Class XII : Maths
Chapter 1 : Relations And Functions

Questions and Solutions | Exercise 1.1 - NCERT Books

Question 1:

Determine whether each of the following relations are reflexive, symmetric and transitive:

(i)Relation R in the set $A = \{1, 2, 3 \ldots 13, 14\}$ defined as

$R = \{(x, y): 3x - y = 0\}$

(ii) Relation R in the set **N** of natural numbers defined as

$R = \{(x, y): y = x + 5 \text{ and } x < 4\}$

(iii) Relation R in the set $A = \{1, 2, 3, 4, 5, 6\}$ as

$R = \{(x, y): y \text{ is divisible by } x\}$

(iv) Relation R in the set **Z** of all integers defined as

$R = \{(x, y): x - y \text{ is as integer}\}$

(v) Relation R in the set A of human beings in a town at a particular time given by

(a) $R = \{(x, y): x \text{ and } y \text{ work at the same place}\}$

(b) $R = \{(x, y): x \text{ and } y \text{ live in the same locality}\}$

(c) $R = \{(x, y): x \text{ is exactly 7 cm taller than } y\}$

(d) $R = \{(x, y): x \text{ is wife of } y\}$

(e) $R = \{(x, y): x \text{ is father of } y\}$

Answer

(i) $A = \{1, 2, 3 \ldots 13, 14\}$

$R = \{(x, y): 3x - y = 0\}$

$\therefore R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$

R is not reflexive since $(1, 1), (2, 2) \ldots (14, 14) \notin R$.

Also, R is not symmetric as $(1, 3) \in R$, but $(3, 1) \notin R$. $[3(3) - 1 \neq 0]$

Also, R is not transitive as $(1, 3), (3, 9) \in R$, but $(1, 9) \notin R$.

$[3(1) - 9 \neq 0]$

Hence, R is neither reflexive, nor symmetric, nor transitive.

(ii) $R = \{(x, y): y = x + 5 \text{ and } x < 4\} = \{(1, 6), (2, 7), (3, 8)\}$

It is seen that $(1, 1) \notin R$.

$\therefore R$ is not reflexive.

$(1, 6) \in R$

But,

(1, 6) ∉ R.

∴R is not symmetric.

Now, since there is no pair in R such that (x, y) and (y, z) ∈R, then (x, z) cannot belong to R.

∴ R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

(iii) $A = \{1, 2, 3, 4, 5, 6\}$

$R = \{(x, y): y$ is divisible by $x\}$

We know that any number (x) is divisible by itself.

⇒ (x, x) ∈R

∴R is reflexive.

Now,

(2, 4) ∈R [as 4 is divisible by 2]

But,

(4, 2) ∉ R. [as 2 is not divisible by 4]

∴R is not symmetric.

Let $(x, y), (y, z) \in R$. Then, y is divisible by x and z is divisible by y.

∴z is divisible by x.

⇒ (x, z) ∈R

∴R is transitive.

Hence, R is reflexive and transitive but not symmetric.

(iv) $R = \{(x, y): x - y$ is an integer$\}$

Now, for every $x \in \mathbf{Z}$, (x, x) ∈R as $x - x = 0$ is an integer.

∴R is reflexive.

Now, for every $x, y \in \mathbf{Z}$ if $(x, y) \in R$, then $x - y$ is an integer.

⇒ $-(x - y)$ is also an integer.

⇒ $(y - x)$ is an integer.

∴ $(y, x) \in R$

∴R is symmetric.

Now,

Let (x, y) and (y, z) ∈R, where $x, y, z \in \mathbf{Z}$.

⇒ $(x - y)$ and $(y - z)$ are integers.

⇒ $x - z = (x - y) + (y - z)$ is an integer.

∴ $(x, z) \in R$

∴R is transitive.

Hence, R is reflexive, symmetric, and transitive.

(v) (a) R = {(x, y): x and y work at the same place}

⇒ $(x, x) \in R$

∴ R is reflexive.

If $(x, y) \in R$, then x and y work at the same place.

⇒ y and x work at the same place.

⇒ $(y, x) \in R$.

∴R is symmetric.

Now, let $(x, y), (y, z) \in R$

⇒ x and y work at the same place and y and z work at the same place.

⇒ x and z work at the same place.

⇒ $(x, z) \in R$

∴ R is transitive.

Hence, R is reflexive, symmetric, and transitive.

(b) R = {(x, y): x and y live in the same locality}

Clearly $(x, x) \in R$ as x and x is the same human being.

∴ R is reflexive.

If $(x, y) \in R$, then x and y live in the same locality.

⇒ y and x live in the same locality.

⇒ $(y, x) \in R$

∴R is symmetric.

Now, let $(x, y) \in R$ and $(y, z) \in R$.

⇒ x and y live in the same locality and y and z live in the same locality.

⇒ x and z live in the same locality.

⇒ $(x, z) \in R$

∴ R is transitive.

Hence, R is reflexive, symmetric, and transitive.

(c) R = {(x, y): x is exactly 7 cm taller than y}

Now,

$(x, x) \notin R$

Since human being x cannot be taller than himself.

∴R is not reflexive.

Now, let (x, y) ∈R.

⇒ x is exactly 7 cm taller than y.

Then, y is not taller than x.

∴ $(y, x) \notin$R

Indeed if x is exactly 7 cm taller than y, then y is exactly 7 cm shorter than x.

∴R is not symmetric.

Now,

Let $(x, y), (y, z) \in$ R.

⇒ x is exactly 7 cm taller than y and y is exactly 7 cm taller than z.

⇒ x is exactly 14 cm taller than z .

∴$(x, z) \notin$R

∴ R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

(d) R = {(x, y): x is the wife of y}

Now,

$(x, x) \notin$ R

Since x cannot be the wife of herself.

∴R is not reflexive.

Now, let $(x, y) \in$ R

⇒ x is the wife of y.

Clearly y is not the wife of x.

∴$(y, x) \notin$ R

Indeed if x is the wife of y, then y is the husband of x.

∴ R is not transitive.

Let $(x, y), (y, z) \in$ R

⇒ x is the wife of y and y is the wife of z.

