Class X : Math
Chapter 12 : Surface Area and Volumes
Questions & Answers - Ex : 12.2 - NCERT Book

NOTE: Unless stated otherwise, take $\pi = \frac{22}{7}$

Q1. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius.
Find the volume of the solid in terms of π.

Sol. Here, r = 1 cm and h = 1 cm.

Volume of the conical part = $\frac{1}{3}\pi r^2 h$

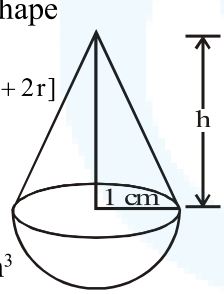
and volume of the hemispherical part $\frac{2}{3}\pi r^3$

$\therefore$ Volume of the solid shape

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^2 [h + 2r]$

$= \frac{1}{3}\pi(1)^2[1 + 2(1)]\ cm^3$

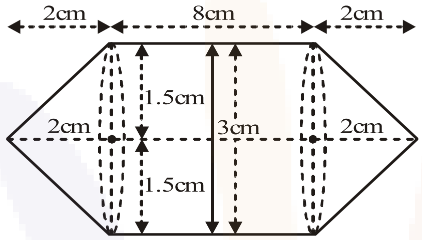
$= \frac{1}{3}\pi \times 1 \times 3\ cm^3 = \pi cm^3$



Q2. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same).

Sol. Volume of the cylindrical part

$= \pi \times (1.5)^2 \times 8 \text{ cm}^3 = 18\,\pi \text{ cm}^3$



Volume of each conical part

$= \frac{1}{3}\pi \times (1.5)^2 \times 2 \text{ cm}^3 = \frac{3}{2}\pi \text{ cm}^3$

Therefore, the volume of the air

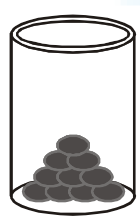
= The volume of cylindrical part

+ The volumes of two conical parts

$= 18\,\pi + 2 \times \frac{3}{2}\pi \text{ cm}^3 = 21\,\pi \text{ cm}^3$

$= 21 \times \frac{22}{7} \text{ cm}^3 = 66 \text{ cm}^3$

3. A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm. (see fig.)



Sol. Since, a gulab jamun is like a cylinder with hemispherical ends.

Total height of the
gulab jamun = 5 cm.
Diameter = 2.8 cm
$\Rightarrow$ Radius = 1.4 cm
$\therefore$ Length (height) of the cylindrical part
= 5cm – (1.4 + 1.4) cm = 5 cm – 2.8 cm = 2.2 cm
Now, volume of the cylindrical part = $\pi r^2 h$ and volume of both the hemispherical ends

$$= 2\left(\frac{2}{3}\pi r^3\right) = \frac{4}{3}\pi r^3$$

$\therefore$ Volume of a gulab jamun

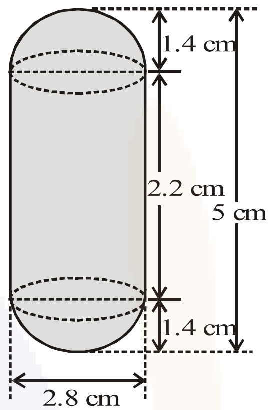
$$= \pi r^2 h + \frac{4}{3}\pi r^3 = \pi r^2\left[h + \frac{4}{3}r\right]$$

$$= \frac{22}{7} \times (1.4)^2 \left[2.2 + \frac{4}{3}(1.4)\right] cm^3$$

$$= \frac{22}{7} \times \frac{14}{10} \times \frac{14}{10}\left[\frac{22}{10} + \frac{56}{30}\right] cm^3$$

$$= \frac{22 \times 2 \times 14}{10 \times 10}\left[\frac{66 + 56}{30}\right] cm^2$$

$$= \frac{44 \times 14}{100} \times \frac{122}{30} cm^3$$



Volume of 45 gulab jamuns

$= 45 \times \left[\frac{44 \times 14}{100} \times \frac{122}{30}\right] \text{cm}^3$

$= \frac{15 \times 44 \times 14 \times 122}{1000} \text{cm}^3$

Since, the quantity of syrup in gulab jamuns

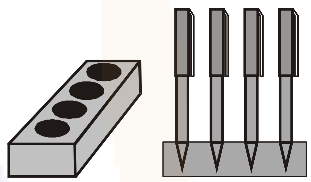
= 30% of [volume]

$= 30\% \text{ of } \left[\frac{15 \times 44 \times 14 \times 122}{1000}\right] \text{cm}^3$

$= \frac{30}{100} \times \frac{15 \times 44 \times 14 \times 122}{1000} \text{ cm}^3 = 338.184 \text{ cm}^3$

$= 338 \text{ cm}^2$ (approx)

Q4. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand (see fig.).



