Class XII : Maths
Chapter 4 : Determinants

Questions and Solutions | Exercise 4.1 - NCERT Books

Question 1:

Evaluate the determinants in Exercises 1 and 2.

$$\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$$

Answer

$\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix} = 2(-1) - 4(-5) = -2 + 20 = 18$

Question 2:

Evaluate the determinants in Exercises 1 and 2.

(i) $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$ (ii) $\begin{vmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{vmatrix}$

Answer

(i) $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} = (\cos\theta)(\cos\theta) - (-\sin\theta)(\sin\theta) = \cos^2\theta + \sin^2\theta = 1$

(ii) $\begin{vmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{vmatrix}$

$= (x^2 - x + 1)(x + 1) - (x - 1)(x + 1)$

$= x^3 - x^2 + x + x^2 - x + 1 - (x^2 - 1)$

$= x^3 + 1 - x^2 + 1$

$= x^3 - x^2 + 2$

Question 3:

If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then show that $|2A| = 4|A|$

Answer

The given matrix is $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$.

$$\therefore 2A = 2\begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 8 & 4 \end{bmatrix}$$

$$\therefore \text{L.H.S.} = |2A| = \begin{vmatrix} 2 & 4 \\ 8 & 4 \end{vmatrix} = 2\times4 - 4\times8 = 8 - 32 = -24$$

$$\text{Now, } |A| = \begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix} = 1\times2 - 2\times4 = 2 - 8 = -6$$

$$\therefore \text{R.H.S.} = 4|A| = 4\times(-6) = -24$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

Question 4:

If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, then show that $|3A| = 27|A|$.

Answer

The given matrix is $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$.

It can be observed that in the first column, two entries are zero. Thus, we expand along the first column (C_1) for easier calculation.

$$|A| = 1\begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix} - 0\begin{vmatrix} 0 & 1 \\ 0 & 4 \end{vmatrix} + 0\begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix} = 1(4-0) - 0 + 0 = 4$$

$$\therefore 27|A| = 27(4) = 108 \qquad \text{...(i)}$$

$$\text{Now, } 3A = 3\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{bmatrix}$$

$$\therefore |3A| = 3\begin{vmatrix} 3 & 6 \\ 0 & 12 \end{vmatrix} - 0\begin{vmatrix} 0 & 3 \\ 0 & 12 \end{vmatrix} + 0\begin{vmatrix} 0 & 3 \\ 3 & 6 \end{vmatrix}$$

$$= 3(36-0) = 3(36) = 108 \qquad \text{...(ii)}$$

From equations (i) and (ii), we have:

$$|3A| = 27|A|$$

Hence, the given result is proved.

Question 5:

Evaluate the determinants

(i) $\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$ (iii) $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

(ii) $\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$ (iv) $\begin{bmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix}$

Answer

(i) Let $A = \begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$.

It can be observed that in the second row, two entries are zero. Thus, we expand along the second row for easier calculation.

$$|A| = -0\begin{vmatrix} -1 & -2 \\ -5 & 0 \end{vmatrix} + 0\begin{vmatrix} 3 & -2 \\ 3 & 0 \end{vmatrix} - (-1)\begin{vmatrix} 3 & -1 \\ 3 & -5 \end{vmatrix} = (-15+3) = -12$$

(ii) Let $A = \begin{bmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{bmatrix}$.

By expanding along the first row, we have:

$$\begin{aligned} |A| &= 3\begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} + 4\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} + 5\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} \\ &= 3(1+6) + 4(1+4) + 5(3-2) \\ &= 3(7) + 4(5) + 5(1) \\ &= 21 + 20 + 5 = 46 \end{aligned}$$

(iii) Let $A=\begin{bmatrix}0 & 1 & 2\\ -1 & 0 & -3\\ -2 & 3 & 0\end{bmatrix}$.

By expanding along the first row, we have:

$$|A|=0\begin{vmatrix}0 & -3\\ 3 & 0\end{vmatrix}-1\begin{vmatrix}-1 & -3\\ -2 & 0\end{vmatrix}+2\begin{vmatrix}-1 & 0\\ -2 & 3\end{vmatrix}$$
$$=0-1(0-6)+2(-3-0)$$
$$=-1(-6)+2(-3)$$
$$=6-6=0$$

(iv) Let $A=\begin{bmatrix}2 & -1 & -2\\ 0 & 2 & -1\\ 3 & -5 & 0\end{bmatrix}$.

By expanding along the first column, we have:

$$|A|=2\begin{vmatrix}2 & -1\\ -5 & 0\end{vmatrix}-0\begin{vmatrix}-1 & -2\\ -5 & 0\end{vmatrix}+3\begin{vmatrix}-1 & -2\\ 2 & -1\end{vmatrix}$$
$$=2(0-5)-0+3(1+4)$$
$$=-10+15=5$$

Question 6:

If $A=\begin{bmatrix}1 & 1 & -2\\ 2 & 1 & -3\\ 5 & 4 & -9\end{bmatrix}$, find $|\mathrm{A}|$.

Answer

Let $A=\begin{bmatrix}1 & 1 & -2\\ 2 & 1 & -3\\ 5 & 4 & -9\end{bmatrix}$.

By expanding along the first row, we have:

$$|A| = 1\begin{vmatrix} 1 & -3 \\ 4 & -9 \end{vmatrix} - 1\begin{vmatrix} 2 & -3 \\ 5 & -9 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 5 & 4 \end{vmatrix}$$
$$= 1(-9+12) - 1(-18+15) - 2(8-5)$$
$$= 1(3) - 1(-3) - 2(3)$$
$$= 3 + 3 - 6$$
$$= 6 - 6$$
$$= 0$$

Question 7:

Find values of x, if

(i) $\begin{vmatrix} 2 & 4 \\ 2 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$ (ii) $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

Answer

(i) $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

$$\Rightarrow 2\times 1 - 5\times 4 = 2x\times x - 6\times 4$$
$$\Rightarrow 2 - 20 = 2x^2 - 24$$
$$\Rightarrow 2x^2 = 6$$
$$\Rightarrow x^2 = 3$$
$$\Rightarrow x = \pm\sqrt{3}$$

(ii) $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

$$\Rightarrow 2\times 5 - 3\times 4 = x\times 5 - 3\times 2x$$
$$\Rightarrow 10 - 12 = 5x - 6x$$
$$\Rightarrow -2 = -x$$
$$\Rightarrow x = 2$$

Question 8:

If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, then x is equal to

(A) 6 (B) ±6 (C) −6 (D) 0

Answer

Answer: B

$\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$

$\Rightarrow x^2 - 36 = 36 - 36$

$\Rightarrow x^2 - 36 = 0$

$\Rightarrow x^2 = 36$

$\Rightarrow x = \pm 6$

Hence, the correct answer is B.

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Questions and Solutions | Exercise 4.2 - NCERT Books

Question 1:

Find area of the triangle with vertices at the point given in each of the following:

(i) (1, 0), (6, 0), (4, 3) (ii) (2, 7), (1, 1), (10, 8)

(iii) (−2, −3), (3, 2), (−1, −8)

Answer

(i) The area of the triangle with vertices (1, 0), (6, 0), (4, 3) is given by the relation,

$$\Delta=\frac{1}{2}\begin{vmatrix}1 & 0 & 1\\ 6 & 0 & 1\\ 4 & 3 & 1\end{vmatrix}$$

$$=\frac{1}{2}\left[1(0-3)-0(6-4)+1(18-0)\right]$$

$$=\frac{1}{2}\left[-3+18\right]=\frac{15}{2}\text{ square units}$$

(ii) The area of the triangle with vertices (2, 7), (1, 1), (10, 8) is given by the relation,

$$\Delta=\frac{1}{2}\begin{vmatrix}2 & 7 & 1\\ 1 & 1 & 1\\ 10 & 8 & 1\end{vmatrix}$$

$$=\frac{1}{2}\left[2(1-8)-7(1-10)+1(8-10)\right]$$

$$=\frac{1}{2}\left[2(-7)-7(-9)+1(-2)\right]$$

$$=\frac{1}{2}\left[-14+63-2\right]=\frac{1}{2}\left[-16+63\right]$$

$$=\frac{47}{2}\text{ square units}$$

(iii) The area of the triangle with vertices (−2, −3), (3, 2), (−1, −8)

is given by the relation,

$$\Delta=\frac{1}{2}\begin{vmatrix}-2 & -3 & 1\\ 3 & 2 & 1\\ -1 & -8 & 1\end{vmatrix}$$

$$=\frac{1}{2}\left[-2(2+8)+3(3+1)+1(-24+2)\right]$$

$$=\frac{1}{2}\left[-2(10)+3(4)+1(-22)\right]$$

$$=\frac{1}{2}\left[-20+12-22\right]$$

$$=-\frac{30}{2}=-15$$

Hence, the area of the triangle is $|-15|=15$ square units .

Question 2:

Show that points

$A(a,b+c),B(b,c+a),C(c,a+b)$ are collinear

Answer

Area of ΔABC is given by the relation,

$$\Delta=\frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$$

$$=\frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b-a & a-b & 0 \\ c-a & a-c & 0 \end{vmatrix} \quad (\text{Applying } R_2 \to R_2-R_1 \text{ and } R_3 \to R_3-R_1)$$

$$=\frac{1}{2}(a-b)(c-a)\begin{vmatrix} a & b+c & 1 \\ -1 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix}$$

$$=\frac{1}{2}(a-b)(c-a)\begin{vmatrix} a & b+c & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} \quad (\text{Applying } R_3 \to R_3+R_2)$$

$$=0 \qquad (\text{All elements of } R_3 \text{ are } 0)$$

Thus, the area of the triangle formed by points A, B, and C is zero.

Hence, the points A, B, and C are collinear.

Question 3:

Find values of k if area of triangle is 4 square units and vertices are

(i) $(k, 0), (4, 0), (0, 2)$ (ii) $(-2, 0), (0, 4), (0, k)$

Answer

We know that the area of a triangle whose vertices are (x_1, y_1), (x_2, y_2), and (x_3, y_3) is the absolute value of the determinant (Δ), where

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

It is given that the area of triangle is 4 square units.

$\therefore \Delta = \pm 4.$

(i) The area of the triangle with vertices $(k, 0), (4, 0), (0, 2)$ is given by the relation,

$$\Delta = \frac{1}{2}\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[k(0-2) - 0(4-0) + 1(8-0)\right]$$

$$= \frac{1}{2}\left[-2k+8\right] = -k+4$$

$\therefore -k + 4 = \pm 4$

When $-k + 4 = -4$, $k = 8$.

