frac{35}{10} (12.5 + 2 \times 3.5)\ cm^2$

$= 11 \times (12.5 + 7)\ cm^2 = 11 \times 19.5\ cm^2$

$= 214.5\ cm^2$

Q4. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

Sol. On 7 cm × 7 cm base of the cubical block, we can mount hemisphere having greatest diameter equal to 7 cm.

Here, the radius of the hemisphere = 3.5 cm.

Now, the surface area of the solid made in figure.

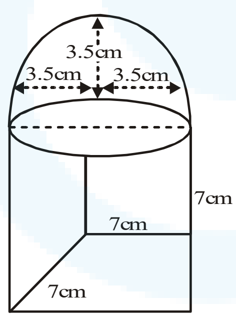
= The surface area of the cube + The curved surface area of the hemisphere – The area of the base of the hemisphere.

$= \{6 \times (7)^2 + 2\pi \times (3.5)^2 - \pi \times (3.5)^2\}\ cm^2$

($\because$ the part of the top of the cubical part which is covered by the hemisphere is not visible outside)

$= \left\{6 \times 49 + \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10}\right\} cm^2$

$= \left\{294 + 11 \times \frac{35}{10}\right\} cm^2 = 332.5\ cm^2$



Q5. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter ℓ of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

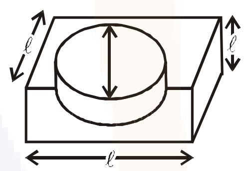
Sol. Let ℓ be the side of the cube.

$\therefore$ The greatest diameter of the hemisphere = ℓ

$\Rightarrow$ Radius of the hemisphere = $\frac{\ell}{2}$

$\therefore$ Surface area of hemisphere = $2\pi r^2$

$= 2 \times \pi \times \frac{\ell}{2} \times \frac{\ell}{2} = \frac{\pi\ell^2}{2}$



Base area of the hemisphere = $\pi\left(\frac{\ell}{2}\right)^2 = \frac{\pi\ell^2}{4}$

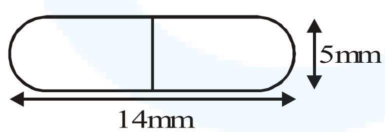
Surface area of the cube = $6 \times \ell^2 = 6\ell^2$

$\therefore$ Surface area of the remaining solid

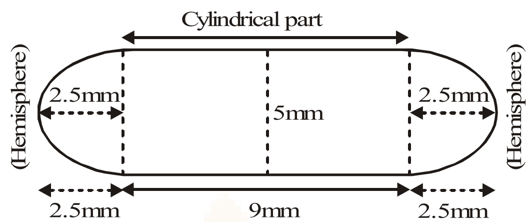
$= 6\ell^2 + \frac{\pi\ell^2}{2} - \frac{\pi\ell^2}{4} = \frac{24\ell^2 + 2\pi\ell^2 - \pi\ell^2}{4} = \frac{24\ell^2 + \pi\ell^2}{4}$

$= \frac{\ell^2}{4}(24 + \pi)$ sq. units.

Q6. A medicine capsule is in the shape of a cylinder with two hemispheres struck to each of its ends (see figure). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.



Sol. Surface area of the cylindrical part = $2\pi \times r \times h$



$= 2\pi \times \left(\frac{5}{2}\right) \times 9\ mm^2 = 45\ \pi\ mm^2$

Sum of the curved surface areas of two hemispherical parts.

$= 2\left\{2\pi \times \left(\frac{5}{2}\right)^2\right\} mm^2 = 25\ \pi\ mm^2$

Total surface area of the capsule

$= 45\pi + 25\pi\ mm^2 = 70\pi\ mm^2$

$= 70 \times \frac{22}{7}\ mm^2 = 220\ mm^2$

7. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2·1 m and 4 m respectively, and the slant height of the top is 2·8 m, find the area of the canvas used for making the tent. Also find the cost of the canvas of the tent at the rate of Rs.500 per m^2 (Note that the base of the tent will not be covered with canvas).