This case is not possible. Also, this does not imply that x is the wife of z.

∴$(x, z) \notin$ R

∴R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

(e) R = {(x, y): x is the father of y}

$(x, x) \notin$ R

As x cannot be the father of himself.

∴R is not reflexive.

Now, let $(x, y) \in R$.

⇒ x is the father of y.

⇒ y cannot be the father of y.

Indeed, y is the son or the daughter of y.

∴$(y, x) \notin R$

∴ R is not symmetric.

Now, let $(x, y) \in R$ and $(y, z) \in R$.

⇒ x is the father of y and y is the father of z.

⇒ x is not the father of z.

Indeed x is the grandfather of z.

∴ $(x, z) \notin R$

∴R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

Question 2:

Show that the relation R in the set **R** of real numbers, defined as

$R = \{(a, b): a \le b^2\}$ is neither reflexive nor symmetric nor transitive.

Answer

$R = \{(a, b): a \le b^2\}$

It can be observed that $\left(\frac{1}{2}, \frac{1}{2}\right) \notin \mathbf{R}$, since $\frac{1}{2} > \left(\frac{1}{2}\right)^2 = \frac{1}{4}$.

∴R is not reflexive.

Now, $(1, 4) \in R$ as $1 < 4^2$

But, 4 is not less than 1^2.

∴$(4, 1) \notin R$

∴R is not symmetric.

Now,

$(3, 2), (2, 1.5) \in R$

(as $3 < 2^2 = 4$ and $2 < (1.5)^2 = 2.25$)

But, $3 > (1.5)^2 = 2.25$

∴(3, 1.5) ∉ R

∴ R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

Question 3:

Check whether the relation R defined in the set {1, 2, 3, 4, 5, 6} as

R = {(a, b): $b = a + 1$} is reflexive, symmetric or transitive.

Answer

Let A = {1, 2, 3, 4, 5, 6}.

A relation R is defined on set A as:

R = {(a, b): $b = a + 1$}

∴R = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)}

We can find (a, a) ∉ R, where a ∈ A.

For instance,

(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) ∉ R

∴R is not reflexive.

It can be observed that (1, 2) ∈ R, but (2, 1) ∉ R.

∴R is not symmetric.

Now, (1, 2), (2, 3) ∈ **R**

But,

(1, 3) ∉ R

∴R is not transitive

Hence, R is neither reflexive, nor symmetric, nor transitive.

Question 4:

Show that the relation R in **R** defined as R = {(a, b): $a \le b$}, is reflexive and transitive but not symmetric.

Answer

R = {(a, b); $a \le b$}

Clearly (a, a) ∈ R as $a = a$.

∴R is reflexive.

Now,

(2, 4) ∈ R (as 2 < 4)

But, (4, 2) ∉ R as 4 is greater than 2.

∴ R is not symmetric.

Now, let (a, b), (b, c) ∈ R.

Then,

$a \leq b$ and $b \leq c$

$\Rightarrow a \leq c$

$\Rightarrow (a, c) \in$ R

∴R is transitive.

Hence,R is reflexive and transitive but not symmetric.

Question 5:

Check whether the relation R in **R** defined as R = {(a, b): $a \leq b^3$} is reflexive, symmetric or transitive.

Answer

R = {(a, b): $a \leq b^3$}

It is observed that $\left(\frac{1}{2}, \frac{1}{2}\right) \notin R$ as $\frac{1}{2} > \left(\frac{1}{2}\right)^3 = \frac{1}{8}$.

∴ R is not reflexive.

Now,

(1, 2) ∈ R (as $1 < 2^3 = 8$)

But,

(2, 1) ∉ R (as $2^3 > 1$)

∴ R is not symmetric.

We have $\left(3, \frac{3}{2}\right), \left(\frac{3}{2}, \frac{6}{5}\right) \in R$ as $3 < \left(\frac{3}{2}\right)^3$ and $\frac{3}{2} < \left(\frac{6}{5}\right)^3$.

But $\left(3, \frac{6}{5}\right) \notin R$ as $3 > \left(\frac{6}{5}\right)^3$.

∴ R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

Question 6:

Show that the relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} is symmetric but neither reflexive nor transitive.

Answer

Let A = {1, 2, 3}.

A relation R on A is defined as R = {(1, 2), (2, 1)}.

It is seen that (1, 1), (2, 2), (3, 3) ∉R.

∴ R is not reflexive.

Now, as (1, 2) ∈ R and (2, 1) ∈ R, then R is symmetric.

Now, (1, 2) and (2, 1) ∈ R

However,

(1, 1) ∉ R

∴ R is not transitive.

Hence, R is symmetric but neither reflexive nor transitive.

Question 7:

Show that the relation R in the set A of all the books in a library of a college, given by R = {(x, y): x and y have same number of pages} is an equivalence relation.

Answer

Set A is the set of all books in the library of a college.

R = {x, y): x and y have the same number of pages}

Now, R is reflexive since (x, x) ∈ R as x and x has the same number of pages.

Let (x, y) ∈ R ⇒ x and y have the same number of pages.

⇒ y and x have the same number of pages.

⇒ (y, x) ∈ R

∴R is symmetric.

Now, let (x, y) ∈R and (y, z) ∈ R.

⇒ x and y and have the same number of pages and y and z have the same number of pages.

⇒ x and z have the same number of pages.

⇒ (x, z) ∈ R

∴R is transitive.

Hence, R is an equivalence relation.

Question 8:

Show that the relation R in the set A = {1, 2, 3, 4, 5} given by

$R=\{(a, b):|a-b| \text{ is even}\}$, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element of 2, 4}.

Answer

A = {1, 2, 3, 4, 5}

$R=\{(a, b):|a-b| \text{ is even}\}$

It is clear that for any element $a \in A$, we have $|a-a|=0$ (which is even).

∴R is reflexive.

Let $(a, b) \in$ R.

$\Rightarrow |a-b| \text{is even.}$

$\Rightarrow |-(a-b)|=|b-a| \text{is also even.}$

$\Rightarrow (b, a) \in R$

∴R is symmetric.