When $-k + 4 = 4$, $k = 0$.

Hence, $k = 0, 8$.

(ii) The area of the triangle with vertices (−2, 0), (0, 4), (0, k) is given by the relation,

$$\Delta = \frac{1}{2}\begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[-2(4-k)\right]$$

$$= k - 4$$

$\therefore k - 4 = \pm 4$

When $k - 4 = -4$, $k = 0$.

When $k - 4 = 4$, $k = 8$.

Hence, $k = 0, 8$.

Question 4:

(i) Find equation of line joining (1, 2) and (3, 6) using determinants

(ii) Find equation of line joining (3, 1) and (9, 3) using determinants

Answer

(i) Let P (x, y) be any point on the line joining points A (1, 2) and B (3, 6). Then, the points A, B, and P are collinear. Therefore, the area of triangle ABP will be zero.

$$\therefore \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 3 & 6 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{2}\left[1(6-y) - 2(3-x) + 1(3y-6x)\right] = 0$$

$$\Rightarrow 6 - y - 6 + 2x + 3y - 6x = 0$$

$$\Rightarrow 2y - 4x = 0$$

$$\Rightarrow y = 2x$$

Hence, the equation of the line joining the given points is $y = 2x$.

(ii) Let P (x, y) be any point on the line joining points A (3, 1) and

B (9, 3). Then, the points A, B, and P are collinear. Therefore, the area of triangle ABP will be zero.

$$\therefore \frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ 9 & 3 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{2}\left[3(3-y)-1(9-x)+1(9y-3x)\right]=0$$

$$\Rightarrow 9-3y-9+x+9y-3x=0$$

$$\Rightarrow 6y-2x=0$$

$$\Rightarrow x-3y=0$$

Hence, the equation of the line joining the given points is $x - 3y = 0$.

Question 5:

If area of triangle is 35 square units with vertices (2, −6), (5, 4), and (k, 4). Then k is

A. 12 **B.** −2 **C.** −12, −2 **D.** 12, −2

Answer

Answer: D

The area of the triangle with vertices (2, −6), (5, 4), and (k, 4) is given by the relation,

$$\Delta = \frac{1}{2}\begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[2(4-4)+6(5-k)+1(20-4k)\right]$$

$$= \frac{1}{2}\left[30-6k+20-4k\right]$$

$$= \frac{1}{2}\left[50-10k\right]$$

$$= 25-5k$$

It is given that the area of the triangle is ±35.

Therefore, we have:

$$\Rightarrow 25-5k=\pm 35$$

$$\Rightarrow 5(5-k)=\pm 35$$

$$\Rightarrow 5-k=\pm 7$$

When $5 - k = -7$, $k = 5 + 7 = 12$.

When $5 - k = 7$, $k = 5 - 7 = -2$.

Hence, $k = 12, -2$.

The correct answer is D.

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Chapter 4 : Determinants

Questions and Solutions | Exercise 4.3 - NCERT Books

Question 1:

Write Minors and Cofactors of the elements of following determinants:

(i) $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$ (ii) $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$

Answer

(i) The given determinant is $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$.

Minor of element a_{ij} is M_{ij}.

$\therefore M_{11}$ = minor of element $a_{11} = 3$

M_{12} = minor of element $a_{12} = 0$

M_{21} = minor of element $a_{21} = -4$

M_{22} = minor of element $a_{22} = 2$

Cofactor of a_{ij} is $A_{ij} = (-1)^{i+j} M_{ij}$.

$\therefore A_{11} = (-1)^{1+1} M_{11} = (-1)^2 (3) = 3$

$A_{12} = (-1)^{1+2} M_{12} = (-1)^3 (0) = 0$

$A_{21} = (-1)^{2+1} M_{21} = (-1)^3 (-4) = 4$

$A_{22} = (-1)^{2+2} M_{22} = (-1)^4 (2) = 2$

(ii) The given determinant is $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$.

Minor of element a_{ij} is M_{ij}.

$\therefore M_{11}$ = minor of element $a_{11} = d$

M_{12} = minor of element $a_{12} = b$

M_{21} = minor of element $a_{21} = c$

M_{22} = minor of element $a_{22} = a$

Cofactor of a_{ij} is $A_{ij} = (-1)^{i+j} M_{ij}$.

$\therefore A_{11} = (-1)^{1+1} M_{11} = (-1)^2 (d) = d$

$A_{12} = (-1)^{1+2} M_{12} = (-1)^3 (b) = -b$

$A_{21} = (-1)^{2+1} M_{21} = (-1)^3 (c) = -c$

$A_{22} = (-1)^{2+2} M_{22} = (-1)^4 (a) = a$

Question 2:

(i) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ (ii) $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$

Answer

(i) The given determinant is $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$.

By the definition of minors and cofactors, we have:

M_{11} = minor of $a_{11} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

M_{12} = minor of $a_{12} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0$

M_{13} = minor of a_{13} = $\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$

M_{21} = minor of a_{21} = $\begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0$

M_{22} = minor of a_{22} = $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

M_{23} = minor of a_{23} = $\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$

M_{31} = minor of a_{31}= $\begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0$

M_{32} = minor of a_{32} = $\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$

M_{33} = minor of a_{33} = $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

A_{11} = cofactor of a_{11}= $(-1)^{1+1}$ M_{11} = 1

A_{12} = cofactor of a_{12} = $(-1)^{1+2}$ M_{12} = 0

A_{13} = cofactor of a_{13} = $(-1)^{1+3}$ M_{13} = 0

A_{21} = cofactor of a_{21} = $(-1)^{2+1}$ M_{21} = 0

A_{22} = cofactor of a_{22} = $(-1)^{2+2}$ M_{22} = 1

A_{23} = cofactor of a_{23} = $(-1)^{2+3}$ M_{23} = 0

A_{31} = cofactor of a_{31} = $(-1)^{3+1}$ M_{31} = 0

A_{32} = cofactor of a_{32} = $(-1)^{3+2}$ M_{32} = 0

A_{33} = cofactor of a_{33} = $(-1)^{3+3}$ M_{33} = 1

(ii) The given determinant is $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$.

By definition of minors and cofactors, we have:

M_{11} = minor of a_{11}= $\begin{vmatrix} 5 & -1 \\ 1 & 2 \end{vmatrix} = 10 + 1 = 11$

M_{12} = minor of a_{12}= $\begin{vmatrix} 3 & -1 \\ 0 & 2 \end{vmatrix} = 6 - 0 = 6$

M_{13} = minor of a_{13} = $\begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3 - 0 = 3$

M_{21} = minor of a_{21} = $\begin{vmatrix} 0 & 4 \\ 1 & 2 \end{vmatrix} = 0 - 4 = -4$

M_{22} = minor of a_{22} = $\begin{vmatrix} 1 & 4 \\ 0 & 2 \end{vmatrix} = 2 - 0 = 2$

M_{23} = minor of a_{23} = $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$

M_{31} = minor of a_{31}= $\begin{vmatrix} 0 & 4 \\ 5 & -1 \end{vmatrix} = 0 - 20 = -20$

M_{32} = minor of a_{32} = $\begin{vmatrix} 1 & 4 \\ 3 & -1 \end{vmatrix} = -1 - 12 = -13$

M_{33} = minor of a_{33} = $\begin{vmatrix} 1 & 0 \\ 3 & 5 \end{vmatrix} = 5 - 0 = 5$

A_{11} = cofactor of a_{11}= $(-1)^{1+1}$ M_{11} = 11

A_{12} = cofactor of a_{12} = $(-1)^{1+2}$ M_{12} = -6

A_{13} = cofactor of a_{13} = $(-1)^{1+3}$ M_{13} = 3

A_{21} = cofactor of a_{21} = $(-1)^{2+1}$ M_{21} = 4

A_{22} = cofactor of a_{22} = $(-1)^{2+2}$ M_{22} = 2

A_{23} = cofactor of a_{23} = $(-1)^{2+3}$ M_{23} = -1

A_{31} = cofactor of a_{31} = $(-1)^{3+1}$ M_{31} = -20

A_{32} = cofactor of a_{32} = $(-1)^{3+2}$ M_{32} = 13

A_{33} = cofactor of a_{33} = $(-1)^{3+3}$ M_{33} = 5

Question 3:

Using Cofactors of elements of second row, evaluate $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$.

Answer

The given determinant is $\begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$.

We have:

$M_{21} = \begin{vmatrix} 3 & 8 \\ 2 & 3 \end{vmatrix} = 9 - 16 = -7$

$\therefore A_{21}$ = cofactor of $a_{21} = (-1)^{2+1} M_{21} = 7$

$M_{22} = \begin{vmatrix} 5 & 8 \\ 1 & 3 \end{vmatrix} = 15 - 8 = 7$

$\therefore A_{22}$ = cofactor of $a_{22} = (-1)^{2+2} M_{22} = 7$

$M_{23} = \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = 10 - 3 = 7$

$\therefore A_{23}$ = cofactor of $a_{23} = (-1)^{2+3} M_{23} = -7$

We know that Δ is equal to the sum of the product of the elements of the second row with their corresponding cofactors.

$\therefore \Delta = a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23} = 2(7) + 0(7) + 1(-7) = 14 - 7 = 7$

Question 4:

Using Cofactors of elements of third column, evaluate $\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$

Answer

The given determinant is $\begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$.

We have:

$M_{13} = \begin{vmatrix} 1 & y \\ 1 & z \end{vmatrix} = z - y$

$M_{23} = \begin{vmatrix} 1 & x \\ 1 & z \end{vmatrix} = z - x$

$M_{33} = \begin{vmatrix} 1 & x \\ 1 & y \end{vmatrix} = y - x$

$\therefore A_{13}$ = cofactor of $a_{13} = (-1)^{1+3} M_{13} = (z - y)$

A_{23} = cofactor of $a_{23} = (-1)^{2+3} M_{23} = -(z - x) = (x - z)$

A_{33} = cofactor of $a_{33} = (-1)^{3+3} M_{33} = (y - x)$

We know that Δ is equal to the sum of the product of the elements of the second row with their corresponding cofactors.

$$\therefore \Delta = a_{13}A_{13} + a_{23}A_{23} + a_{33}A_{33}$$
$$= yz(z-y) + zx(x-z) + xy(y-x)$$
$$= yz^2 - y^2z + x^2z - xz^2 + xy^2 - x^2y$$
$$= (x^2z - y^2z) + (yz^2 - xz^2) + (xy^2 - x^2y)$$
$$= z(x^2 - y^2) + z^2(y-x) + xy(y-x)$$
$$= z(x-y)(x+y) + z^2(y-x) + xy(y-x)$$
$$= (x-y)[zx + zy - z^2 - xy]$$
$$= (x-y)[z(x-z) + y(z-x)]$$
$$= (x-y)(z-x)[-z+y]$$
$$= (x-y)(y-z)(z-x)$$

Hence, $\Delta = (x-y)(y-z)(z-x)$.

