Class XII : Maths
Chapter 8 : Application Of Integrals

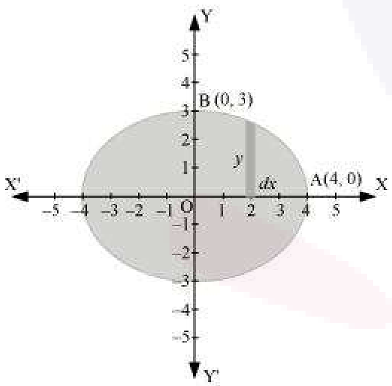
Questions and Solutions | Exercise 8.1 - NCERT Books

Question 1:

Find the area of the region bounded by the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$

Answer

The given equation of the ellipse, $\frac{x^2}{16}+\frac{y^2}{9}=1$, can be represented as



It can be observed that the ellipse is symmetrical about *x*-axis and *y*-axis.

∴ Area bounded by ellipse = 4 × Area of OAB

$$\text{Area of OAB} = \int_0^4 y\,dx$$
$$= \int_0^4 3\sqrt{1-\frac{x^2}{16}}dx$$
$$= \frac{3}{4}\int_0^4 \sqrt{16-x^2}dx$$
$$= \frac{3}{4}\left[\frac{x}{2}\sqrt{16-x^2}+\frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^4$$
$$= \frac{3}{4}\left[2\sqrt{16-16}+8\sin^{-1}(1)-0-8\sin^{-1}(0)\right]$$
$$= \frac{3}{4}\left[\frac{8\pi}{2}\right]$$
$$= \frac{3}{4}[4\pi]$$
$$= 3\pi$$

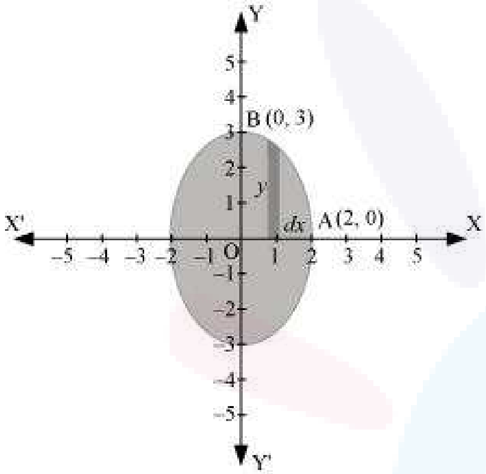
herefore, area bounded by the ellipse = 4 × 3π = 12π units

Question 2:

Find the area of the region bounded by the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$

Answer

The given equation of the ellipse can be represented as



$$\frac{x^2}{4}+\frac{y^2}{9}=1$$

$$\Rightarrow y=3\sqrt{1-\frac{x^2}{4}} \qquad ...(1)$$

It can be observed that the ellipse is symmetrical about *x*-axis and *y*-axis.

∴ Area bounded by ellipse = 4 × Area OAB

$$\therefore \text{Area of OAB} = \int_0^2 y\,dx$$

$$= \int_0^2 3\sqrt{1-\frac{x^2}{4}}\,dx \qquad \text{[Using (1)]}$$

$$= \frac{3}{2}\int_0^2 \sqrt{4-x^2}\,dx$$

$$= \frac{3}{2}\left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$$

$$= \frac{3}{2}\left[\frac{2\pi}{2}\right]$$

$$= \frac{3\pi}{2}$$

Therefore, area bounded by the ellipse $= 4\times\frac{3\pi}{2} = 6\pi$ units

Question 3:

Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$ is

A. π

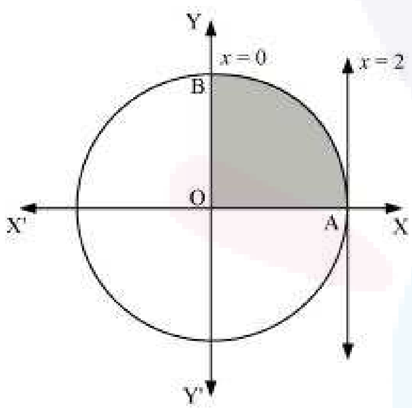
B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{4}$

Answer

The area bounded by the circle and the lines, $x = 0$ and $x = 2$, in the first quadrant is represented as



$$\therefore \text{Area OAB} = \int_0^2 y\,dx$$

$$= \int_0^2 \sqrt{4-x^2}\,dx$$

$$= \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$$

$$= 2\left(\frac{\pi}{2}\right)$$

$$= \pi \text{ units}$$

Thus, the correct answer is A.