Now, let $(a, b) \in$ R and $(b, c) \in$ R.

$\Rightarrow |a-b| \text{ is even and } |b-c| \text{ is even.}$

$\Rightarrow (a-b) \text{ is even and } (b-c) \text{ is even.}$

$\Rightarrow (a-c)=(a-b)+(b-c) \text{ is even.}$ [Sum of two even integers is even]

$\Rightarrow |a-c| \text{ is even.}$

$\Rightarrow (a, c) \in$ R

∴R is transitive.

Hence, R is an equivalence relation.

Now, all elements of the set {1, 2, 3} are related to each other as all the elements of this subset are odd. Thus, the modulus of the difference between any two elements will be even.

Similarly, all elements of the set {2, 4} are related to each other as all the elements of this subset are even.

Also, no element of the subset {1, 3, 5} can be related to any element of {2, 4} as all elements of {1, 3, 5} are odd and all elements of {2, 4} are even. Thus, the modulus of the difference between the two elements (from each of these two subsets) will not be even.

Question 9:

Show that each of the relation R in the set $A = \{x \in \mathbf{Z} : 0 \le x \le 12\}$, given by

(i) $R = \{(a, b) : |a - b| \text{ is a multiple of } 4\}$

(ii) $R = \{(a, b) : a = b\}$

is an equivalence relation. Find the set of all elements related to 1 in each case.

Answer

$A = \{x \in \mathbf{Z} : 0 \le x \le 12\} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

(i) $R = \{(a, b) : |a - b| \text{ is a multiple of } 4\}$

For any element *a* ∈A, we have (*a*, *a*) ∈ R as $|a - a| = 0$ is a multiple of 4.

∴R is reflexive.

Now, let (*a*, *b*) ∈ R ⇒ $|a - b|$ is a multiple of 4.

$\Rightarrow |-(a-b)| = |b - a|$ is a multiple of 4.

⇒ (*b*, *a*) ∈ R

∴R is symmetric.

Now, let (*a*, *b*), (*b*, *c*) ∈ R.

$\Rightarrow |a - b|$ is a multiple of 4 and $|b - c|$ is a multiple of 4.

$\Rightarrow (a - b)$ is a multiple of 4 and $(b - c)$ is a multiple of 4.

$\Rightarrow (a - c) = (a - b) + (b - c)$ is a multiple of 4.

$\Rightarrow |a - c|$ is a multiple of 4.

⇒ (*a*, *c*) ∈R

∴ R is transitive.

Hence, R is an equivalence relation.

The set of elements related to 1 is {1, 5, 9} since

$|1-1| = 0$ is a multiple of 4,

$|5-1| = 4$ is a multiple of 4, and

$|9-1| = 8$ is a multiple of 4.

(ii) R = {(a, b): $a = b$}

For any element a ∈A, we have (a, a) ∈ R, since $a = a$.

∴R is reflexive.

Now, let (a, b) ∈ R.

⇒ $a = b$

⇒ $b = a$

⇒ (b, a) ∈ R

∴R is symmetric.

Now, let (a, b) ∈ R and (b, c) ∈ R.

⇒ $a = b$ and $b = c$

⇒ $a = c$

⇒ (a, c) ∈ R

∴ R is transitive.

Hence, R is an equivalence relation.

The elements in R that are related to 1 will be those elements from set A which are equal to 1.

Hence, the set of elements related to 1 is {1}.

Question 10:

Given an example of a relation. Which is

(i) Symmetric but neither reflexive nor transitive.

(ii) Transitive but neither reflexive nor symmetric.

(iii) Reflexive and symmetric but not transitive.

(iv) Reflexive and transitive but not symmetric.

(v) Symmetric and transitive but not reflexive.

Answer

(i) Let A = {5, 6, 7}.

Define a relation R on A as R = {(5, 6), (6, 5)}.

Relation R is not reflexive as (5, 5), (6, 6), (7, 7) ∉ R.

Now, as (5, 6) ∈ R and also (6, 5) ∈ R, R is symmetric.

⇒ (5, 6), (6, 5) ∈ R, but (5, 5) ∉ R

∴R is not transitive.

Hence, relation R is symmetric but not reflexive or transitive.

(ii)Consider a relation R in **R** defined as:

R = {(a, b): $a < b$}

For any a ∈ R, we have (a, a) ∉ R since a cannot be strictly less than a itself. In fact, a = a.

∴ R is not reflexive.

Now,

(1, 2) ∈ R (as 1 < 2)

But, 2 is not less than 1.

∴ (2, 1) ∉ R

∴ R is not symmetric.

Now, let (a, b), (b, c) ∈ R.

⇒ $a < b$ and $b < c$

⇒ $a < c$

⇒ (a, c) ∈ R

∴R is transitive.

Hence, relation R is transitive but not reflexive and symmetric.

(iii)Let A = {4, 6, 8}.

Define a relation R on A as:

A = {(4, 4), (6, 6), (8, 8), (4, 6), (6, 4), (6, 8), (8, 6)}

Relation R is reflexive since for every a ∈ A, (a, a) ∈R i.e., (4, 4), (6, 6), (8, 8)} ∈ R.

Relation R is symmetric since (a, b) ∈ R ⇒ (b, a) ∈ R for all a, b ∈ R.

Relation R is not transitive since (4, 6), (6, 8) ∈ R, but (4, 8) ∉ R.

Hence, relation R is reflexive and symmetric but not transitive.

(iv) Define a relation R in **R** as:

R = {(a, b): $a^3 \geq b^3$}

Clearly (a, a) ∈ R as $a^3 = a^3$.

∴R is reflexive.

Now,

(2, 1) ∈ R (as $2^3 \geq 1^3$)

But,

(1, 2) ∉ R (as $1^3 < 2^3$)

∴ R is not symmetric.

Now,

Let (a, b), (b, c) ∈ R.

⇒ $a^3 \geq b^3$ and $b^3 \geq c^3$

⇒ $a^3 \geq c^3$

⇒ (a, c) ∈ R

∴R is transitive.

Hence, relation R is reflexive and transitive but not symmetric.

(v) Let $A = \{-5, -6\}$.