Question 5:

If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ A_{ij} is Cofactors of a_{ij}, then value of Δ is given by

(A) $a_{11}A_{11} + a_{12}A_{32} + a_{13}A_{33}$
(B) $a_{11}A_{11} + a_{12}A_{21} + a_{13}A_{31}$
(C) $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$
(D) $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

Answer 5:

The value of $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ is given by: $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

Hence, the option (D) is correct.

Class XII : Maths
Chapter 4 : Determinants

Questions and Solutions | Exercise 4.4 - NCERT Books

Question 1:

Find adjoint of each of the matrices.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Answer

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

We have,

$A_{11} = 4,\ A_{12} = -3,\ A_{21} = -2,\ A_{22} = 1$

$$\therefore adjA = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

Question 2:

Find adjoint of each of the matrices.

$$\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$$

Answer

Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$.

We have,

$$A_{11} = \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3 - 0 = 3$$

$$A_{12} = -\begin{vmatrix} 2 & 5 \\ -2 & 1 \end{vmatrix} = -(2 + 10) = -12$$

$$A_{13} = \begin{vmatrix} 2 & 3 \\ -2 & 0 \end{vmatrix} = 0 + 6 = 6$$

$$A_{21} = -\begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} = -(-1-0) = 1$$

$$A_{22} = \begin{vmatrix} 1 & 2 \\ -2 & 1 \end{vmatrix} = 1+4 = 5$$

$$A_{23} = -\begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} = -(0-2) = 2$$

$$A_{31} = \begin{vmatrix} -1 & 2 \\ 3 & 5 \end{vmatrix} = -5-6 = -11$$

$$A_{32} = -\begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} = -(5-4) = -1$$

$$A_{33} = \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 3+2 = 5$$

$$\text{Hence, } adjA = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} 3 & 1 & -11 \\ -12 & 5 & -1 \\ 6 & 2 & 5 \end{bmatrix}.$$

Question 3:

Verify A (adj A) = (adj A) A = $|A| I$.

$$\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$$

Answer

$$A=\begin{bmatrix}2 & 3\\ -4 & -6\end{bmatrix}$$

we have,

$$|A|=-12-(-12)=-12+12=0$$

$$\therefore |A|I=0\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

Now,

$$A_{11}=-6, A_{12}=4, A_{21}=-3, A_{22}=2$$

$$\therefore adjA=\begin{bmatrix}-6 & -3\\ 4 & 2\end{bmatrix}$$

Now,

$$A(adjA)=\begin{bmatrix}2 & 3\\ -4 & -6\end{bmatrix}\begin{bmatrix}-6 & -3\\ 4 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-12+12 & -6+6\\ 24-24 & 12-12\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

Also, $(adjA)A=\begin{bmatrix}-6 & -3\\ 4 & 2\end{bmatrix}\begin{bmatrix}2 & 3\\ -4 & -6\end{bmatrix}$

$$=\begin{bmatrix}-12+12 & -18+18\\ 8-8 & 12-12\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

Hence, $A(adjA)=(adjA)A=|A|I$.

Question 4:

Verify *A* (*adj A*) = (*adj A*) *A* = $|A|I$.

$$\begin{bmatrix}1 & -1 & 2\\ 3 & 0 & -2\\ 1 & 0 & 3\end{bmatrix}$$

Answer

$$A=\begin{bmatrix}1 & -1 & 2\\ 3 & 0 & -2\\ 1 & 0 & 3\end{bmatrix}$$

$$|A|=1(0-0)+1(9+2)+2(0-0)=11$$

$$\therefore |A|I=11\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}11 & 0 & 0\\ 0 & 11 & 0\\ 0 & 0 & 11\end{bmatrix}$$

Now,

$A_{11}=0, A_{12}=-(9+2)=-11, A_{13}=0$

$A_{21}=-(-3-0)=3, A_{22}=3-2=1, A_{23}=-(0+1)=-1$

$A_{31}=2-0=2, A_{32}=-(-2-6)=8, A_{33}=0+3=3$

$$\therefore adjA=\begin{bmatrix}0 & 3 & 2\\ -11 & 1 & 8\\ 0 & -1 & 3\end{bmatrix}$$

Now,

$$A(adjA)=\begin{bmatrix}1 & -1 & 2\\ 3 & 0 & -2\\ 1 & 0 & 3\end{bmatrix}\begin{bmatrix}0 & 3 & 2\\ -11 & 1 & 8\\ 0 & -1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}0+11+0 & 3-1-2 & 2-8+6\\ 0+0+0 & 9+0+2 & 6+0-6\\ 0+0+0 & 3+0-3 & 2+0+9\end{bmatrix}$$

$$=\begin{bmatrix}11 & 0 & 0\\ 0 & 11 & 0\\ 0 & 0 & 11\end{bmatrix}$$

Also,

$$(adjA)\cdot A=\begin{bmatrix}0 & 3 & 2\\ -11 & 1 & 8\\ 0 & -1 & 3\end{bmatrix}\begin{bmatrix}1 & -1 & 2\\ 3 & 0 & -2\\ 1 & 0 & 3\end{bmatrix}$$

$$=\begin{bmatrix}0+9+2 & 0+0+0 & 0-6+6\\ -11+3+8 & 11+0+0 & -22-2+24\\ 0-3+3 & 0+0+0 & 0+2+9\end{bmatrix}$$

$$=\begin{bmatrix}11 & 0 & 0\\ 0 & 11 & 0\\ 0 & 0 & 11\end{bmatrix}$$

Hence, $A(adjA)=(adjA)A=|A|I$.

Question 5:

Find the inverse of each of the matrices (if it exists): $\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$

Answer 5:

Here, $A = \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$,

Therefore, $A_{11} = 3 \quad A_{12} = -4 \quad A_{21} = 2 \quad A_{22} = 2 \; |A| = 6 + 8 = 14 \neq 0 \Rightarrow A^{-1}$ exists.

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{|A|} \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \frac{1}{14} \begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$$

Question 6:

Find the inverse of each of the matrices (if it exists).

$\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$

Answer

Let $A = \begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$.

we have,

$|A| = -2 + 15 = 13$

Now,

$A_{11} = 2, A_{12} = 3, A_{21} = -5, A_{22} = -1$

$\therefore adjA = \begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|} adjA = \frac{1}{13} \begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$

Question 7:

Find the inverse of each of the matrices (if it exists).

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$$

Answer

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$.

We have,

$|A| = 1(10-0) - 2(0-0) + 3(0-0) = 10$

Now,

$A_{11} = 10 - 0 = 10, A_{12} = -(0-0) = 0, A_{13} = 0 - 0 = 0$

$A_{21} = -(10-0) = -10, A_{22} = 5 - 0 = 5, A_{23} = -(0-0) = 0$

$A_{31} = 8 - 6 = 2, A_{32} = -(4-0) = -4, A_{33} = 2 - 0 = 2$

$$\therefore adjA = \begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\therefore \; A^{-1} = \frac{1}{|A|} adjA = \frac{1}{10}\begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

Question 8:

Find the inverse of each of the matrices (if it exists).

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$$

Answer

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$.

We have,

$|A| = 1(-3-0) - 0 + 0 = -3$

Now,

$A_{11} = -3 - 0 = -3, A_{12} = -(-3-0) = 3, A_{13} = 6 - 15 = -9$

$A_{21} = -(0-0) = 0, A_{22} = -1 - 0 = -1, A_{23} = -(2-0) = -2$

$A_{31} = 0 - 0 = 0, A_{32} = -(0-0) = 0, A_{33} = 3 - 0 = 3$

$$\therefore adjA = \begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

$$\therefore \ A^{-1} = \frac{1}{|A|} adjA = -\frac{1}{3}\begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

Question 9:

Find the inverse of each of the matrices (if it exists).

$$\begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$$

Answer

Let $A=\begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$.

We have,

$$\begin{aligned} |A| &= 2(-1-0)-1(4-0)+3(8-7) \\ &= 2(-1)-1(4)+3(1) \\ &= -2-4+3 \\ &= -3 \end{aligned}$$

Now,

$A_{11}=-1-0=-1, A_{12}=-(4-0)=-4, A_{13}=8-7=1$

$A_{21}=-(1-6)=5, A_{22}=2+21=23, A_{23}=-(4+7)=-11$

$A_{31}=0+3=3, A_{32}=-(0-12)=12, A_{33}=-2-4=-6$

$$\therefore adjA=\begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$$

$$\therefore A^{-1}=\frac{1}{|A|}adjA=-\frac{1}{3}\begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$$

Question 10:

Find the inverse of each of the matrices (if it exists).

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}.$$

Answer

Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$.

By expanding along C_1, we have:

$|A| = 1(8-6) - 0 + 3(3-4) = 2 - 3 = -1$

Now,

$A_{11} = 8-6 = 2, A_{12} = -(0+9) = -9, A_{13} = 0-6 = -6$

$A_{21} = -(-4+4) = 0, A_{22} = 4-6 = -2, A_{23} = -(-2+3) = -1$

$A_{31} = 3-4 = -1, A_{32} = -(-3-0) = 3, A_{33} = 2-0 = 2$

$\therefore adjA = \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|} adjA = -\begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

Question 11:

Find the inverse of each of the matrices (if it exists).

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$

Answer

Let $A=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\alpha & \sin\alpha\\ 0 & \sin\alpha & -\cos\alpha\end{bmatrix}$.

