

Class XI : Maths
Chapter 1 : Trigonometric Functions

Questions and Solutions | Exercise 3.3 - NCERT Books

Question 1:

$$\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4} = -\frac{1}{2}$$

Answer

$$\begin{aligned} \text{L.H.S.} &= \sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4} \\ &= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 - (1)^2 \\ &= \frac{1}{4} + \frac{1}{4} - 1 = -\frac{1}{2} \\ &= \text{R.H.S.} \end{aligned}$$

Question 2:

Prove that $2 \sin^2 \frac{\pi}{6} + \operatorname{cosec}^2 \frac{7\pi}{6} \cos^2 \frac{\pi}{3} = \frac{3}{2}$

Answer

$$\begin{aligned} \text{L.H.S.} &= 2 \sin^2 \frac{\pi}{6} + \operatorname{cosec}^2 \frac{7\pi}{6} \cos^2 \frac{\pi}{3} \\ &= 2 \left(\frac{1}{2}\right)^2 + \operatorname{cosec}^2 \left(\pi + \frac{\pi}{6}\right) \left(\frac{1}{2}\right)^2 \\ &= 2 \times \frac{1}{4} + \left(-\operatorname{cosec} \frac{\pi}{6}\right)^2 \left(\frac{1}{4}\right) \\ &= \frac{1}{2} + (-2)^2 \left(\frac{1}{4}\right) \\ &= \frac{1}{2} + \frac{4}{4} = \frac{1}{2} + 1 = \frac{3}{2} \\ &= \text{R.H.S.} \end{aligned}$$

Question 5:

Find the value of:

(i) $\sin 75^\circ$

(ii) $\tan 15^\circ$

Answer

(i) $\sin 75^\circ = \sin (45^\circ + 30^\circ)$

$= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$

$[\sin (x + y) = \sin x \cos y + \cos x \sin y]$

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

(ii) $\tan 15^\circ = \tan (45^\circ - 30^\circ)$

$$\begin{aligned}
&= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ} && \left[\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y} \right] \\
&= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1\left(\frac{1}{\sqrt{3}}\right)} = \frac{\frac{\sqrt{3} - 1}{\sqrt{3}}}{\frac{\sqrt{3} + 1}{\sqrt{3}}} \\
&= \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = \frac{(\sqrt{3} - 1)^2}{(\sqrt{3} + 1)(\sqrt{3} - 1)} = \frac{3 + 1 - 2\sqrt{3}}{(\sqrt{3})^2 - (1)^2} \\
&= \frac{4 - 2\sqrt{3}}{3 - 1} = 2 - \sqrt{3}
\end{aligned}$$

Question 6:

Prove that: $\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right) = \sin(x + y)$

Answer

$$\begin{aligned}
&\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right) \\
&= \frac{1}{2}\left[2\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right)\right] + \frac{1}{2}\left[-2\sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right)\right] \\
&= \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4} - x\right) + \left(\frac{\pi}{4} - y\right)\right\} + \cos\left\{\left(\frac{\pi}{4} - x\right) - \left(\frac{\pi}{4} - y\right)\right\}\right] \\
&\quad + \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4} - x\right) + \left(\frac{\pi}{4} - y\right)\right\} - \cos\left\{\left(\frac{\pi}{4} - x\right) - \left(\frac{\pi}{4} - y\right)\right\}\right] \\
&\quad \left[\begin{aligned} \because 2\cos A \cos B &= \cos(A + B) + \cos(A - B) \\ -2\sin A \sin B &= \cos(A + B) - \cos(A - B) \end{aligned} \right] \\
&= 2 \times \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4} - x\right) + \left(\frac{\pi}{4} - y\right)\right\}\right] \\
&= \cos\left[\frac{\pi}{2} - (x + y)\right] \\
&= \sin(x + y) \\
&= \text{R.H.S}
\end{aligned}$$

Question 7:

$$\frac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)} = \left(\frac{1 + \tan x}{1 - \tan x}\right)^2$$

Prove that:

Answer

It is known that $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ and $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$$\frac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)} = \frac{\left(\frac{\tan\frac{\pi}{4} + \tan x}{1 - \tan\frac{\pi}{4} \tan x}\right)}{\left(\frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4} \tan x}\right)} = \frac{\left(\frac{1 + \tan x}{1 - \tan x}\right)}{\left(\frac{1 - \tan x}{1 + \tan x}\right)} = \left(\frac{1 + \tan x}{1 - \tan x}\right)^2 = \text{R.H.S.}$$

□ L.H.S. =

Question 8:

$$\frac{\cos(\pi + x)\cos(-x)}{\sin(\pi - x)\cos\left(\frac{\pi}{2} + x\right)} = \cot^2 x$$

Prove that

Answer

$$\begin{aligned} \text{L.H.S.} &= \frac{\cos(\pi + x)\cos(-x)}{\sin(\pi - x)\cos\left(\frac{\pi}{2} + x\right)} \\ &= \frac{[-\cos x][\cos x]}{(\sin x)(-\sin x)} \\ &= \frac{-\cos^2 x}{-\sin^2 x} \\ &= \cot^2 x \\ &= \text{R.H.S.} \end{aligned}$$

Question 9:

$$\cos\left(\frac{3\pi}{2} + x\right)\cos(2\pi + x)\left[\cot\left(\frac{3\pi}{2} - x\right) + \cot(2\pi + x)\right] = 1$$

Answer

$$\begin{aligned} \text{L.H.S.} &= \cos\left(\frac{3\pi}{2} + x\right)\cos(2\pi + x)\left[\cot\left(\frac{3\pi}{2} - x\right) + \cot(2\pi + x)\right] \\ &= \sin x \cos x [\tan x + \cot x] \\ &= \sin x \cos x \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right) \\ &= (\sin x \cos x) \left[\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}\right] \\ &= 1 = \text{R.H.S.} \end{aligned}$$

Question 10:

Prove that $\sin(n+1)x \sin(n+2)x + \cos(n+1)x \cos(n+2)x = \cos x$

Answer

$$\begin{aligned} \text{L.H.S.} &= \sin(n+1)x \sin(n+2)x + \cos(n+1)x \cos(n+2)x \\ &= \frac{1}{2} [2 \sin(n+1)x \sin(n+2)x + 2 \cos(n+1)x \cos(n+2)x] \\ &= \frac{1}{2} \left[\cos\{(n+1)x - (n+2)x\} - \cos\{(n+1)x + (n+2)x\} \right. \\ &\quad \left. + \cos\{(n+1)x + (n+2)x\} + \cos\{(n+1)x - (n+2)x\} \right] \\ &\quad \left[\begin{array}{l} \because -2 \sin A \sin B = \cos(A+B) - \cos(A-B) \\ 2 \cos A \cos B = \cos(A+B) + \cos(A-B) \end{array} \right] \\ &= \frac{1}{2} \times 2 \cos\{(n+1)x - (n+2)x\} \\ &= \cos(-x) = \cos x = \text{R.H.S.} \end{aligned}$$

Question 11:

Prove that $\cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right) = -\sqrt{2} \sin x$

Answer

$$\cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \cdot \sin\left(\frac{A-B}{2}\right)$$

It is known that

$$\therefore \text{L.H.S.} = \cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right)$$

$$= -2 \sin\left\{\frac{\left(\frac{3\pi}{4} + x\right) + \left(\frac{3\pi}{4} - x\right)}{2}\right\} \cdot \sin\left\{\frac{\left(\frac{3\pi}{4} + x\right) - \left(\frac{3\pi}{4} - x\right)}{2}\right\}$$

$$= -2 \sin\left(\frac{3\pi}{4}\right) \sin x$$

$$= -2 \sin\left(\pi - \frac{\pi}{4}\right) \sin x$$

$$= -2 \sin \frac{\pi}{4} \sin x$$

$$= -2 \times \frac{1}{\sqrt{2}} \times \sin x$$

$$= -\sqrt{2} \sin x$$

$$= \text{R.H.S.}$$

Question 12:

Prove that $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

Answer

It is known that

$$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right), \quad \sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

$$\therefore \text{L.H.S.} = \sin^2 6x - \sin^2 4x$$

$$= (\sin 6x + \sin 4x) (\sin 6x - \sin 4x)$$

$$= \left[2 \sin\left(\frac{6x+4x}{2}\right) \cos\left(\frac{6x-4x}{2}\right)\right] \left[2 \cos\left(\frac{6x+4x}{2}\right) \sin\left(\frac{6x-4x}{2}\right)\right]$$

$$= (2 \sin 5x \cos x) (2 \cos 5x \sin x)$$

$$= (2 \sin 5x \cos 5x) (2 \sin x \cos x)$$

$$= \sin 10x \sin 2x$$

= R.H.S.

Question 13:

Prove that $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

Answer

It is known that

$$\cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right), \quad \cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

$$\therefore \text{L.H.S.} = \cos^2 2x - \cos^2 6x$$

$$= (\cos 2x + \cos 6x) (\cos 2x - \cos 6x)$$

$$= \left[2 \cos\left(\frac{2x+6x}{2}\right) \cos\left(\frac{2x-6x}{2}\right) \right] \left[-2 \sin\left(\frac{2x+6x}{2}\right) \sin\left(\frac{2x-6x}{2}\right) \right]$$

$$= \left[2 \cos 4x \cos(-2x) \right] \left[-2 \sin 4x \sin(-2x) \right]$$

$$= [2 \cos 4x \cos 2x] [-2 \sin 4x (-\sin 2x)]$$

$$= (2 \sin 4x \cos 4x) (2 \sin 2x \cos 2x)$$

$$= \sin 8x \sin 4x$$

= R.H.S.

Question 14:

Prove that $\sin 2x + 2 \sin 4x + \sin 6x = 4 \cos^2 x \sin 4x$

Answer

$$\text{L.H.S.} = \sin 2x + 2 \sin 4x + \sin 6x$$

$$= [\sin 2x + \sin 6x] + 2 \sin 4x$$

$$= \left[2 \sin\left(\frac{2x+6x}{2}\right) \cos\left(\frac{2x-6x}{2}\right) \right] + 2 \sin 4x$$

$$\left[\because \sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right) \right]$$

$$= 2 \sin 4x \cos(-2x) + 2 \sin 4x$$

$$= 2 \sin 4x \cos 2x + 2 \sin 4x$$

$$= 2 \sin 4x (\cos 2x + 1)$$

$$= 2 \sin 4x (2 \cos^2 x - 1 + 1)$$

$$= 2 \sin 4x (2 \cos^2 x)$$

$$= 4\cos^2 x \sin 4x$$

$$= \text{R.H.S.}$$

Question 15:

Prove that $\cot 4x (\sin 5x + \sin 3x) = \cot x (\sin 5x - \sin 3x)$

Answer

$$\text{L.H.S} = \cot 4x (\sin 5x + \sin 3x)$$

$$= \frac{\cot 4x}{\sin 4x} \left[2 \sin \left(\frac{5x+3x}{2} \right) \cos \left(\frac{5x-3x}{2} \right) \right]$$

$$\left[\because \sin A + \sin B = 2 \sin \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right) \right]$$

$$= \left(\frac{\cos 4x}{\sin 4x} \right) [2 \sin 4x \cos x]$$

$$= 2 \cos 4x \cos x$$

$$\text{R.H.S.} = \cot x (\sin 5x - \sin 3x)$$

$$= \frac{\cos x}{\sin x} \left[2 \cos \left(\frac{5x+3x}{2} \right) \sin \left(\frac{5x-3x}{2} \right) \right]$$

$$\left[\because \sin A - \sin B = 2 \cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right) \right]$$

$$= \frac{\cos x}{\sin x} [2 \cos 4x \sin x]$$

$$= 2 \cos 4x \cdot \cos x$$

$$\text{L.H.S.} = \text{R.H.S.}$$

Question 16:

Prove that $\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\frac{\sin 2x}{\cos 10x}$