Sol. Radius of conical cavity = 0.5 cm and depth (i.e., vertical height) = 1.4 cm

Volume of wood taken out to make one cavity

$$= \frac{1}{3}\pi r^2 \times h = \frac{1}{3} \times \frac{22}{7} \times (0.5)^2 \times (1.4)\ cm^3$$

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{1}{4} \times \frac{14}{10}\ cm^3 = \frac{11}{30}\ cm^3$$

Volume of wood taken out to make four cavities

$$= 4 \times \frac{11}{30}\ cm^3 = \frac{44}{30}\ cm^3$$

volume of the wood in the pen stand

$$= (15 \times 10 \times 3.5) - \frac{44}{30}\ cm^3 = (525 - 1.47)\ cm^3\ (approx.) = 523.53\ cm^3\ (approx.)$$

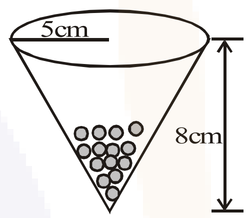
Q5. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water upto the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Sol. Height of the conical vessel (h) = 8 cm

Base radius (r) = 5 cm

Volume of water in conical vessel $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times (5)^2 \times 8 \text{ cm}^3$



$= \frac{4400}{21} \text{ cm}^3$

Now, Total volume of lead shots

$= \frac{1}{4} \times \frac{4400}{21} \text{ cm}^3 = \frac{1100}{21} \text{ cm}^3$

Since, radius of spherical lead shot (r) = 0.5 cm

$\therefore$ Volume of 1 lead shot $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \frac{5}{10} \times \frac{5}{10} \times \frac{5}{10} \text{ cm}^3$

$\therefore$ Number of lead shots

$= \frac{\text{Total volume of lead shots}}{\text{Volume of 1 lead shot}} = \frac{\left[\frac{1100}{21}\right]}{\left[\frac{4 \times 22 \times 5 \times 5 \times 5}{3 \times 7 \times 1000}\right]}$

= 100

Thus, the required number of lead shots = 100.

Q6. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 g mass. (Use $\pi = 3.14$)

Sol. First cylindrical part has height 220 cm and radius 12 cm.

Its volume = $\pi \times (12)^2 \times 220 \text{ cm}^3$.

Second cylindrical part has height 60 cm and radius 8 cm.

Its volume = $\pi \times (8)^2 \times 60 \text{ cm}^3$

Total volume = $\{144 \times 220 + 64 \times 60\} \pi \text{ cm}^3$

$= 35520 \pi \text{ cm}^3 = 35520 \times 3.14 \text{ cm}^3$

$= 111532.8 \text{ cm}^3$

Total weight (at the rate of 8 gm per 1 cm^3)

$$= \frac{111532.8 \times 8}{1000} \text{ kg} = 111.5328 \times 8 \text{ kg} = 892.2624 \text{ kg}$$

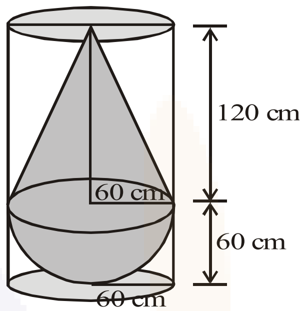
Q7. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius is 60 cm and its height is 180 cm.

Sol. Height of the conical part = 120 cm

Base radius of the conical part = 60 cm

$\therefore$ Volume of the conical part = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 60^2 \times 120 \text{ cm}^3$$



Radius of the hemispherical part = 60 cm.