We have,

$|A|=1\left(-\cos^2\alpha-\sin^2\alpha\right)=-\left(\cos^2\alpha+\sin^2\alpha\right)=-1$

Now,

$A_{11}=-\cos^2\alpha-\sin^2\alpha=-1, A_{12}=0, A_{13}=0$

$A_{21}=0, A_{22}=-\cos\alpha, A_{23}=-\sin\alpha$

$A_{31}=0, A_{32}=-\sin\alpha, A_{33}=\cos\alpha$

$\therefore adjA=\begin{bmatrix}-1 & 0 & 0\\ 0 & -\cos\alpha & -\sin\alpha\\ 0 & -\sin\alpha & \cos\alpha\end{bmatrix}$

$\therefore\ A^{-1}=\frac{1}{|A|}\cdot adjA=-\begin{bmatrix}-1 & 0 & 0\\ 0 & -\cos\alpha & -\sin\alpha\\ 0 & -\sin\alpha & \cos\alpha\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\alpha & \sin\alpha\\ 0 & \sin\alpha & -\cos\alpha\end{bmatrix}$

Question 12:

Let $A=\begin{bmatrix}3 & 7\\ 2 & 5\end{bmatrix}$ and $B=\begin{bmatrix}6 & 8\\ 7 & 9\end{bmatrix}$. Verify that $(AB)^{-1}=B^{-1}A^{-1}$

Answer

Let $A=\begin{bmatrix}3 & 7\\ 2 & 5\end{bmatrix}$.

We have,

$|A|=15-14=1$

Now,

$A_{11}=5, A_{12}=-2, A_{21}=-7, A_{22}=3$

$\therefore adjA=\begin{bmatrix}5 & -7\\ -2 & 3\end{bmatrix}$

$\therefore A^{-1}=\frac{1}{|A|}\cdot adjA=\begin{bmatrix}5 & -7\\ -2 & 3\end{bmatrix}$

Now, let $B=\begin{bmatrix}6 & 8\\ 7 & 9\end{bmatrix}$.

We have,

$|B|=54-56=-2$

$\therefore adjB=\begin{bmatrix}9 & -8\\ -7 & 6\end{bmatrix}$

$$\therefore B^{-1}=\frac{1}{|B|}adjB=-\frac{1}{2}\begin{bmatrix}9 & -8\\ -7 & 6\end{bmatrix}=\begin{bmatrix}-\frac{9}{2} & 4\\ \frac{7}{2} & -3\end{bmatrix}$$

Now,

$$B^{-1}A^{-1}=\begin{bmatrix}-\frac{9}{2} & 4\\ \frac{7}{2} & -3\end{bmatrix}\begin{bmatrix}5 & -7\\ -2 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-\frac{45}{2}-8 & \frac{63}{2}+12\\ \frac{35}{2}+6 & -\frac{49}{2}-9\end{bmatrix}=\begin{bmatrix}-\frac{61}{2} & \frac{87}{2}\\ \frac{47}{2} & -\frac{67}{2}\end{bmatrix} \quad ...(1)$$

Then,

$$AB=\begin{bmatrix}3 & 7\\ 2 & 5\end{bmatrix}\begin{bmatrix}6 & 8\\ 7 & 9\end{bmatrix}$$

$$=\begin{bmatrix}18+49 & 24+63\\ 12+35 & 16+45\end{bmatrix}$$

$$=\begin{bmatrix}67 & 87\\ 47 & 61\end{bmatrix}$$

Therefore,we have $|AB|=67\times 61-87\times 47=4087-4089=-2.$

Also,

$$adj(AB)=\begin{bmatrix}61 & -87\\ -47 & 67\end{bmatrix}$$

$$\therefore (AB)^{-1}=\frac{1}{|AB|}adj(AB)=-\frac{1}{2}\begin{bmatrix}61 & -87\\ -47 & 67\end{bmatrix}$$

$$=\begin{bmatrix}-\frac{61}{2} & \frac{87}{2}\\ \frac{47}{2} & -\frac{67}{2}\end{bmatrix} \quad ...(2)$$

From (1) and (2), we have:

$(AB)^{-1} = B^{-1}A^{-1}$

Hence, the given result is proved.

Question 13:

If $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$, show that $A^2-5A+7I=O$. Hence find A^{-1}.

Answer

$$A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$$

$$A^2=A\cdot A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}=\begin{bmatrix}9-1 & 3+2\\ -3-2 & -1+4\end{bmatrix}=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}$$

$$\therefore A^2-5A+7I$$

$$=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}-5\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}+7\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}-\begin{bmatrix}15 & 5\\ -5 & 10\end{bmatrix}+\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$$

$$=\begin{bmatrix}-7 & 0\\ 0 & -7\end{bmatrix}+\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

Hence, $A^2-5A+7I=O.$

$$\therefore A\cdot A-5A=-7I$$

$$\Rightarrow A\cdot A\left(A^{-1}\right)-5AA^{-1}=-7IA^{-1} \qquad \left[\text{Post-multiplying by } A^{-1} \text{ as } |A|\neq 0\right]$$

$$\Rightarrow A\left(AA^{-1}\right)-5I=-7A^{-1}$$

$$\Rightarrow AI-5I=-7A^{-1}$$

$$\Rightarrow A^{-1}=-\frac{1}{7}(A-5I)$$

$$\Rightarrow A^{-1}=\frac{1}{7}(5I-A)$$

$$=\frac{1}{7}\left(\begin{bmatrix}5 & 0\\ 0 & 5\end{bmatrix}-\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}\right)=\frac{1}{7}\begin{bmatrix}2 & -1\\ 1 & 3\end{bmatrix}$$

$$\therefore A^{-1}=\frac{1}{7}\begin{bmatrix}2 & -1\\ 1 & 3\end{bmatrix}$$

Question 14:

For the matrix $A=\begin{bmatrix}3 & 2\\ 1 & 1\end{bmatrix}$, find the numbers *a* and *b* such that $A^2 + aA + bI = O$.

Answer

$$A=\begin{bmatrix}3 & 2\\ 1 & 1\end{bmatrix}$$

$$\therefore A^2=\begin{bmatrix}3 & 2\\ 1 & 1\end{bmatrix}\begin{bmatrix}3 & 2\\ 1 & 1\end{bmatrix}=\begin{bmatrix}9+2 & 6+2\\ 3+1 & 2+1\end{bmatrix}=\begin{bmatrix}11 & 8\\ 4 & 3\end{bmatrix}$$

Now,

$$A^2+aA+bI=O$$

$$\Rightarrow (AA)A^{-1}+aAA^{-1}+bIA^{-1}=O \qquad \left[\text{Post-multiplying by } A^{-1} \text{ as } |A|\neq 0\right]$$

$$\Rightarrow A\left(AA^{-1}\right)+aI+b\left(IA^{-1}\right)=O$$

$$\Rightarrow AI+aI+bA^{-1}=O$$

$$\Rightarrow A+aI=-bA^{-1}$$

$$\Rightarrow A^{-1}=-\frac{1}{b}(A+aI)$$

Now,

$$A^{-1}=\frac{1}{|A|}adjA=\frac{1}{1}\begin{bmatrix}1 & -2\\ -1 & 3\end{bmatrix}=\begin{bmatrix}1 & -2\\ -1 & 3\end{bmatrix}$$

We have:

$$\begin{bmatrix}1 & -2\\ -1 & 3\end{bmatrix}=-\frac{1}{b}\left(\begin{bmatrix}3 & 2\\ 1 & 1\end{bmatrix}+\begin{bmatrix}a & 0\\ 0 & a\end{bmatrix}\right)=-\frac{1}{b}\begin{bmatrix}3+a & 2\\ 1 & 1+a\end{bmatrix}=\begin{bmatrix}\frac{-3-a}{b} & -\frac{2}{b}\\ -\frac{1}{b} & \frac{-1-a}{b}\end{bmatrix}$$

Comparing the corresponding elements of the two matrices, we have:

$$-\frac{1}{b}=-1\Rightarrow b=1$$

$$\frac{-3-a}{b}=1\Rightarrow -3-a=1\Rightarrow a=-4$$

Hence, −4 and 1 are the required values of *a* and *b* respectively.

Question 15:

For the matrix $A=\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{bmatrix}$ show that $A^3 - 6A^2 + 5A + 11I = O$. Hence, find A^{-1}.

Answer

$$A=\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{bmatrix}$$

$$A^2=\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{bmatrix}\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}1+1+2 & 1+2-1 & 1-3+3\\ 1+2-6 & 1+4+3 & 1-6-9\\ 2-1+6 & 2-2-3 & 2+3+9\end{bmatrix}=\begin{bmatrix}4 & 2 & 1\\ -3 & 8 & -14\\ 7 & -3 & 14\end{bmatrix}$$

$$A^3=A^2\cdot A=\begin{bmatrix}4 & 2 & 1\\ -3 & 8 & -14\\ 7 & -3 & 14\end{bmatrix}\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}4+2+2 & 4+4-1 & 4-6+3\\ -3+8-28 & -3+16+14 & -3-24-42\\ 7-3+28 & 7-6-14 & 7+9+42\end{bmatrix}$$

$$=\begin{bmatrix}8 & 7 & 1\\ -23 & 27 & -69\\ 32 & -13 & 58\end{bmatrix}$$

$\therefore A^3-6A^2+5A+11I$

$$=\begin{bmatrix}8&7&1\\-23&27&-69\\32&-13&58\end{bmatrix}-6\begin{bmatrix}4&2&1\\-3&8&-14\\7&-3&14\end{bmatrix}+5\begin{bmatrix}1&1&1\\1&2&-3\\2&-1&3\end{bmatrix}+11\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}8&7&1\\-23&27&-69\\32&-13&58\end{bmatrix}-\begin{bmatrix}24&12&6\\-18&48&-84\\42&-18&84\end{bmatrix}+\begin{bmatrix}5&5&5\\5&10&-15\\10&-5&15\end{bmatrix}+\begin{bmatrix}11&0&0\\0&11&0\\0&0&11\end{bmatrix}$$

$$=\begin{bmatrix}24&12&6\\-18&48&-84\\42&-18&84\end{bmatrix}-\begin{bmatrix}24&12&6\\-18&48&-84\\42&-18&84\end{bmatrix}$$

$$=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}=O$$

Thus, $A^3-6A^2+5A+11I=O$.