Define a relation R on A as:

R = {(−5, −6), (−6, −5), (−5, −5)}

Relation R is not reflexive as (−6, −6) ∉ R.

Relation R is symmetric as (−5, −6) ∈ R and (−6, −5}∈R.

It is seen that (−5, −6), (−6, −5) ∈ R. Also, (−5, −5) ∈ R.

∴The relation R is transitive.

Hence, relation R is symmetric and transitive but not reflexive.

Question 11:

Show that the relation R in the set A of points in a plane given by R = {(P, Q): distance of the point P from the origin is same as the distance of the point Q from the origin}, is an equivalence relation. Further, show that the set of all point related to a point P ≠ (0, 0) is the circle passing through P with origin as centre.

Answer

R = {(P, Q): distance of point P from the origin is the same as the distance of point Q from the origin}

Clearly, (P, P) ∈ R since the distance of point P from the origin is always the same as the distance of the same point P from the origin.

∴R is reflexive.

Now,

Let (P, Q) ∈ R.

⇒ The distance of point P from the origin is the same as the distance of point Q from the origin.

⇒ The distance of point Q from the origin is the same as the distance of point P from the origin.

⇒ (Q, P) ∈ R

∴R is symmetric.

Now,

Let (P, Q), (Q, S) ∈ R.

⇒ The distance of points P and Q from the origin is the same and also, the distance of points Q and S from the origin is the same.

⇒ The distance of points P and S from the origin is the same.

⇒ (P, S) ∈ R

∴R is transitive.

Therefore, R is an equivalence relation.

The set of all points related to P ≠ (0, 0) will be those points whose distance from the origin is the same as the distance of point P from the origin.

In other words, if O (0, 0) is the origin and OP = k, then the set of all points related to P is at a distance of k from the origin.

Hence, this set of points forms a circle with the centre as the origin and this circle passes through point P.

Question 12:

Show that the relation R defined in the set A of all triangles as R = {(T_1, T_2): T_1 is similar to T_2}, is equivalence relation. Consider three right angle triangles T_1 with sides 3, 4, 5, T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are related?

Answer

R = {(T_1, T_2): T_1 is similar to T_2}

R is reflexive since every triangle is similar to itself.

Further, if (T_1, T_2) ∈ R, then T_1 is similar to T_2.

⇒ T_2 is similar to T_1.

⇒ (T_2, T_1) ∈R

∴R is symmetric.

Now,

Let $(T_1, T_2), (T_2, T_3) \in R$.

$\Rightarrow T_1$ is similar to T_2 and T_2 is similar to T_3.

$\Rightarrow T_1$ is similar to T_3.

$\Rightarrow (T_1, T_3) \in R$

$\therefore$ R is transitive.

Thus, R is an equivalence relation.

Now, we can observe that:

$$\frac{3}{6}=\frac{4}{8}=\frac{5}{10}\left(=\frac{1}{2}\right)$$

$\therefore$The corresponding sides of triangles T_1 and T_3 are in the same ratio.

Then, triangle T_1 is similar to triangle T_3.

Hence, T_1 is related to T_3.

Question 13:

Show that the relation R defined in the set A of all polygons as R = $\{(P_1, P_2)$: P_1 and P_2 have same number of sides$\}$, is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3, 4 and 5?

Answer

R = $\{(P_1, P_2)$: P_1 and P_2 have same the number of sides$\}$

R is reflexive since $(P_1, P_1) \in R$ as the same polygon has the same number of sides with itself.

Let $(P_1, P_2) \in R$.

$\Rightarrow P_1$ and P_2 have the same number of sides.

$\Rightarrow P_2$ and P_1 have the same number of sides.

$\Rightarrow (P_2, P_1) \in R$

$\therefore$R is symmetric.

Now,

Let $(P_1, P_2), (P_2, P_3) \in R$.

$\Rightarrow P_1$ and P_2 have the same number of sides. Also, P_2 and P_3 have the same number of sides.

$\Rightarrow P_1$ and P_3 have the same number of sides.

$\Rightarrow (P_1, P_3) \in R$

∴R is transitive.

Hence, R is an equivalence relation.

The elements in A related to the right-angled triangle (T) with sides 3, 4, and 5 are those polygons which have 3 sides (since T is a polygon with 3 sides).

Hence, the set of all elements in A related to triangle T is the set of all triangles.

Question 14:

Let L be the set of all lines in XY plane and R be the relation in L defined as R = $\{(L_1, L_2)$: L_1 is parallel to $L_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line $y = 2x + 4$.

Answer

R = $\{(L_1, L_2)$: L_1 is parallel to $L_2\}$

R is reflexive as any line L_1 is parallel to itself i.e., $(L_1, L_1) \in$ R.

Now,

Let $(L_1, L_2) \in$ R.

⇒ L_1 is parallel to L_2.

⇒ L_2 is parallel to L_1.

⇒ $(L_2, L_1) \in$ R

∴ R is symmetric.

Now,

Let (L_1, L_2), $(L_2, L_3) \in$R.

⇒ L_1 is parallel to L_2. Also, L_2 is parallel to L_3.

⇒ L_1 is parallel to L_3.

∴R is transitive.

Hence, R is an equivalence relation.

The set of all lines related to the line $y = 2x + 4$ is the set of all lines that are parallel to the line $y = 2x + 4$.

Slope of line $y = 2x + 4$ is $m = 2$

It is known that parallel lines have the same slopes.

The line parallel to the given line is of the form $y = 2x + c$, where $c \in \mathbf{R}$.

Hence, the set of all lines related to the given line is given by $y = 2x + c$, where $c \in \mathbf{R}$.

Question 15:

Let R be the relation in the set {1, 2, 3, 4} given by R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)}. Choose the correct answer.

(A) R is reflexive and symmetric but not transitive.

(B) R is reflexive and transitive but not symmetric.

(C) R is symmetric and transitive but not reflexive.

(D) R is an equivalence relation.

Answer

R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)}

It is seen that $(a, a) \in$ **R,** for every $a \in \{1, 2, 3, 4\}$.

∴ R is reflexive.