Answer

It is known that

$$\cos A - \cos B = -2 \sin \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right), \quad \sin A - \sin B = 2 \cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right)$$

$$\begin{aligned}
& \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} \\
\therefore \text{L.H.S} &= \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} \\
&= \frac{-2 \sin\left(\frac{9x+5x}{2}\right) \cdot \sin\left(\frac{9x-5x}{2}\right)}{2 \cos\left(\frac{17x+3x}{2}\right) \cdot \sin\left(\frac{17x-3x}{2}\right)} \\
&= \frac{-2 \sin 7x \cdot \sin 2x}{2 \cos 10x \cdot \sin 7x} \\
&= -\frac{\sin 2x}{\cos 10x} \\
&= \text{R.H.S.}
\end{aligned}$$

Question 17:

$$\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$$

Prove that

Answer

It is known that

$$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right), \quad \cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\therefore \text{L.H.S.} = \frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$$

$$\begin{aligned}
&= \frac{2 \sin\left(\frac{5x+3x}{2}\right) \cdot \cos\left(\frac{5x-3x}{2}\right)}{2 \cos\left(\frac{5x+3x}{2}\right) \cdot \cos\left(\frac{5x-3x}{2}\right)} \\
&= \frac{2 \sin 4x \cdot \cos x}{2 \cos 4x \cdot \cos x} \\
&= \frac{\sin 4x}{\cos 4x} \\
&= \tan 4x = \text{R.H.S.}
\end{aligned}$$

Question 18:

$$\frac{\sin x - \sin y}{\cos x + \cos y} = \tan \frac{x-y}{2}$$

Prove that

Answer

It is known that

$$\sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right), \quad \cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\therefore \text{L.H.S.} = \frac{\sin x - \sin y}{\cos x + \cos y}$$

$$= \frac{2 \cos\left(\frac{x+y}{2}\right) \cdot \sin\left(\frac{x-y}{2}\right)}{2 \cos\left(\frac{x+y}{2}\right) \cdot \cos\left(\frac{x-y}{2}\right)}$$

$$= \frac{\sin\left(\frac{x-y}{2}\right)}{\cos\left(\frac{x-y}{2}\right)}$$

$$= \tan\left(\frac{x-y}{2}\right) = \text{R.H.S.}$$

Question 19:

$$\text{Prove that } \frac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$$

Answer

It is known that

$$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right), \quad \cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\therefore \text{L.H.S.} = \frac{\sin x + \sin 3x}{\cos x + \cos 3x}$$

$$\begin{aligned}
&= \frac{2 \sin\left(\frac{x+3x}{2}\right) \cos\left(\frac{x-3x}{2}\right)}{2 \cos\left(\frac{x+3x}{2}\right) \cos\left(\frac{x-3x}{2}\right)} \\
&= \frac{\sin 2x}{\cos 2x} \\
&= \tan 2x \\
&= \text{R.H.S}
\end{aligned}$$

Question 20:

Prove that $\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2 \sin x$

Answer

It is known that

$$\sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right), \quad \cos^2 A - \sin^2 A = \cos 2A$$

$$\begin{aligned}
\therefore \text{L.H.S.} &= \frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} \\
&= \frac{2 \cos\left(\frac{x+3x}{2}\right) \sin\left(\frac{x-3x}{2}\right)}{-\cos 2x} \\
&= \frac{2 \cos 2x \sin(-x)}{-\cos 2x} \\
&= -2 \times (-\sin x) \\
&= 2 \sin x = \text{R.H.S.}
\end{aligned}$$

Question 21:

Prove that $\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$

Answer

$$\text{L.H.S.} = \frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$$

$$\begin{aligned}
&= \frac{(\cos 4x + \cos 2x) + \cos 3x}{(\sin 4x + \sin 2x) + \sin 3x} \\
&= \frac{2 \cos \left(\frac{4x+2x}{2} \right) \cos \left(\frac{4x-2x}{2} \right) + \cos 3x}{2 \sin \left(\frac{4x+2x}{2} \right) \cos \left(\frac{4x-2x}{2} \right) + \sin 3x} \\
&\left[\because \cos A + \cos B = 2 \cos \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right), \sin A + \sin B = 2 \sin \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right) \right] \\
&= \frac{2 \cos 3x \cos x + \cos 3x}{2 \sin 3x \cos x + \sin 3x} \\
&= \frac{\cos 3x (2 \cos x + 1)}{\sin 3x (2 \cos x + 1)} \\
&= \cot 3x = \text{R.H.S.}
\end{aligned}$$