Now,

$A^3-6A^2+5A+11I=O$

$\Rightarrow (AAA)A^{-1}-6(AA)A^{-1}+5AA^{-1}+11IA^{-1}=0$ [Post-multiplying by A^{-1} as $|A|\neq 0$]

$\Rightarrow AA(AA^{-1})-6A(AA^{-1})+5(AA^{-1})=-11(IA^{-1})$

$\Rightarrow A^2-6A+5I=-11A^{-1}$

$\Rightarrow A^{-1}=-\frac{1}{11}(A^2-6A+5I)$ ...(1)

Now,

$A^2 - 6A + 5I$

$$= \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} - 6\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} + 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} - \begin{bmatrix} 6 & 6 & 6 \\ 6 & 12 & -18 \\ 12 & -6 & 18 \end{bmatrix} + \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 2 & 1 \\ -3 & 13 & -14 \\ 7 & -3 & 19 \end{bmatrix} - \begin{bmatrix} 6 & 6 & 6 \\ 6 & 12 & -18 \\ 12 & -6 & 18 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix}$$

From equation (1), we have:

$$A^{-1} = -\frac{1}{11}\begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$$

Question 16:

If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ verify that $A^3 - 6A^2 + 9A - 4I = O$ and hence find A^{-1}

Answer

$$A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$$

$$A^2=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}4+1+1 & -2-2-1 & 2+1+2\\ -2-2-1 & 1+4+1 & -1-2-2\\ 2+1+2 & -1-2-2 & 1+1+4\end{bmatrix}$$

$$=\begin{bmatrix}6 & -5 & 5\\ -5 & 6 & -5\\ 5 & -5 & 6\end{bmatrix}$$

$$A^3=A^2A=\begin{bmatrix}6 & -5 & 5\\ -5 & 6 & -5\\ 5 & -5 & 6\end{bmatrix}\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}12+5+5 & -6-10-5 & 6+5+10\\ -10-6-5 & 5+12+5 & -5-6-10\\ 10+5+6 & -5-10-6 & 5+5+12\end{bmatrix}$$

$$=\begin{bmatrix}22 & -21 & 21\\ -21 & 22 & -21\\ 21 & -21 & 22\end{bmatrix}$$

Now,

$A^3-6A^2+9A-4I$

$$=\begin{bmatrix}22 & -21 & 21\\ -21 & 22 & -21\\ 21 & -21 & 22\end{bmatrix}-6\begin{bmatrix}6 & -5 & 5\\ -5 & 6 & -5\\ 5 & -5 & 6\end{bmatrix}+9\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}-4\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}22 & -21 & 21\\ -21 & 22 & -21\\ 21 & -21 & 22\end{bmatrix}-\begin{bmatrix}36 & -30 & 30\\ -30 & 36 & -30\\ 30 & -30 & 36\end{bmatrix}+\begin{bmatrix}18 & -9 & 9\\ -9 & 18 & -9\\ 9 & -9 & 18\end{bmatrix}-\begin{bmatrix}4 & 0 & 0\\ 0 & 4 & 0\\ 0 & 0 & 4\end{bmatrix}$$

$$=\begin{bmatrix}40 & -30 & 30\\ -30 & 40 & -30\\ 30 & -30 & 40\end{bmatrix}-\begin{bmatrix}40 & -30 & 30\\ -30 & 40 & -30\\ 30 & -30 & 40\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$\therefore A^3-6A^2+9A-4I=O$

Now,

$A^3-6A^2+9A-4I=O$

$\Rightarrow (AAA)A^{-1}-6(AA)A^{-1}+9AA^{-1}-4IA^{-1}=O$ $\quad$ $\left[\text{Post-multiplying by } A^{-1} \text{ as } |A|\neq 0\right]$

$\Rightarrow AA\left(AA^{-1}\right)-6A\left(AA^{-1}\right)+9\left(AA^{-1}\right)=4\left(IA^{-1}\right)$

$\Rightarrow AAI-6AI+9I=4A^{-1}$

$\Rightarrow A^2-6A+9I=4A^{-1}$

$\Rightarrow A^{-1}=\frac{1}{4}\left(A^2-6A+9I\right) \qquad \ldots(1)$

$A^2-6A+9I$

$$=\begin{bmatrix}6 & -5 & 5\\ -5 & 6 & -5\\ 5 & -5 & 6\end{bmatrix}-6\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}+9\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$=\begin{bmatrix}6 & -5 & 5\\ -5 & 6 & -5\\ 5 & -5 & 6\end{bmatrix}-\begin{bmatrix}12 & -6 & 6\\ -6 & 12 & -6\\ 6 & -6 & 12\end{bmatrix}+\begin{bmatrix}9 & 0 & 0\\ 0 & 9 & 0\\ 0 & 0 & 9\end{bmatrix}$$

$$=\begin{bmatrix}3 & 1 & -1\\ 1 & 3 & 1\\ -1 & 1 & 3\end{bmatrix}$$

From equation (1), we have:

$$A^{-1}=\frac{1}{4}\begin{bmatrix}3 & 1 & -1\\ 1 & 3 & 1\\ -1 & 1 & 3\end{bmatrix}$$

Question 17:

Let A be a nonsingular square matrix of order 3 × 3. Then $|adjA|$ is equal to

A. $|A|$ **B.** $|A|^2$ **C.** $|A|^3$ **D.** $3|A|$

Answer **B**

We know that,

$$(adjA)A=|A|I=\begin{bmatrix}|A| & 0 & 0\\ 0 & |A| & 0\\ 0 & 0 & |A|\end{bmatrix}$$

$$\Rightarrow |(adjA)A|=\begin{vmatrix}|A| & 0 & 0\\ 0 & |A| & 0\\ 0 & 0 & |A|\end{vmatrix}$$

$$\Rightarrow |adjA||A|=|A|^3\begin{vmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{vmatrix}=|A|^3(I)$$

$$\therefore |adjA|=|A|^2$$

Hence, the correct answer is B.

Question 18:

If A is an invertible matrix of order 2, then det (A^{-1}) is equal to

A. det (A) **B.** $\frac{1}{\det(A)}$ **C.** 1 **D.** 0

Answer

Since A is an invertible matrix, A^{-1} exists and $A^{-1}=\frac{1}{|A|}adjA.$

As matrix A is of order 2, let $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

Then, $|A| = ad - bc$ and $adjA=\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Now,

$$A^{-1} = \frac{1}{|A|} adjA = \begin{bmatrix} \frac{d}{|A|} & \frac{-b}{|A|} \\ \frac{-c}{|A|} & \frac{a}{|A|} \end{bmatrix}$$

$$\therefore |A^{-1}| = \begin{vmatrix} \frac{d}{|A|} & \frac{-b}{|A|} \\ \frac{-c}{|A|} & \frac{a}{|A|} \end{vmatrix} = \frac{1}{|A|^2}\begin{vmatrix} d & -b \\ -c & a \end{vmatrix} = \frac{1}{|A|^2}(ad-bc) = \frac{1}{|A|^2}.|A| = \frac{1}{|A|}$$

$$\therefore \det\left(A^{-1}\right) = \frac{1}{\det(A)}$$

Hence, the correct answer is B.

Class XII : Maths
Chapter 4 : Determinants

Questions and Solutions | Exercise 4.5 - NCERT Books

Question 1:

Examine the consistency of the system of equations.

$x + 2y = 2$

$2x + 3y = 3$

Answer

The given system of equations is:

$x + 2y = 2$

$2x + 3y = 3$

The given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}.$$

Now,

$$|A| = 1(3) - 2(2) = 3 - 4 = -1 \neq 0$$

$\therefore$ A is non-singular.

Therefore, A^{-1} exists.

Hence, the given system of equations is consistent.

Question 2:

Examine the consistency of the system of equations.

$2x - y = 5$

$x + y = 4$

Answer

The given system of equations is:

$2x - y = 5$

$x + y = 4$

The given system of equations can be written in the form of $AX = B$, where

$$A=\begin{bmatrix}2 & -1\\1 & 1\end{bmatrix},\ X=\begin{bmatrix}x\\z\end{bmatrix}\text{ and }B=\begin{bmatrix}5\\4\end{bmatrix}.$$

Now,

$$|A|=2(1)-(-1)(1)=2+1=3\neq 0$$

∴ A is non-singular.

Therefore, A^{-1} exists.

Hence, the given system of equations is consistent.

Question 3:

Examine the consistency of the system of equations.

$x + 3y = 5$

$2x + 6y = 8$

Answer

The given system of equations is:

$x + 3y = 5$

$2x + 6y = 8$

The given system of equations can be written in the form of $AX = B$, where

$$A=\begin{bmatrix}1 & 3\\2 & 6\end{bmatrix},\ X=\begin{bmatrix}x\\y\end{bmatrix}\text{ and }B=\begin{bmatrix}5\\8\end{bmatrix}.$$

Now,

$$|A|=1(6)-3(2)=6-6=0$$

∴ A is a singular matrix.

Now,

$$(adjA)=\begin{bmatrix}6 & -3\\ -2 & 1\end{bmatrix}$$

$$(adjA)B=\begin{bmatrix}6 & -3\\ -2 & 1\end{bmatrix}\begin{bmatrix}5\\ 8\end{bmatrix}=\begin{bmatrix}30-24\\ -10+8\end{bmatrix}=\begin{bmatrix}6\\ -2\end{bmatrix}\neq O$$

Thus, the solution of the given system of equations does not exist. Hence, the system of equations is inconsistent.

Question 4:

Examine the consistency of the system of equations.

$x + y + z = 1$

$2x + 3y + 2z = 2$

$ax + ay + 2az = 4$

Answer

The given system of equations is:

$x + y + z = 1$

$2x + 3y + 2z = 2$

$ax + ay + 2az = 4$

This system of equations can be written in the form $AX = B$, where

$$A=\begin{bmatrix}1 & 1 & 1\\ 2 & 3 & 2\\ a & a & 2a\end{bmatrix},\ X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}\text{ and }B=\begin{bmatrix}1\\ 2\\ 4\end{bmatrix}.$$

Now,

$$|A|=1(6a-2a)-1(4a-2a)+1(2a-3a)$$
$$=4a-2a-a=4a-3a=a\neq 0$$

$\therefore$ A is non-singular.

Therefore, A^{-1} exists.

Hence, the given system of equations is consistent.

Question 5:

Examine the consistency of the system of equations.

$3x - y - 2z = 2$

$2y - z = -1$

$3x - 5y = 3$

Answer

The given system of equations is:

$3x - y - 2z = 2$

$2y - z = -1$

$3x - 5y = 3$

This system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}.$$

Now,

$$|A| = 3(0-5) - 0 + 3(1+4) = -15 + 15 = 0$$

$\therefore$ A is a singular matrix.