It is seen that (1, 2) ∈ R, but (2, 1) ∉ R.

∴R is not symmetric.

Also, it is observed that $(a, b), (b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b, c \in \{1, 2, 3, 4\}$.

∴ R is transitive.

Hence, R is reflexive and transitive but not symmetric.

The correct answer is B.

Question 16:

Let R be the relation in the set **N** given by $R = \{(a, b): a = b - 2, b > 6\}$. Choose the correct answer.

(A) (2, 4) ∈ R (B) (3, 8) ∈R (C) (6, 8) ∈R (D) (8, 7) ∈ R

Answer

$R = \{(a, b): a = b - 2, b > 6\}$

Now, since $b > 6$, (2, 4) ∉ R

Also, as $3 \neq 8 - 2$, (3, 8) ∉ R

And, as $8 \neq 7 - 2$

∴ (8, 7) ∉ R

Now, consider (6, 8).

We have $8 > 6$ and also, $6 = 8 - 2$.

∴(6, 8) ∈ R

The correct answer is C.

Class XII : Maths
Chapter 1 : Relations And Functions

Questions and Solutions | Exercise 1.2 - NCERT Books

Question 1:

Show that the function *f*: **R**∗ → **R**∗ defined by $f(x)=\frac{1}{x}$ is one-one and onto, where **R**∗ is the set of all non-zero real numbers. Is the result true, if the domain **R**∗ is replaced by **N** with co-domain being same as **R**∗? Answer

It is given that *f*: **R**∗ → **R**∗ is defined by $f(x)=\frac{1}{x}$. One-one:

$$f(x)=f(y)$$
$$\Rightarrow \frac{1}{x}=\frac{1}{y}$$
$$\Rightarrow x=y$$

∴*f* is one-one.

Onto:

It is clear that for *y*∈ **R**∗, there exists $x=\frac{1}{y}\in R_*$ (Exists as $y\neq 0$) such that

$$f(x)=\frac{1}{\left(\frac{1}{y}\right)}=y.$$

∴*f* is onto.

Thus, the given function (*f*) is one-one and onto. Now, consider function *g*: **N** → **R**∗defined by

$$g(x)=\frac{1}{x}.$$

We have,

$$g(x_1)=g(x_2)\Rightarrow\frac{1}{x_1}=\frac{1}{x_2}\Rightarrow x_1=x_2$$

∴*g* is one-one.

Further, it is clear that g is not onto as for $1.2 \in \mathbf{R}_*$ there does not exit any x in **N** such that $g(x) = \frac{1}{1.2}$.

Hence, function g is one-one but not onto.

Question 2:

Check the injectivity and surjectivity of the following functions:

(i) f: $\mathbf{N} \rightarrow \mathbf{N}$ given by $f(x) = x^2$

(ii) f: $\mathbf{Z} \rightarrow \mathbf{Z}$ given by $f(x) = x^2$

(iii) f: $\mathbf{R} \rightarrow \mathbf{R}$ given by $f(x) = x^2$

(iv) f: $\mathbf{N} \rightarrow \mathbf{N}$ given by $f(x) = x^3$

(v) f: $\mathbf{Z} \rightarrow \mathbf{Z}$ given by $f(x) = x^3$

Answer

(i) f: $\mathbf{N} \rightarrow \mathbf{N}$ is given by,

$f(x) = x^2$

It is seen that for $x, y \in \mathbf{N}$, $f(x) = f(y) \Rightarrow x^2 = y^2 \Rightarrow x = y$.

$\therefore f$ is injective.

Now, $2 \in \mathbf{N}$. But, there does not exist any x in **N** such that $f(x) = x^2 = 2$.

$\therefore$ f is not surjective.

Hence, function f is injective but not surjective.

(ii) f: $\mathbf{Z} \rightarrow \mathbf{Z}$ is given by,

$f(x) = x^2$

It is seen that $f(-1) = f(1) = 1$, but $-1 \neq 1$.

$\therefore$ f is not injective.

Now, $-2 \in \mathbf{Z}$. But, there does not exist any element $x \in \mathbf{Z}$ such that $f(x) = x^2 = -2$.

$\therefore$ f is not surjective.

Hence, function f is neither injective nor surjective.

(iii) f: $\mathbf{R} \rightarrow \mathbf{R}$ is given by, $f(x) = x^2$

It is seen that $f(-1) = f(1) = 1$, but $-1 \neq 1$.

$\therefore$ f is not injective.

Now, $-2 \in \mathbf{R}$. But, there does not exist any element $x \in \mathbf{R}$ such that $f(x) = x^2 = -2$.

$\therefore$ f is not surjective.

Hence, function f is neither injective nor surjective.

(iv) f: $\mathbf{N} \rightarrow \mathbf{N}$ given by,

$f(x) = x^3$

It is seen that for $x, y \in \mathbf{N}$, $f(x) = f(y) \Rightarrow x^3 = y^3 \Rightarrow x = y$.

$\therefore f$ is injective.

Now, $2 \in \mathbf{N}$. But, there does not exist any element x in domain $\mathbf{N}$ such that $f(x) = x^3 = 2$.

$\therefore$ f is not surjective

Hence, function f is injective but not surjective.

(v) f: $\mathbf{Z} \rightarrow \mathbf{Z}$ is given by,

$f(x) = x^3$

It is seen that for $x, y \in \mathbf{Z}$, $f(x) = f(y) \Rightarrow x^3 = y^3 \Rightarrow x = y$.

$\therefore$ f is injective.

Now, $2 \in \mathbf{Z}$. But, there does not exist any element x in domain $\mathbf{Z}$ such that $f(x) = x^3 = 2$.

$\therefore$ f is not surjective.

Hence, function f is injective but not surjective.

Question 3:

Prove that the Greatest Integer Function f: $\mathbf{R} \rightarrow \mathbf{R}$ given by $f(x) = [x]$, is neither oneonce nor onto, where $[x]$ denotes the greatest integer less than or equal to x.

Answer f: $\mathbf{R} \rightarrow \mathbf{R}$ is given by, $f(x) = [x]$

It is seen that $f(1.2) = [1.2] = 1$, $f(1.9) = [1.9] = 1$.