$\therefore$ Volume of the hemispherical part $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times (60)^3 \text{ cm}^3$

$\therefore$ Volume of the solid = [Volume of conical part] + [Volume of hemispherical part]

$= \left[\frac{1}{3} \times \frac{22}{7} \times 60^2 \times 120\right] + \left[\frac{2}{3} \times \frac{22}{7} \times 60^3\right] \text{cm}^3$

$= \frac{2}{3} \times \frac{22}{7} \times 60^2 \, [60 + 60] \text{ cm}^3$

$= \frac{2}{3} \times \frac{22}{7} \times 60 \times 60 \times 120 \text{ cm}^3 = \frac{6336000}{7} \text{ cm}^3$

$\Rightarrow$ Volume of water in the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times 60 \times 60 \times 180$

$= \frac{14256000}{7} \text{ cm}^3$

$\therefore$ Volume of water left in the cylinder

$= \left[\frac{14256000}{7} - \frac{6336000}{7}\right] \text{cm}^3$

$= \frac{7920000}{7}$ cm^3

$= 1131428.57142$ cm^3 $= \frac{1131428.57142}{1000000}$ m^3

$= 1.13142857142$ m^3 $= 1.131$ m^3 (approx).

Q8. A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diamter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurements, and π = 3.14.

Sol. The cylinder neck has length = 8 cm

and radius = 1 cm

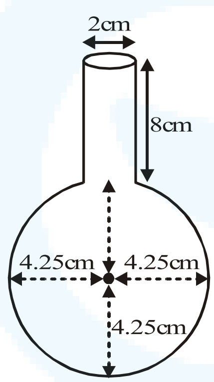
Volume of the cylinder part

$= \pi (1)^2 \times 8$ cm^3

$= 8\pi$ cm^3

The radius of the spherical part

$= \frac{8.5}{2}$ cm $= 4.25$ cm



Volume of the spherical part

$= \frac{4}{3}\pi \times (4.25)^3$

Total volume of water

$= 8\pi + \frac{4}{3} \times (4.25)^3 \pi$ cm^3

$= 8 \times 3.14 + \frac{4}{3} \times 3.14 \times (4.25)^3$ cm^3

$= 25.12 + 321.38$ (approx.)

$= 346.5\ cm^3$ (approx.),

So, $345\ cm^3$ is not correct.

$= \frac{22}{7} \times H \times 7 \times 4 m^3$

Since, Volume of the embankment = Volume of the cylindrical well

$\therefore \left[\frac{22}{7} \times H \times 7 \times 4\right] = 99$

$\Rightarrow H = 99 \times \frac{7}{22} \times \frac{1}{7} \times \frac{1}{4} m = \frac{9}{8} m = 1.125 m$

$= \frac{9}{8} m = 1.125 m$

Thus, the required height of the embankment

= 1.125 m

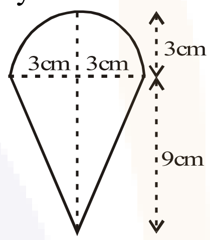
Q5. A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice-cream. The ice-cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice-cream.

Sol. Volume of one ice-cream cone as shown in figure.

$= \frac{1}{3}\pi \times (3)^2 \times 9 + \frac{2}{3}\pi \times (3)^3 \text{ cm}^3$

$= 27\pi + 18\pi = 45\pi \text{ cm}^3$

volume of the ice-cream in the cylindrical container (of height 15 cm and diameter 12 cm) $= \pi \times (6)^2 \times 15$



Let the number of cones made be n.

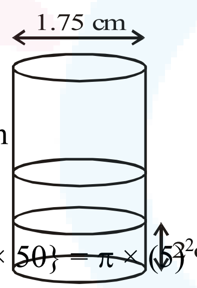
Then, $n \times 45\pi = \pi \times (6)^2 \times 15$

$\Rightarrow 45n = 36 \times 15 \Rightarrow n = 12$

Q6. How many silver coins 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions 5.5 cm × 10 cm × 3.5 cm?

Sol. For a circular coin :

Diameter = 1.75 cm



$\Rightarrow$ Radius (r) $= \frac{175}{200}$ cm

$\Rightarrow \pi \times \left(\frac{1}{10}\right)^2 \times \{n \times 50\} = \pi \times (5)^2 \times 2$

$\frac{1}{100} \times n \times 50 = 50 \Rightarrow n = 100$

Hence, the required time is 100 minutes.