Now,

$$(adjA) = \begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix}$$

$$\therefore (adjA)B = \begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -10-10+15 \\ -6-6+9 \\ -12-12+18 \end{bmatrix} = \begin{bmatrix} -5 \\ -3 \\ -6 \end{bmatrix} \neq O$$

Thus, the solution of the given system of equations does not exist. Hence, the system of equations is inconsistent.

Question 6:

Examine the consistency of the system of equations.

$5x - y + 4z = 5$

$2x + 3y + 5z = 2$

$5x - 2y + 6z = -1$

Answer

The given system of equations is:

$5x - y + 4z = 5$

$2x + 3y + 5z = 2$

$5x - 2y + 6z = -1$

This system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}.$$

Now,

$$\begin{aligned} |A| &= 5(18+10)+1(12-25)+4(-4-15) \\ &= 5(28)+1(-13)+4(-19) \\ &= 140-13-76 \\ &= 51 \neq 0 \end{aligned}$$

$\therefore$ A is non-singular.

Therefore, A^{-1} exists.

Hence, the given system of equations is consistent.

Question 7:

Solve system of linear equations, using matrix method.

$5x + 2y = 4$

$7x + 3y = 5$

Answer

The given system of equations can be written in the form of $AX = B$, where

$$A=\begin{bmatrix}5 & 2\\ 7 & 3\end{bmatrix},\ X=\begin{bmatrix}x\\ y\end{bmatrix} \text{ and } B=\begin{bmatrix}4\\ 5\end{bmatrix}.$$

Now, $|A| = 15-14 = 1 \neq 0$.

Thus, A is non-singular. Therefore, its inverse exists.

Now,

$$A^{-1}=\frac{1}{|A|}(adjA)$$

$$\therefore A^{-1}=\begin{bmatrix}3 & -2\\ -7 & 5\end{bmatrix}$$

$$\therefore X = A^{-1}B=\begin{bmatrix}3 & -2\\ -7 & 5\end{bmatrix}\begin{bmatrix}4\\ 5\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}12-10\\ -28+25\end{bmatrix}=\begin{bmatrix}2\\ -3\end{bmatrix}$$

Hence, $x = 2$ and $y = -3$.

Question 8:

Solve system of linear equations, using matrix method.

$2x - y = -2$

$3x + 4y = 3$

Answer

The given system of equations can be written in the form of $AX = B$, where

$$A=\begin{bmatrix}2 & -1\\ 3 & 4\end{bmatrix},\ X=\begin{bmatrix}x\\ y\end{bmatrix} \text{ and } B=\begin{bmatrix}-2\\ 3\end{bmatrix}.$$

Now,

$|A| = 8+3 = 11 \neq 0$

Thus, A is non-singular. Therefore, its inverse exists.

Now,

$$A^{-1}=\frac{1}{|A|}adjA=\frac{1}{11}\begin{bmatrix}4 & 1\\ -3 & 2\end{bmatrix}$$

$$\therefore X=A^{-1}B=\frac{1}{11}\begin{bmatrix}4 & 1\\ -3 & 2\end{bmatrix}\begin{bmatrix}-2\\ 3\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{11}\begin{bmatrix}-8+3\\ 6+6\end{bmatrix}=\frac{1}{11}\begin{bmatrix}-5\\ 12\end{bmatrix}=\begin{bmatrix}-\frac{5}{11}\\ \frac{12}{11}\end{bmatrix}$$

Hence, $x=\frac{-5}{11}$ and $y=\frac{12}{11}$.

Question 9:

Solve system of linear equations, using matrix method.

$4x-3y=3$

$3x-5y=7$

Answer

The given system of equations can be written in the form of $AX = B$, where

$$A=\begin{bmatrix}4 & -3\\ 3 & -5\end{bmatrix},\ X=\begin{bmatrix}x\\ y\end{bmatrix}\text{and } B=\begin{bmatrix}3\\ 7\end{bmatrix}.$$

Now,

$|A|=-20+9=-11\neq 0$

Thus, A is non-singular. Therefore, its inverse exists.

Now,

$$A^{-1}=\frac{1}{|A|}(adjA)=-\frac{1}{11}\begin{bmatrix}-5 & 3\\ -3 & 4\end{bmatrix}=\frac{1}{11}\begin{bmatrix}5 & -3\\ 3 & -4\end{bmatrix}$$

$$\therefore X=A^{-1}B=\frac{1}{11}\begin{bmatrix}5 & -3\\ 3 & -4\end{bmatrix}\begin{bmatrix}3\\ 7\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{11}\begin{bmatrix}5 & -3\\ 3 & -4\end{bmatrix}\begin{bmatrix}3\\ 7\end{bmatrix}=\frac{1}{11}\begin{bmatrix}15-21\\ 9-28\end{bmatrix}=\frac{1}{11}\begin{bmatrix}-6\\ -19\end{bmatrix}=\begin{bmatrix}-\frac{6}{11}\\ -\frac{19}{11}\end{bmatrix}$$

Hence, $x=\frac{-6}{11}$ and $y=\frac{-19}{11}$.

Question 10:

Solve system of linear equations, using matrix method.

$5x + 2y = 3$

$3x + 2y = 5$

Answer

The given system of equations can be written in the form of $AX = B$, where

$$A=\begin{bmatrix}5 & 2\\ 3 & 2\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix} \text{ and } B=\begin{bmatrix}3\\ 5\end{bmatrix}.$$

Now,

$$|A|=10-6=4\neq 0$$

Thus, A is non-singular. Therefore, its inverse exists.

Question 11:

Solve system of linear equations, using matrix method.

$$2x+y+z=1$$

$$x-2y-z=\frac{3}{2}$$

$$3y-5z=9$$

Answer

The given system of equations can be written in the form of $AX = B$, where

$$A=\begin{bmatrix}2 & 1 & 1\\ 1 & -2 & -1\\ 0 & 3 & -5\end{bmatrix},\ X=\begin{bmatrix}x\\ y\\ z\end{bmatrix} \text{ and } B=\begin{bmatrix}1\\ \frac{3}{2}\\ 9\end{bmatrix}.$$

Now,

$$|A|=2(10+3)-1(-5-3)+0=2(13)-1(-8)=26+8=34\neq 0$$

Thus, A is non-singular. Therefore, its inverse exists.

Now, $A_{11}=13, A_{12}=5, A_{13}=3$

$A_{21}=8, A_{22}=-10, A_{23}=-6$

$A_{31}=1, A_{32}=3, A_{33}=-5$

$$\therefore\ A^{-1}=\frac{1}{|A|}(adjA)=\frac{1}{34}\begin{bmatrix}13 & 8 & 1\\ 5 & -10 & 3\\ 3 & -6 & -5\end{bmatrix}$$

$$\therefore\ X=A^{-1}B=\frac{1}{34}\begin{bmatrix}13 & 8 & 1\\ 5 & -10 & 3\\ 3 & -6 & -5\end{bmatrix}\begin{bmatrix}1\\ \frac{3}{2}\\ 9\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{34}\begin{bmatrix}13+12+9\\ 5-15+27\\ 3-9-45\end{bmatrix}$$

$$=\frac{1}{34}\begin{bmatrix}34\\ 17\\ -51\end{bmatrix}=\begin{bmatrix}1\\ \frac{1}{2}\\ -\frac{3}{2}\end{bmatrix}$$

Hence, $x=1, y=\frac{1}{2}$, and $z=-\frac{3}{2}$.

Question 12:

Solve system of linear equations, using matrix method.

$x - y + z = 4$

$2x + y - 3z = 0$

$x + y + z = 2$

Answer

The given system of equations can be written in the form of $AX = B$, where

$$A=\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix},\ X=\begin{bmatrix}x\\ y\\ z\end{bmatrix} \text{ and } B=\begin{bmatrix}4\\ 0\\ 2\end{bmatrix}.$$

Now,

$$|A| = 1(1+3)+1(2+3)+1(2-1) = 4+5+1 = 10 \neq 0$$

Thus, A is non-singular. Therefore, its inverse exists.

$$\begin{aligned}\text{Now, } & A_{11}=4, A_{12}=-5, A_{13}=1\\ & A_{21}=2, A_{22}=0, A_{23}=-2\\ & A_{31}=2, A_{32}=5, A_{33}=3\end{aligned}$$

$$\therefore\ A^{-1}=\frac{1}{|A|}(adjA)=\frac{1}{10}\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & 5\\ 1 & -2 & 3\end{bmatrix}$$

$$\therefore X = A^{-1}B=\frac{1}{10}\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & 5\\ 1 & -2 & 3\end{bmatrix}\begin{bmatrix}4\\ 0\\ 2\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{10}\begin{bmatrix}16+0+4\\ -20+0+10\\ 4+0+6\end{bmatrix}$$

$$=\frac{1}{10}\begin{bmatrix}20\\ -10\\ 10\end{bmatrix}$$

$$=\begin{bmatrix}2\\ -1\\ 1\end{bmatrix}$$

Hence, $x=2, y=-1,$ and $z=1$.

Question 13:

Solve system of linear equations, using matrix method.

$2x + 3y + 3z = 5$

$x - 2y + z = -4$

$3x - y - 2z = 3$

Answer

The given system of equations can be written in the form $AX = B$, where

$$A=\begin{bmatrix}2 & 3 & 3\\ 1 & -2 & 1\\ 3 & -1 & -2\end{bmatrix}, X=\begin{bmatrix}x\\ y\\ z\end{bmatrix} \text{and } B=\begin{bmatrix}5\\ -4\\ 3\end{bmatrix}.$$

Now,

$$|A| = 2(4+1)-3(-2-3)+3(-1+6) = 2(5)-3(-5)+3(5) = 10+15+15 = 40 \neq 0$$

Thus, A is non-singular. Therefore, its inverse exists.

$$\text{Now, } A_{11}=5, A_{12}=5, A_{13}=5$$
$$A_{21}=3, A_{22}=-13, A_{23}=11$$
$$A_{31}=9, A_{32}=1, A_{33}=-7$$

$$\therefore A^{-1}=\frac{1}{|A|}(adjA)=\frac{1}{40}\begin{bmatrix}5 & 3 & 9\\ 5 & -13 & 1\\ 5 & 11 & -7\end{bmatrix}$$

$$\therefore X = A^{-1}B=\frac{1}{40}\begin{bmatrix}5 & 3 & 9\\ 5 & -13 & 1\\ 5 & 11 & -7\end{bmatrix}\begin{bmatrix}5\\ -4\\ 3\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{40}\begin{bmatrix}25-12+27\\ 25+52+3\\ 25-44-21\end{bmatrix}$$

$$=\frac{1}{40}\begin{bmatrix}40\\ 80\\ -40\end{bmatrix}$$

$$=\begin{bmatrix}1\\ 2\\ -1\end{bmatrix}$$

Hence, $x=1, y=2, \text{and } z=-1$.