$\therefore$ $f(1.2) = f(1.9)$, but $1.2 \neq 1.9$.

$\therefore$ f is not one-one.

Now, consider 0.7 ∈ **R**.

It is known that $f(x)$ = $[x]$ is always an integer. Thus, there does not exist any element x ∈ **R** such that $f(x)$ = 0.7.

∴ f is not onto.

Hence, the greatest integer function is neither one-one nor onto.

Question 4:

Show that the Modulus Function f: **R** → **R** given by $f(x)=|x|$, is neither one-one nor onto, where $|x|$ is x, if x is positive or 0 and $|x|$ is $-x$, if x is negative.

Answer f: **R** → **R** is given by,

$$f(x)=|x|=\begin{cases}x, & \text{if } x\geq 0\\ -x, & \text{if } x<0\end{cases}$$

It is seen that $f(-1)=|-1|=1, f(1)=|1|=1$.

∴$f(-1) = f(1)$, but $-1 \neq 1$.

∴ f is not one-one.

Now, consider -1 ∈ **R**.

It is known that $f(x)$ = $|x|$ is always non-negative. Thus, there does not exist any element x in domain **R** such that $f(x)$ = $|x|$ = -1.

∴ f is not onto.

Hence, the modulus function is neither one-one nor onto.

Question 5:

Show that the Signum Function f: **R** → **R**, given by

$$f(x)=\begin{cases}1, & \text{if } x>0\\ 0, & \text{if } x=0\\ -1, & \text{if } x<0\end{cases}$$

is neither one-one nor onto.

Answer f: **R** → **R** is given by,

$$f(x)=\begin{cases}1, \text{ if } x>0\\0, \text{ if } x=0\\-1, \text{ if } x<0\end{cases}$$

It is seen that $f(1) = f(2) = 1$, but $1 \neq 2$.

∴f is not one-one.

Now, as $f(x)$ takes only 3 values (1, 0, or −1) for the element −2 in co-domain **R**, there does not exist any x in domain **R** such that $f(x) = -2$. ∴ f is not onto.

Hence, the signum function is neither one-one nor onto.

Question 6:

Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and let $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B. Show that f is one-one. Answer

It is given that $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$. f: $A \to B$ is defined as $f = \{(1, 4), (2, 5), (3, 6)\}$.

∴ $f(1) = 4$, $f(2) = 5$, $f(3) = 6$

It is seen that the images of distinct elements of A under f are distinct.

Hence, function f is one-one.

Question 7:

In each of the following cases, state whether the function is one-one, onto or bijective. Justify your answer.

(i) f: **R** → **R** defined by $f(x) = 3 - 4x$ (ii) f: **R** → **R** defined by $f(x) = 1 + x^2$ Answer (i) f: **R** → **R** is defined as $f(x) = 3 - 4x$.

Let $x_1, x_2 \in \mathbf{R}$ such that $f(x_1) = f(x_2)$

$\Rightarrow 3 - 4x_1 = 3 - 4x_2$
$\Rightarrow -4x_1 = -4x_2$
$\Rightarrow x_1 = x_2$

∴ f is one-one.

For any real number (y) in **R**, there exists $\frac{3-y}{4}$ in **R** such that

$f\left(\frac{3-y}{4}\right) = 3 - 4\left(\frac{3-y}{4}\right) = y.$

∴ f is onto.

Hence, f is bijective.

(ii) f: **R** → **R** is defined as

$f(x) = 1 + x^2$

Let $x_1, x_2 \in \mathbf{R}$ such that $f(x_1) = f(x_2)$

$\Rightarrow 1 + x_1^2 = 1 + x_2^2$
$\Rightarrow x_1^2 = x_2^2$
$\Rightarrow x_1 = \pm x_2$

∴ $f(x_1) = f(x_2)$ does not imply that $x_1 = x_2$. For instance, $f(1) = f(-1) = 2$

∴ f is not one-one.

Consider an element −2 in co-domain **R**.

It is seen that $f(x) = 1 + x^2$ is positive for all $x \in$ **R**.

Thus, there does not exist any x in domain **R** such that $f(x) = -2$.

∴ f is not onto.

Hence, f is neither one-one nor onto.

Question 8:

Let *A* and *B* be sets. Show that *f*: *A* × *B* → *B* × *A* such that (*a*, *b*) = (*b*, *a*) is bijective function. Answer *f*: *A* × *B* → *B* × *A* is defined as *f*(*a*, *b*) = (*b*, *a*).

Let $(a_1, b_1), (a_2, b_2) \in A \times B$ such that $f(a_1, b_1) = f(a_2, b_2)$

$$\Rightarrow (b_1, a_1) = (b_2, a_2)$$
$$\Rightarrow b_1 = b_2 \text{ and } a_1 = a_2$$
$$\Rightarrow (a_1, b_1) = (a_2, b_2)$$

∴ *f* is one-one.

Now, let (*b*, *a*) ∈ *B* × *A* be any element.

Then, there exists (*a*, *b*) ∈*A* × *B* such that *f*(*a*, *b*) = (*b*, *a*). [By definition of *f*]

∴ *f* is onto.

Hence, *f* is bijective.

Question 9:

Let *f*: **N** → **N** be defined by

$$f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases} \quad \text{for all } n \in \mathbf{N}.$$

State whether the function f is bijective. Justify your answer. Answer

f: **N** → **N** is defined as

$$f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases} \quad \text{for all } n \in \mathbf{N}.$$

It can be observed that:

$$f(1) = \frac{1+1}{2} = 1 \text{ and } f(2) = \frac{2}{2} = 1 \qquad [\text{By definition of } f]$$

$$\therefore f(1) = f(2), \text{ where } 1 \neq 2.$$

∴ *f* is not one-one.

Consider a natural number (*n*) in co-domain **N.**

Case **I:** *n* is odd

∴*n* = 2*r* + 1 for some *r* ∈ **N.** Then, there exists 4*r* + 1∈**N** such that

$$f(4r+1)=\frac{4r+1+1}{2}=2r+1$$.