Sol. Radius of the cylindrical base = 2m and

height = 2.1 m. The curved surface area of the cylindrical part

$= 2\ \pi \times (2) \times (2.1)\ m^2$ (i.e., $2\pi rh$)

$= 4 \times \frac{22}{7} \times 2.1\ m^2$

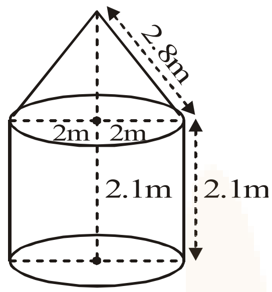
$= 26.4\ m^2$

Now, for the conical part,

we have r = 2m and

ℓ (slant height) = 2.8 m

The curved surface area of the conical part = $\pi r\ell$



$= \frac{22}{7} \times 2 \times 2.8\ m^2$

$= 17.6\ m^2$

Then the area of the canvas

$= 26.4\ m^2 + 17.6\ m^2 = 44\ m^2$

Total cost of the canvas at the rate of Rs. 500 per m^2

$=$ Rs. $500 \times 44 =$ Rs. 22000

8. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2.

Sol. For cylinder part :

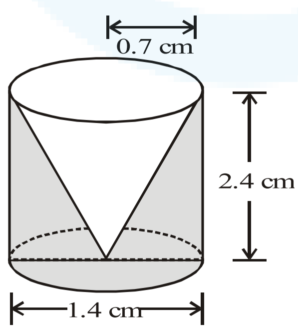
Height = 2.4 cm and diameter = 1.4 cm

$\Rightarrow$ Radius (r) = 0.7 cm

$\therefore$ Total surface area of the cylindrical part

$= 2 \times \frac{22}{7} \times \frac{7}{10}[2.4 + 0.7]\ cm^2$

$= \frac{44}{10} \times 3.1\ cm^2 = \frac{44 \times 31}{100} = \frac{1364}{100} cm^2$



For conical part :

Base radius (r) = 0.7 cm and height (h) = 2.4 cm

$\therefore$ Slant height (ℓ) = $\sqrt{r^2 + h^2} = \sqrt{(0.7)^2 + (2.4)^2}$

$= \sqrt{0.49 + 5.76} = \sqrt{6.25} = 2.5$ cm

$\therefore$ Curved surface area of the conical part

$= \pi r\ell = \frac{22}{7} \times 0.7 \times 2.5 \text{ cm}^2 = \frac{550}{100} \text{ cm}^2$

Base area of the conical part

$\pi r^2 = \frac{22}{7} \times \left(\frac{7}{10}\right)^2 \text{cm}^2 = \frac{22 \times 7}{100} \text{cm}^2 = \frac{154}{100} \text{cm}^2$

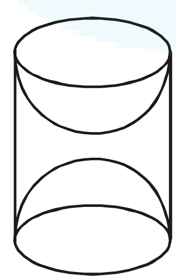
Total surface area of the remaining solid

= [(Total surface area of cylinderical part) + (Curved surface area of conical part) – (Base area of the conical part)] $= \left[\frac{1364}{100} + \frac{550}{100} - \frac{154}{100}\right] \text{cm}^2$

$= \frac{1760}{100} \text{cm}^2 = 17.6 \text{ cm}^2$.

Hence, total surface area to the nearest cm^2 is 18 cm^2.

9. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in fig. If the height of the cylinder is 10 cm, and its base is of radius 3·5 cm, find the total surface area of the article.



Sol. Radius of the cylinder (r) = 3.5 cm

Height of the cylinder (h) = 10 cm

$\therefore$ Curved surface area of cylinder = $2\pi rh$

$= 2 \times \frac{22}{7} \times \frac{35}{10} \times 10 \text{ cm}^2 = 220 \text{ cm}^2$

Curved surface area of a hemisphere = $2\pi r^2$

$\therefore$ Curved surface area of both hemispheres

$= 2 \times 2\pi r^2 = 4\pi r^2 = 4 \times \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10} \text{ cm}^2 = 154 \text{ cm}^2$

Total surface area of the remaining solid

$= (220 + 154) \text{ cm}^2 = 374 \text{ cm}^2$.