Question 14:

Solve system of linear equations, using matrix method.

$x - y + 2z = 7$

$3x + 4y - 5z = -5$

$2x - y + 3z = 12$

Answer

The given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}.$$

Now,

$$|A| = 1(12-5) + 1(9+10) + 2(-3-8) = 7 + 19 - 22 = 4 \neq 0$$

Thus, A is non-singular. Therefore, its inverse exists.

$$\text{Now, } A_{11} = 7, A_{12} = -19, A_{13} = -11$$
$$A_{21} = 1, A_{22} = -1, A_{23} = -1$$
$$A_{31} = -3, A_{32} = 11, A_{33} = 7$$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}\begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 49-5-36 \\ -133+5+132 \\ -77+5+84 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

Hence, $x = 2, y = 1, \text{and } z = 3.$

Question 15:

If $A=\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations

$$2x-3y+5z=11$$
$$3x+2y-4z=-5$$
$$x+y-2z=-3$$

Answer

$$A=\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}$$

$$\therefore |A| = 2(-4+4)+3(-6+4)+5(3-2) = 0-6+5 = -1 \neq 0$$

Now, $A_{11}=0,\ A_{12}=2,\ A_{13}=1$

$A_{21}=-1,\ A_{22}=-9,\ A_{23}=-5$

$A_{31}=2,\ A_{32}=23,\ A_{33}=13$

$$\therefore\ A^{-1}=\frac{1}{|A|}(adjA)=-\begin{bmatrix}0 & -1 & 2\\ 2 & -9 & 23\\ 1 & -5 & 13\end{bmatrix}=\begin{bmatrix}0 & 1 & -2\\ -2 & 9 & -23\\ -1 & 5 & -13\end{bmatrix} \quad ...(1)$$

Now, the given system of equations can be written in the form of $AX = B$, where

$$A=\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}, X=\begin{bmatrix}x\\ y\\ z\end{bmatrix} \text{ and } B=\begin{bmatrix}11\\ -5\\ -3\end{bmatrix}.$$

The solution of the system of equations is given by $X = A^{-1}B$.

$X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} \qquad [\text{Using } (1)]$$

$$= \begin{bmatrix} 0-5+6 \\ -22-45+69 \\ -11-25+39 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence, $x = 1,\ y = 2,$ and $z = 3$.

Question 16:

The cost of 4 kg onion, 3 kg wheat and 2 kg rice is Rs 60. The cost of 2 kg onion, 4 kg wheat and 6 kg rice is Rs 90. The cost of 6 kg onion 2 kg wheat and 3 kg rice is Rs 70. Find cost of each item per kg by matrix method.

Answer

Let the cost of onions, wheat, and rice per kg be Rs *x*, Rs *y*,and Rs *z* respectively.

Then, the given situation can be represented by a system of equations as:

$$4x + 3y + 2z = 60$$
$$2x + 4y + 6z = 90$$
$$6x + 2y + 3z = 70$$

This system of equations can be written in the form of *AX* = *B*, where

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}.$$

$$|A| = 4(12-12) - 3(6-36) + 2(4-24) = 0 + 90 - 40 = 50 \neq 0$$

Now, $A_{11} = 0, A_{12} = 30, A_{13} = -20$

$A_{21} = -5, A_{22} = 0, A_{23} = 10$

$A_{31} = 10, A_{32} = -20, A_{33} = 10$

$$\therefore adjA = \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} adjA = \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

Now,

$X = A^{-1} B$

$$\Rightarrow X = \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}\begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 0-450+700 \\ 1800+0-1400 \\ -1200+900+700 \end{bmatrix}$$

$$= \frac{1}{50}\begin{bmatrix} 250 \\ 400 \\ 400 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

$\therefore x = 5, y = 8, \text{and } z = 8.$

Hence, the cost of onions is Rs 5 per kg, the cost of wheat is Rs 8 per kg, and the cost of rice is Rs 8 per kg.

Class XII : Maths
Chapter 4 : Determinants

Questions and Solutions | Miscellaneous Exercise 4 - NCERT Books

Question 1:

Prove that the determinant $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of θ.

Answer

$$\Delta = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$$

$$= x\left(x^2-1\right)-\sin\theta\left(-x\sin\theta-\cos\theta\right)+\cos\theta\left(-\sin\theta+x\cos\theta\right)$$

$$= x^3-x+x\sin^2\theta+\sin\theta\cos\theta-\sin\theta\cos\theta+x\cos^2\theta$$

$$= x^3-x+x\left(\sin^2\theta+\cos^2\theta\right)$$

$$= x^3-x+x$$

$$= x^3 \text{ (Independent of } \theta)$$

Hence, Δ is independent of θ.

Question 2:

Evaluate $\begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$

Answer

$$\Delta = \begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$$

Expanding along C_3, we have:

$$\Delta = -\sin\alpha\left(-\sin\alpha\sin^2\beta-\cos^2\beta\sin\alpha\right)+\cos\alpha\left(\cos\alpha\cos^2\beta+\cos\alpha\sin^2\beta\right)$$

$$= \sin^2\alpha\left(\sin^2\beta+\cos^2\beta\right)+\cos^2\alpha\left(\cos^2\beta+\sin^2\beta\right)$$

$$= \sin^2\alpha(1)+\cos^2\alpha(1)$$

$$= 1$$

Question 3:

If $A^{-1}=\begin{bmatrix}3 & -1 & 1\\ -15 & 6 & -5\\ 5 & -2 & 2\end{bmatrix}$ and $B=\begin{bmatrix}1 & 2 & -2\\ -1 & 3 & 0\\ 0 & -2 & 1\end{bmatrix}$, find $(AB)^{-1}$.

Answer 3:

Here, $B=\begin{bmatrix}1 & 2 & -2\\ -1 & 3 & 0\\ 0 & -2 & 1\end{bmatrix}$,

Therefore, $|B|=1(3-0)-2(-1-0)-2(2-0)=1\neq 0 \Rightarrow B^{-1}$ exists.

$$\left.\begin{aligned}
B_{11} &= 3\\
B_{21} &= 2\\
B_{31} &= 6\\
B^{-1} &= \frac{1}{|B|}\operatorname{adj} B=\frac{1}{1}\begin{bmatrix}B_{11} & B_{21} & B_{31}\\ B_{12} & B_{22} & B_{32}\\ B_{13} & B_{23} & B_{33}\end{bmatrix}=\begin{bmatrix}3 & 2 & 6\\ 1 & 1 & 2\\ 2 & 2 & 5\end{bmatrix}
\end{aligned}\right)$$

We know that: $(AB)^{-1}=B^{-1}A^{-1}$, therefore

$$(AB)^{-1}=B^{-1}A^{-1}=\begin{bmatrix}3 & 2 & 6\\ 1 & 1 & 2\\ 2 & 2 & 5\end{bmatrix}\begin{bmatrix}3 & -1 & 1\\ -15 & 6 & -5\\ 5 & -2 & 2\end{bmatrix}$$

$$=\begin{bmatrix}9-30+30 & -3+12-12 & 3-10+12\\ 3-15+10 & -1+6-4 & 1-5+4\\ 6-30+25 & -2+12-10 & 2-10+10\end{bmatrix}=\begin{bmatrix}9 & -3 & 5\\ -2 & 1 & 0\\ 1 & 0 & 2\end{bmatrix}$$

Question 4:

Let $A=\begin{bmatrix}1 & -2 & 1\\ -2 & 3 & 1\\ 1 & 1 & 5\end{bmatrix}$. Verify that

(i) $(\operatorname{adj} A)^{-1}=\operatorname{adj}\left(A^{-1}\right)$

(ii) $\left(A^{-1}\right)^{-1}=A$

Answer 4:

(i) Here, $A=\begin{bmatrix}1 & -2 & 1\\ -2 & 3 & 1\\ 1 & 1 & 5\end{bmatrix}$, therefore

$$|A|=1(15-1)+2(-10-1)+1(-2-3)=-13\neq 0\Rightarrow A^{-1}\text{ exists.}$$

$$A_{11}=14$$
$$A_{12}=11$$
$$A_{13}=-5$$
$$A_{21}=11$$
$$A_{22}=4$$
$$A_{23}=-3$$
$$A_{31}=-5$$
$$A_{32}=-3$$
$$A_{32}=-3$$
$$A_{33}=-1$$

$$\text{dj } A=\begin{bmatrix}A_{11} & A_{21} & A_{31}\\ A_{12} & A_{22} & A_{32}\\ A_{13} & A_{23} & A_{33}\end{bmatrix}=\begin{bmatrix}14 & 11 & -5\\ 11 & 4 & -3\\ -5 & -3 & -1\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}\operatorname{adj} A=\frac{1}{|A|}\begin{bmatrix}A_{11} & A_{21} & A_{31}\\ A_{12} & A_{22} & A_{32}\\ A_{13} & A_{23} & A_{33}\end{bmatrix}=\frac{1}{-13}\begin{bmatrix}14 & 11 & -5\\ 11 & 4 & -3\\ -5 & -3 & -1\end{bmatrix}$$

adj

Let, $B=\operatorname{adj} A$, so, $B=\begin{bmatrix}14 & 11 & -5\\ 11 & 4 & -3\\ -5 & -3 & -1\end{bmatrix}$, therefore

$$|B|=14(-4-9)-11(-11-15)-5(-33+20)=-182+286+65=169\neq 0\Rightarrow B^{-1}\text{ exists.}$$

$$B_{11}=-13 \quad B_{12}=26 \quad B_{13}=-13$$
$$B_{21}=26 \quad B_{22}=-39 \quad B_{23}=-13$$
$$B_{31}=-13 \quad B_{32}=-13$$

$$B^{-1}=\frac{1}{|B|}\begin{bmatrix}B_{11} & B_{21} & B_{31}\\ B_{12} & B_{22} & B_{32}\\ B_{13} & B_{23} & B_{33}\end{bmatrix}=\frac{1}{169}\begin{bmatrix}-13 & 26 & -13\\ 26 & -39 & -13\\ -13 & -13 & -65\end{bmatrix}=\frac{1}{13}\begin{bmatrix}-1 & 2 & -1\\ 2 & -3 & -1\\ -1 & -1 & -5\end{bmatrix}$$

$$\Rightarrow (\text{adj } A)^{-1} = \frac{1}{13}\begin{bmatrix} -1 & 2 & -1 \\ 2 & -3 & -1 \\ -1 & -1 & -5 \end{bmatrix}$$