Case **II:** *n* is even

∴*n* = 2*r* for some *r* ∈ **N.** Then,there exists 4*r* ∈**N** such that $f(4r)=\frac{4r}{2}=2r$.

∴ *f* is onto.

Hence, *f* is not a bijective function.

Question 10:

Let A = **R** − {3} and B = **R** − {1}. Consider the function *f*: A → B defined by $f(x)=\left(\frac{x-2}{x-3}\right)$. Is *f* one-one and onto? Justify your answer.

Answer

A = **R** − {3}, B = **R** − {1}

f: A → B is defined as $f(x)=\left(\frac{x-2}{x-3}\right)$

Let $x,\ y \in$ A such that $f(x)=f(y)$.

$$\Rightarrow \frac{x-2}{x-3}=\frac{y-2}{y-3}$$
$$\Rightarrow (x-2)(y-3)=(y-2)(x-3)$$
$$\Rightarrow xy-3x-2y+6=xy-3y-2x+6$$
$$\Rightarrow -3x-2y=-3y-2x$$
$$\Rightarrow 3x-2x=3y-2y$$
$$\Rightarrow x=y$$
.

∴ *f* is one-one.

Let $y \in B = \mathbf{R} - \{1\}$. Then, $y \neq 1$.

The function f is onto if there exists $x \in A$ such that $f(x) = y$. Now,

$$f(x) = y$$
$$\Rightarrow \frac{x-2}{x-3} = y$$
$$\Rightarrow x - 2 = xy - 3y$$
$$\Rightarrow x(1-y) = -3y + 2$$
$$\Rightarrow x = \frac{2-3y}{1-y} \in A \qquad [y \neq 1]$$

Thus, for any $y \in B$, there exists $\frac{2-3y}{1-y} \in A$ such that

$$f\left(\frac{2-3y}{1-y}\right) = \frac{\left(\frac{2-3y}{1-y}\right) - 2}{\left(\frac{2-3y}{1-y}\right) - 3} = \frac{2-3y-2+2y}{2-3y-3+3y} = \frac{-y}{-1} = y.$$

$\therefore f$ is onto.

Hence, function f is one-one and onto.

Question 11:

Let f: $\mathbf{R} \rightarrow \mathbf{R}$ be defined as $f(x) = x^4$. Choose the correct answer.

(A) f is one-one onto (B) f is many-one onto

(C) f is one-one but not onto (D) f is neither one-one nor onto Answer

f: $\mathbf{R} \rightarrow \mathbf{R}$ is defined as $f(x) = x^4$.

Let $x, y \in \mathbf{R}$ such that $f(x) = f(y)$.

$$\Rightarrow x^4 = y^4$$
$$\Rightarrow x = \pm y$$

$\therefore f(x_1) = f(x_2)$ does not imply that $x_1 = x_2$.

For instance,

$f(1) = f(-1) = 1$

∴ *f* is not one-one.

Consider an element 2 in co-domain **R**. It is clear that there does not exist any *x* in domain **R** such that *f*(*x*) = 2.

∴ *f* is not onto.

Hence, function *f* is neither one-one nor onto.

The correct answer is D.

Question 12:

Let *f*: **R** → **R** be defined as *f*(*x*) = 3*x*. Choose the correct answer.

(A) *f* is one-one onto (B) *f* is many-one onto

(C) *f* is one-one but not onto (D) *f* is neither one-one nor onto

Answer

f: **R** → **R** is defined as *f*(*x*) = 3*x*. Let

x, *y* ∈ **R** such that *f*(*x*) = *f*(*y*).

⇒ 3*x* = 3*y*

⇒ *x* = *y*

∴*f* is one-one.

Also, for any real number (*y*) in co-domain **R**, there exists $\frac{y}{3}$ in **R** such that

$f\left(\frac{y}{3}\right) = 3\left(\frac{y}{3}\right) = y$. ∴*f*

is onto.

Hence, function *f* is one-one and onto.

The correct answer is A.

Class XII : Maths
Chapter 1 : Relations And Functions

Questions and Solutions | Miscellaneous Exercise 1 - NCERT Books

Question 1:

Show that function *f*: **R** → {*x* ∈ **R**: −1 < *x* < 1} defined by $f(x) = \frac{x}{1+|x|}$, *x* ∈**R** is one-one and onto function.

Answer

It is given that *f*: **R** → {*x* ∈ **R**: −1 < *x* < 1} is defined as $f(x) = \frac{x}{1+|x|}$, *x* ∈**R**.

Suppose *f*(*x*) = *f*(*y*), where *x*, *y* ∈ **R**.

$$\Rightarrow \frac{x}{1+|x|} = \frac{y}{1+|y|}$$

It can be observed that if *x* is positive and *y* is negative, then we have:

$$\frac{x}{1+x} = \frac{y}{1-y} \Rightarrow 2xy = x - y$$

Since *x* is positive and *y* is negative:

x > *y* ⇒ *x* − *y* > 0

But, 2*xy* is negative.

Then, $2xy \neq x - y$.

Thus, the case of *x* being positive and *y* being negative can be ruled out.

Under a similar argument, *x* being negative and *y* being positive can also be ruled out

∴ *x* and *y* have to be either positive or negative.

When *x* and *y* are both positive, we have:

$$f(x)=f(y)\Rightarrow \frac{x}{1+x}=\frac{y}{1+y}\Rightarrow x+xy=y+xy\Rightarrow x=y$$

When x and y are both negative, we have:

$$f(x)=f(y)\Rightarrow \frac{x}{1-x}=\frac{y}{1-y}\Rightarrow x-xy=y-yx\Rightarrow x=y$$

∴ f is one-one.

Now, let $y \in \mathbf{R}$ such that $-1 < y < 1$.