Question 4:

Area of the region bounded by the curve $y^2 = 4x$, y-axis and the line $y = 3$ is

A. 2

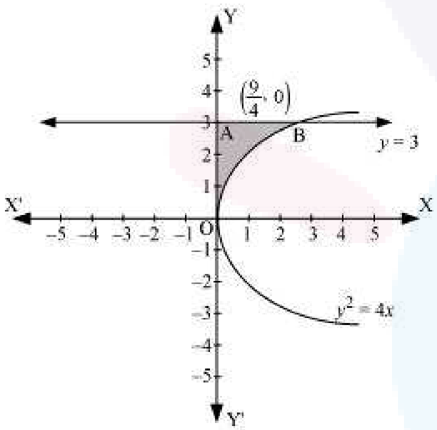
B. $\frac{9}{4}$

C. $\frac{9}{3}$

D. $\frac{9}{2}$

Answer

The area bounded by the curve, $y^2 = 4x$, y-axis, and $y = 3$ is represented as



$$\therefore \text{Area OAB} = \int_0^3 x\,dy$$

$$= \int_0^3 \frac{y^2}{4}dy$$

$$= \frac{1}{4}\left[\frac{y^3}{3}\right]_0^3$$

$$= \frac{1}{12}(27)$$

$$= \frac{9}{4} \text{ units}$$

Thus, the correct answer is B.

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Questions and Solutions | Miscellaneous Exercise 8 - NCERT Books

Question 1:

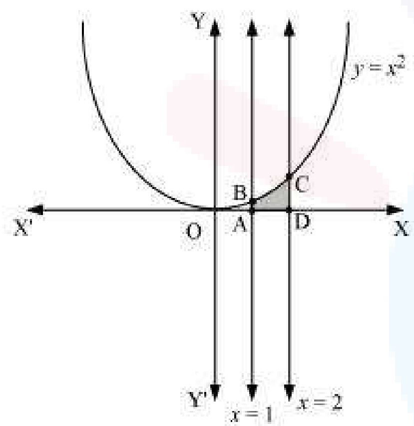
Find the area under the given curves and given lines:

(i) $y = x^2$, $x = 1$, $x = 2$ and x-axis

(ii) $y = x^4$, $x = 1$, $x = 5$ and x –axis

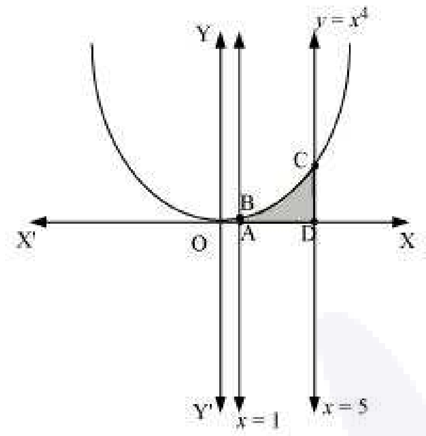
Answer

i. The required area is represented by the shaded area ADCBA as



$$\text{Area ADCBA} = \int_1^2 y\,dx$$
$$= \int_1^2 x^2\,dx$$
$$= \left[\frac{x^3}{3}\right]_1^2$$
$$= \frac{8}{3} - \frac{1}{3}$$
$$= \frac{7}{3} \text{ units}$$

ii. The required area is represented by the shaded area ADCBA as



$$\text{Area ADCBA} = \int_1^5 x^4 dx$$

$$= \left[\frac{x^5}{5}\right]_1^5$$

$$= \frac{(5)^5}{5} - \frac{1}{5}$$

$$= (5)^4 - \frac{1}{5}$$

$$= 625 - \frac{1}{5}$$

$$= 624.8 \text{ units}$$

Question 2:

Sketch the graph of $y=|x+3|$ and evaluate $\int_{-6}^{0}|x+3|dx$

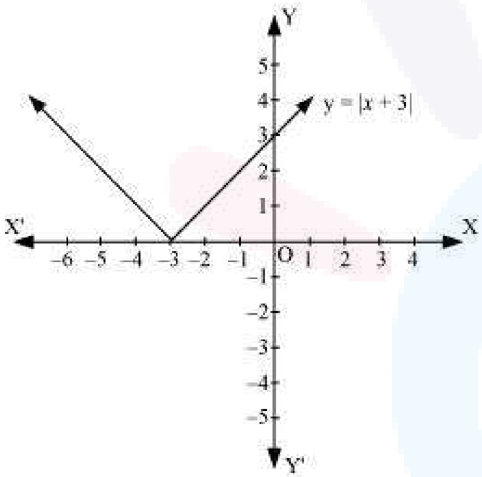
Answer

The given equation is $y=|x+3|$

The corresponding values of *x* and *y* are given in the following table.

x	− 6	− 5	− 4	− 3	− 2	− 1	0
y	3	2	1	0	1	2	3

On plotting these points, we obtain the graph of $y=|x+3|$ as follows.