Let, $C = A^{-1}$, so, $C = \frac{1}{-13}\begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix} = \begin{bmatrix} -\frac{14}{13} & -\frac{11}{13} & \frac{5}{13} \\ -\frac{11}{13} & -\frac{4}{13} & \frac{3}{13} \\ \frac{5}{13} & \frac{3}{13} & \frac{1}{13} \end{bmatrix}$, therefore

$$C_{11} = -\frac{1}{13} \quad C_{12} = \frac{2}{13} \quad C_{13} = -\frac{1}{13}$$
$$C_{21} = \frac{2}{13} \quad C_{22} = -\frac{3}{13} \quad C_{23} = -\frac{1}{13}$$
$$C_{31} = -\frac{1}{13} \quad C_{32} = -\frac{1}{13} \quad C_{33} = -\frac{5}{13}$$

$$\text{Adj } C = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix} = \begin{bmatrix} -\frac{1}{13} & \frac{2}{13} & -\frac{1}{13} \\ \frac{2}{13} & -\frac{3}{13} & -\frac{1}{13} \\ -\frac{1}{13} & -\frac{1}{13} & -\frac{5}{13} \end{bmatrix} = \frac{1}{13}\begin{bmatrix} -1 & 2 & -1 \\ 2 & -3 & -1 \\ -1 & -1 & -5 \end{bmatrix}$$

$$\Rightarrow \text{Adj } C = \text{adj}\left(A^{-1}\right) = \frac{1}{13}\begin{bmatrix} -1 & 2 & -1 \\ 2 & -3 & -1 \\ -1 & -1 & -5 \end{bmatrix}$$

From the equations (2) and (3), we have, $(\text{adj } A)^{-1} = \text{adj}\left(A^{-1}\right)$
(ii) From the equation (1), we have,

$$A^{-1} = \frac{1}{-13}\begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix}$$

Let, $D = A^{-1}$, so, $D = \frac{1}{-13}\begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix} = \begin{bmatrix} -\frac{14}{13} & -\frac{11}{13} & \frac{5}{13} \\ -\frac{11}{13} & -\frac{4}{13} & \frac{3}{13} \\ \frac{5}{13} & \frac{3}{13} & \frac{1}{13} \end{bmatrix}$, therefore

$$|D| = -\left(\frac{1}{13}\right)^3 [14(-4-9) - 11(-11-15) - 5(-33+20)]$$

$$= -\left(\frac{1}{13}\right)^3 (169) = -\frac{1}{13} \neq 0 \Rightarrow D^{-1} \text{ exists.}$$

$$D_{11} = -\frac{1}{13}$$
$$D_{12} = \frac{2}{13}$$
$$D_{13} = -\frac{1}{13}$$
$$D_{21} = \frac{2}{13}$$
$$D_{22} = -\frac{3}{13}$$
$$D_{23} = -\frac{1}{13}$$
$$D_{31} = -\frac{1}{13}$$

$D_{33} = -\frac{5}{13}$

$$D^{-1} = \frac{1}{|D|}\begin{bmatrix} D_{11} & D_{21} & D_{31} \\ D_{12} & D_{22} & D_{32} \\ D_{13} & D_{23} & D_{33} \end{bmatrix} = \frac{1}{-1/13}\begin{bmatrix} -\frac{1}{13} & \frac{2}{13} & -\frac{1}{13} \\ \frac{2}{13} & -\frac{3}{13} & -\frac{1}{13} \\ -\frac{1}{13} & -\frac{1}{13} & -\frac{5}{13} \end{bmatrix} = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$$

$$\Rightarrow D^{-1} = \left(A^{-1}\right)^{-1} = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix} = A$$

Question 5:

Evaluate $\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$

Enswer 5:

Given that: $\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$

$= \begin{vmatrix} 2(x+y) & y & x+y \\ 2(x+y) & x+y & x \\ 2(x+y) & x & y \end{vmatrix}$

[Applying $C_1 \to C_1 + C_2 + C_3$]

$= 2(x+y)\begin{vmatrix} 1 & y & x+y \\ 1 & x+y & x \\ 1 & x & y \end{vmatrix}$

$= 2(x+y)\begin{vmatrix} 0 & -x & y \\ 0 & y & x-y \\ 1 & y & y+k \end{vmatrix}$ [Applying $R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$]

$= 2(x+y)\{(-x)(x-y) - y, y\}$ [Expending along C_1]

$= 2(x+y)\left(-x^2 + xy - y^2\right) = -2(x+y)\left(x^2 - xy + y^2\right) = -2\left(x^3 + y^3\right)$

[Taking $2(x+y)$ as common from C_1]

Question 5:

Evaluate $\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$

Answer 5:

Given that: $\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$

$$= \begin{vmatrix} 2(x+y) & y & x+y \\ 2(x+y) & x+y & x \\ 2(x+y) & x & y \end{vmatrix}$$

[Applying $C_1 \to C_1 + C_2 + C_3$]

$$= 2(x+y)\begin{vmatrix} 1 & y & x+y \\ 1 & x+y & x \\ 1 & x & y \end{vmatrix}$$

$$= 2(x+y)\begin{vmatrix} 0 & -x & y \\ 0 & y & x-y \\ 1 & y & y+k \end{vmatrix}$$

[Applying $R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$]

$= 2(x+y)\{(-x)(x-y) - y \cdot y\}$

$= 2(x+y)\left(-x^2 + xy - y^2\right) = -2(x+y)\left(x^2 - xy + y^2\right) = -2\left(x^3 + y^3\right)$

Question 6:

Evaluate $\begin{vmatrix} 1 & x & y \\ 1 & x+y & y \\ 1 & x & x+y \end{vmatrix}$

Answer 6:

Given that: $\begin{vmatrix} 1 & x & y \\ 1 & x+y & y \\ 1 & x & x+y \end{vmatrix} = \begin{vmatrix} 0 & -y & 0 \\ 0 & y & -x \\ 1 & x & x+y \end{vmatrix}$

[Applying $R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$] $= \{(-y)(-x) - y.0\}$

[Expending along C_1] $= xy$

Question 7:

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$

Solve the system of equations:

$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$
$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

Answer 7:

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$

The given system of equations:

$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$

$$\frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$$

This system of equations can be written as $AX=B$, where

$$A=\begin{bmatrix}2 & 3 & 10\\4 & -6 & 5\\6 & 9 & -20\end{bmatrix}. X=\begin{bmatrix}1/x\\1/y\\1/z\end{bmatrix} \text{ and } B=\begin{bmatrix}4\\1\\2\end{bmatrix}$$

$|A|=2(120-45)-3(-80-30)+10(36+36)=150+330+720=1200\neq 0 \Rightarrow A^{-1}$ exists.

$A_{12}=110$ $A_{23}=0$
$A_{11}=75$ $A_{32}=30$
$A_{21}=150$
$A_{31}=75$

$$A^{-1}=\frac{1}{|A|}\operatorname{adj} A=\frac{1}{1200}\begin{bmatrix}75 & 150 & 75\\110 & -100 & 30\\72 & 0 & -24\end{bmatrix}$$

$$X=A^{-1}B \Rightarrow \begin{bmatrix}1/x\\1/y\\1/z\end{bmatrix}=\frac{1}{1200}\begin{bmatrix}75 & 150 & 75\\110 & -100 & 30\\72 & 0 & -24\end{bmatrix}\begin{bmatrix}4\\1\\2\end{bmatrix}$$

Question 8:

Choose the correct answer.

If x, y, z are nonzero real numbers, then the inverse of matrix $A=\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ is

A. $\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$ **B.** $xyz\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

C. $\frac{1}{xyz}\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ **D.** $\frac{1}{xyz}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Answer

Answer: A

$$A=\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$$

$$\therefore |A| = x(yz-0) = xyz \neq 0$$

$$\text{Now, } A_{11}=yz, A_{12}=0, A_{13}=0$$
$$A_{21}=0, A_{22}=xz, A_{23}=0$$
$$A_{31}=0, A_{32}=0, A_{33}=xy$$

$$\therefore adjA=\begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix}$$

$$\therefore A^{-1}=\frac{1}{|A|}adjA$$

$$=\frac{1}{xyz}\begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix}$$

$$=\begin{bmatrix} \frac{yz}{xyz} & 0 & 0 \\ 0 & \frac{xz}{xyz} & 0 \\ 0 & 0 & \frac{xy}{xyz} \end{bmatrix}$$

$$=\begin{bmatrix} \frac{1}{x} & 0 & 0 \\ 0 & \frac{1}{y} & 0 \\ 0 & 0 & \frac{1}{z} \end{bmatrix}=\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$$

The correct answer is A.

Question 9:

Choose the correct answer.

Let $A=\begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$, where $0 \le \theta \le 2\pi$, then

A. Det (A) = 0

B. Det (A) ∈ (2, ∞)

C. Det (A) ∈ (2, 4)

D. Det (A)∈ [2, 4]

Answer

sAnswer: D

$$A=\begin{bmatrix}1 & \sin\theta & 1\\ -\sin\theta & 1 & \sin\theta\\ -1 & -\sin\theta & 1\end{bmatrix}$$

$$\therefore |A| = 1\left(1+\sin^2\theta\right)-\sin\theta\left(-\sin\theta+\sin\theta\right)+1\left(\sin^2\theta+1\right)$$

$$=1+\sin^2\theta+\sin^2\theta+1$$

$$=2+2\sin^2\theta$$

$$=2\left(1+\sin^2\theta\right)$$

Now, $0\le\theta\le 2\pi$

$\Rightarrow 0\le\sin\theta\le 1$

$\Rightarrow 0\le\sin^2\theta\le 1$

$\Rightarrow 1\le 1+\sin^2\theta\le 2$

$\Rightarrow 2\le 2\left(1+\sin^2\theta\right)\le 4$

$\therefore \operatorname{Det}(A)\in[2,4]$

The correct answer is D.