If y is negative, then there exists $x=\frac{y}{1+y}\in \mathbf{R}$ such that

$$f(x)=f\left(\frac{y}{1+y}\right)=\frac{\left(\frac{y}{1+y}\right)}{1+\left|\frac{y}{1+y}\right|}=\frac{\frac{y}{1+y}}{1+\left(\frac{-y}{1+y}\right)}=\frac{y}{1+y-y}=y.$$

If y is positive, then there exists $x=\frac{y}{1-y}\in \mathbf{R}$ such that

$$f(x)=f\left(\frac{y}{1-y}\right)=\frac{\left(\frac{y}{1-y}\right)}{1+\left|\left(\frac{y}{1-y}\right)\right|}=\frac{\frac{y}{1-y}}{1+\frac{y}{1-y}}=\frac{y}{1-y+y}=y.$$

∴ f is onto.

Hence, f is one-one and onto.

Question 2:

Show that the function f: **R** → **R** given by $f(x) = x^3$ is injective.

Answer

f: **R** → **R** is given as $f(x) = x^3$.

Suppose $f(x) = f(y)$, where $x, y \in$ **R**.

$\Rightarrow x^3 = y^3$ … (1)

Now, we need to show that $x = y$.

Suppose $x \neq y$, their cubes will also not be equal.

$\Rightarrow x^3 \neq y^3$

However, this will be a contradiction to (1).

$\therefore x = y$

Hence, f is injective.

Question 3:

Given a non empty set X, consider P(X) which is the set of all subsets of X.

Define the relation R in P(X) as follows:

For subsets A, B in P(X), ARB if and only if $A \subset B$. Is R an equivalence relation on P(X)?

Justify you answer:

Answer

Since every set is a subset of itself, ARA for all $A \in$ P(X).

∴R is reflexive.

Let AR$B \Rightarrow A \subset B$.

This cannot be implied to $B \subset A$.

For instance, if $A = \{1, 2\}$ and $B = \{1, 2, 3\}$, then it cannot be implied that B is related to A.

∴ R is not symmetric.

Further, if ARB and BRC, then $A \subset B$ and $B \subset C$.

$\Rightarrow A \subset C$

$\Rightarrow A$RC

∴ R is transitive.

Hence, R is not an equivalence relation since it is not symmetric.

Question 4:

Find the number of all onto functions from the set {1, 2, 3, … , *n*) to itself.

Answer

Onto functions from the set {1, 2, 3, … ,*n*} to itself is simply a permutation on *n* symbols 1, 2, …, *n*.

Thus, the total number of onto maps from {1, 2, … , *n*} to itself is the same as the total number of permutations on *n* symbols 1, 2, …, *n*, which is *n*.

Question 5:

Let $A = \{-1, 0, 1, 2\}$, $B = \{-4, -2, 0, 2\}$ and $f, g: A \to B$ be functions defined by $f(x) = x^2 - x, x \in A$ and $g(x) = 2\left|x - \frac{1}{2}\right| - 1, x \in A$. Are *f* and *g* equal?

Justify your answer. (Hint: One may note that two function $f: A \to B$ and $g: A \to B$ such that $f(a) = g(a)$ ☐$a \in A$, are called equal functions).

Answer

It is given that $A = \{-1, 0, 1, 2\}$, $B = \{-4, -2, 0, 2\}$.

Also, it is given that $f, g: A \to B$ are defined by $f(x) = x^2 - x$, $x \in A$ and

$$g(x) = 2\left|x - \frac{1}{2}\right| - 1,\ x \in A$$

It is observed that:

$$f(-1) = (-1)^2 - (-1) = 1 + 1 = 2$$
$$g(-1) = 2\left|(-1) - \frac{1}{2}\right| - 1 = 2\left(\frac{3}{2}\right) - 1 = 3 - 1 = 2$$
$$\Rightarrow f(-1) = g(-1)$$
$$f(0) = (0)^2 - 0 = 0$$
$$g(0) = 2\left|0 - \frac{1}{2}\right| - 1 = 2\left(\frac{1}{2}\right) - 1 = 1 - 1 = 0$$
$$\Rightarrow f(0) = g(0)$$
$$f(1) = (1)^2 - 1 = 1 - 1 = 0$$
$$g(1) = 2\left|1 - \frac{1}{2}\right| - 1 = 2\left(\frac{1}{2}\right) - 1 = 1 - 1 = 0$$
$$\Rightarrow f(1) = g(1)$$
$$f(2) = (2)^2 - 2 = 4 - 2 = 2$$
$$g(2) = 2\left|2 - \frac{1}{2}\right| - 1 = 2\left(\frac{3}{2}\right) - 1 = 3 - 1 = 2$$
$$\Rightarrow f(2) = g(2)$$
$$\therefore f(a) = g(a)\ \forall a \in A$$

Hence, the functions f and g are equal.

Question 6:

Let $A = \{1, 2, 3\}$. Then number of relations containing (1, 2) and (1, 3) which are reflexive and symmetric but not transitive is

(A) 1 (B) 2 (C) 3 (D) 4

Answer

The given set is A = {1, 2, 3}.

The smallest relation containing (1, 2) and (1, 3) which is reflexive and symmetric, but not transitive is given by:

R = {(1, 1), (2, 2), (3, 3), (1, 2), (1, 3), (2, 1), (3, 1)}

This is because relation R is reflexive as (1, 1), (2, 2), (3, 3) ∈ R.

Relation R is symmetric since (1, 2), (2, 1) ∈R and (1, 3), (3, 1) ∈R.

But relation R is not transitive as (3, 1), (1, 2) ∈ R, but (3, 2) ∉ R.

Now, if we add any two pairs (3, 2) and (2, 3) (or both) to relation R, then relation R will become transitive.

Hence, the total number of desired relations is one.

The correct answer is A.

Question 7:

Let *A* = {1, 2, 3}. Then number of equivalence relations containing (1, 2) is

(A) 1 (B) 2 (C) 3 (D) 4

Answer

It is given that *A* = {1, 2, 3}.

The smallest equivalence relation containing (1, 2) is given by,

R_1 = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)}

Now, we are left with only four pairs i.e., (2, 3), (3, 2), (1, 3), and (3, 1).

If we odd any one pair [say (2, 3)] to R_1, then for symmetry we must add (3, 2). Also, for transitivity we are required to add (1, 3) and (3, 1).

Hence, the only equivalence relation (bigger than R_1) is the universal relation.

This shows that the total number of equivalence relations containing (1, 2) is two.

The correct answer is B.