It is known that, $(x+3)\le 0$ for $-6\le x\le -3$ and $(x+3)\ge 0$ for $-3\le x\le 0$

$$\therefore \int_{-6}^{0}|(x+3)|dx=-\int_{-6}^{-3}(x+3)dx+\int_{-3}^{0}(x+3)dx$$

$$=-\left[\frac{x^2}{2}+3x\right]_{-6}^{-3}+\left[\frac{x^2}{2}+3x\right]_{-3}^{0}$$

$$=-\left[\left(\frac{(-3)^2}{2}+3(-3)\right)-\left(\frac{(-6)^2}{2}+3(-6)\right)\right]+\left[0-\left(\frac{(-3)^2}{2}+3(-3)\right)\right]$$

$$=-\left[-\frac{9}{2}\right]-\left[-\frac{9}{2}\right]$$

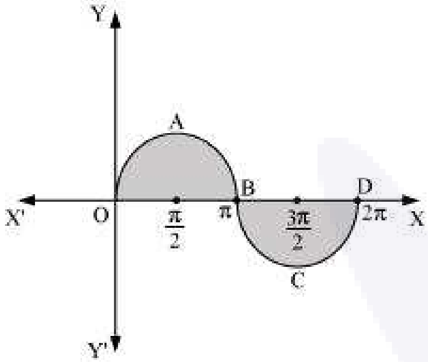
$$=9$$

Question 3:

Find the area bounded by the curve y = sin x between x = 0 and x = 2π

Answer

The graph of y = sin x can be drawn as



∴ Required area = Area OABO + Area BCDB

$$= \int_0^{\pi} \sin x\, dx + \left| \int_{\pi}^{2\pi} \sin x\, dx \right|$$

$$= [-\cos x]_0^{\pi} + \left| [-\cos x]_{\pi}^{2\pi} \right|$$

$$= [-\cos \pi + \cos 0] + |-\cos 2\pi + \cos \pi|$$

$$= 1 + 1 + |(-1-1)|$$

$$= 2 + |-2|$$

$$= 2 + 2 = 4 \text{ units}$$

Question 4:

Area bounded by the curve $y = x^3$, the x-axis and the ordinates $x = -2$ and $x = 1$ is

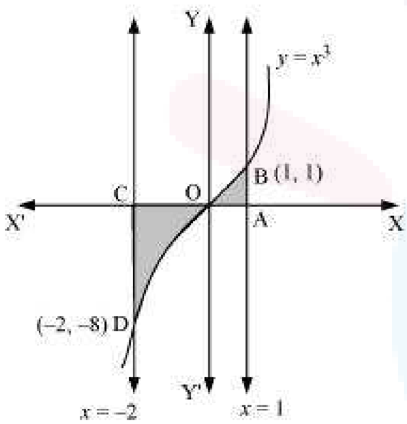
A. – 9

B. $-\frac{15}{4}$

C. $\frac{15}{4}$

D. $\frac{17}{4}$

Answer



$$\text{Required area} = \int_{-2}^{1} y\,dx$$

$$= \int_{-2}^{1} x^3\,dx$$

$$= \left[\frac{x^4}{4}\right]_{-2}^{1}$$

$$= \left[\frac{1}{4} - \frac{(-2)^4}{4}\right]$$

$$= \left(\frac{1}{4} - 4\right) = -\frac{15}{4} \text{ units}$$

Thus, the correct answer is B.

Question 5:

The area bounded by the curve $y = x|x|$, x-axis and the ordinates $x = -1$ and $x = 1$ is given by

[Hint: $y = x^2$ if $x > 0$ and $y = -x^2$ if $x < 0$]

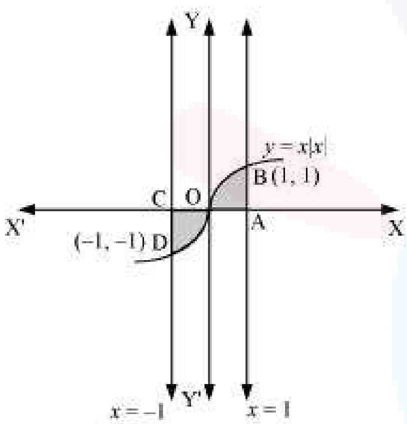
A. 0

B. $\frac{1}{3}$

C. $\frac{2}{3}$

D. $\frac{4}{3}$

Answer



Required area $= \int_{-1}^{1} y\,dx$

$= \int_{-1}^{1} x|x|\,dx$

$= \int_{-1}^{0} x^2 dx + \int_{0}^{1} x^2 dx$

$= \left[\frac{x^3}{3}\right]_{-1}^{0} + \left[\frac{x^3}{3}\right]_{0}^{1}$

$= -\left(-\frac{1}{3}\right) + \frac{1}{3}$

$= \frac{2}{3}$ units

Thus, the correct answer is C.