Question 22:

Prove that $\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$

Answer

$$\begin{aligned}
\text{L.H.S.} &= \cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x \\
&= \cot x \cot 2x - \cot 3x (\cot 2x + \cot x) \\
&= \cot x \cot 2x - \cot (2x + x) (\cot 2x + \cot x) \\
&= \cot x \cot 2x - \left[\frac{\cot 2x \cot x - 1}{\cot x + \cot 2x} \right] (\cot 2x + \cot x) \\
&\left[\because \cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B} \right] \\
&= \cot x \cot 2x - (\cot 2x \cot x - 1) \\
&= 1 = \text{R.H.S.}
\end{aligned}$$

Question 23:

$$\text{Prove that } \tan 4x = \frac{4 \tan x (1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}$$

Answer

$$\text{It is known that } \tan 2A = \frac{2 \tan A}{1 - \tan^2 A} .$$

$$\therefore \text{L.H.S.} = \tan 4x = \tan 2(2x)$$

$$\begin{aligned}
 &= \frac{2 \tan 2x}{1 - \tan^2(2x)} \\
 &= \frac{2 \left(\frac{2 \tan x}{1 - \tan^2 x} \right)}{1 - \left(\frac{2 \tan x}{1 - \tan^2 x} \right)^2} \\
 &= \frac{\left(\frac{4 \tan x}{1 - \tan^2 x} \right)}{\left[1 - \frac{4 \tan^2 x}{(1 - \tan^2 x)^2} \right]} \\
 &= \frac{\left(\frac{4 \tan x}{1 - \tan^2 x} \right)}{\left[\frac{(1 - \tan^2 x)^2 - 4 \tan^2 x}{(1 - \tan^2 x)^2} \right]} \\
 &= \frac{4 \tan x (1 - \tan^2 x)}{(1 - \tan^2 x)^2 - 4 \tan^2 x} \\
 &= \frac{4 \tan x (1 - \tan^2 x)}{1 + \tan^4 x - 2 \tan^2 x - 4 \tan^2 x} \\
 &= \frac{4 \tan x (1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x} = \text{R.H.S.}
 \end{aligned}$$

Question 24:

Prove that $\cos 4x = 1 - 8 \sin^2 x \cos^2 x$

Answer

$$\begin{aligned}
 \text{L.H.S.} &= \cos 4x \\
 &= \cos 2(2x) \\
 &= 1 - 2 \sin^2 2x \quad [\cos 2A = 1 - 2 \sin^2 A] \\
 &= 1 - 2(2 \sin x \cos x)^2 \quad [\sin 2A = 2 \sin A \cos A] \\
 &= 1 - 8 \sin^2 x \cos^2 x \\
 &= \text{R.H.S.}
 \end{aligned}$$

Question 25:

Prove that: $\cos 6x = 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1$

Answer

$$\text{L.H.S.} = \cos 6x$$

$$= \cos 3(2x)$$

$$= 4 \cos^3 2x - 3 \cos 2x \quad [\cos 3A = 4 \cos^3 A - 3 \cos A]$$

$$= 4 [(2 \cos^2 x - 1)^3 - 3 (2 \cos^2 x - 1) \cos 2x] \quad [\cos 2x = 2 \cos^2 x - 1]$$

$$= 4 [(2 \cos^2 x)^3 - (1)^3 - 3 (2 \cos^2 x)^2 + 3 (2 \cos^2 x)] - 6 \cos^2 x + 3$$

$$= 4 [8 \cos^6 x - 1 - 12 \cos^4 x + 6 \cos^2 x] - 6 \cos^2 x + 3$$

$$= 32 \cos^6 x - 4 - 48 \cos^4 x + 24 \cos^2 x - 6 \cos^2 x + 3$$

$$= 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1$$

$$= \text{R.H.S.}$$