Class XII : Physics
Chapter 13 : Nuclei

Questions and Solutions | Exercises - NCERT Books

Question 1:

Obtain the binding energy (in MeV) of a nitrogen nucleus $\left({}^{14}_{7}\text{N}\right)$, given $m\left({}^{14}_{7}\text{N}\right)$ =14.00307 u

Answer

Atomic mass of nitrogen $\left({}_{7}\text{N}^{14}\right)$, m = 14.00307 u

A nucleus of nitrogen ${}_{7}\text{N}^{14}$ contains 7 protons and 7 neutrons.

Hence, the mass defect of this nucleus, $\Delta m = 7m_H + 7m_n - m$

Where,

Mass of a proton, m_H = 1.007825 u

Mass of a neutron, m_n= 1.008665 u

$\therefore \Delta m = 7 \times 1.007825 + 7 \times 1.008665 - 14.00307$

= 7.054775 + 7.06055 − 14.00307

= 0.11236 u

But 1 u = 931.5 MeV/c^2

$\therefore \Delta m = 0.11236 \times 931.5\ \text{MeV}/c^2$

Hence, the binding energy of the nucleus is given as:

$E_b = \Delta mc^2$

Where,

c = Speed of light

$\therefore E_b = 0.11236 \times 931.5 \left(\frac{\text{MeV}}{c^2}\right) \times c^2$

= 104.66334 MeV

Hence, the binding energy of a nitrogen nucleus is 104.66334 MeV.

Question 2:

Obtain the binding energy of the nuclei ${}^{56}_{26}\text{Fe}$ and ${}^{209}_{83}\text{Bi}$ in units of MeV from the following data:

$m\left({}^{56}_{26}\text{Fe}\right) = 55.934939$ u $m\left({}^{209}_{83}\text{Bi}\right) = 208.980388$ u

Answer

Atomic mass of ${}^{56}_{26}\text{Fe}$, m_1 = 55.934939 u

${}^{56}_{26}\text{Fe}$ nucleus has 26 protons and (56 − 26) = 30 neutrons

Hence, the mass defect of the nucleus, $\Delta m = 26 \times m_H + 30 \times m_n - m_1$

Where,

Mass of a proton, $m_H = 1.007825$ u

Mass of a neutron, $m_n = 1.008665$ u

$\therefore \Delta m = 26 \times 1.007825 + 30 \times 1.008665 - 55.934939$

$= 26.20345 + 30.25995 - 55.934939$

$= 0.528461$ u

But 1 u = 931.5 MeV/c^2

$\therefore \Delta m = 0.528461 \times 931.5$ MeV/c^2

The binding energy of this nucleus is given as:

$E_{b1} = \Delta mc^2$

Where,

c = Speed of light

$\therefore E_{b1} = 0.528461 \times 931.5 \left(\frac{\text{MeV}}{c^2}\right) \times c^2$

$= 492.26$ MeV

Average binding energy per nucleon $= \frac{492.26}{56} = 8.79$ MeV

Atomic mass of ${}^{209}_{83}\text{Bi}$, $m_2 = 208.980388$ u

${}^{209}_{83}\text{Bi}$ nucleus has 83 protons and (209 − 83) 126 neutrons.

Hence, the mass defect of this nucleus is given as:

$\Delta m' = 83 \times m_H + 126 \times m_n - m_2$

Where,

Mass of a proton, $m_H = 1.007825$ u

Mass of a neutron, $m_n = 1.008665$ u

$\therefore \Delta m' = 83 \times 1.007825 + 126 \times 1.008665 - 208.980388$

$= 83.649475 + 127.091790 - 208.980388$

= 1.760877 u

But 1 u = 931.5 MeV/c^2

∴$\Delta m'$ = 1.760877 × 931.5 MeV/c^2

Hence, the binding energy of this nucleus is given as:

$E_{b2} = \Delta m' c^2$

$= 1.760877 \times 931.5 \left(\frac{\text{MeV}}{c^2}\right) \times c^2$

= 1640.26 MeV

Average bindingenergy per nucleon = $\frac{1640.26}{209} = 7.848$ MeV

Question 3:

A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of $^{63}_{29}\text{Cu}$ atoms (of mass 62.92960 u).

Answer

Mass of a copper coin, m' = 3 g

Atomic mass of $_{29}\text{Cu}^{63}$ atom, m = 62.92960 u

The total number of $_{29}\text{Cu}^{63}$ atoms in the coin, $N = \frac{N_A \times m'}{\text{Mass number}}$

Where,

N_A = Avogadro's number = 6.023×10^{23} atoms /g

Mass number = 63 g

$\therefore N = \dfrac{6.023 \times 10^{23} \times 3}{63} = 2.868 \times 10^{22}$ atoms

${}_{29}\text{Cu}^{63}$ nucleus has 29 protons and (63 − 29) 34 neutrons

∴Mass defect of this nucleus, $\Delta m' = 29 \times m_H + 34 \times m_n - m$

Where,

Mass of a proton, $m_H = 1.007825$ u

Mass of a neutron, $m_n = 1.008665$ u

$\therefore \Delta m' = 29 \times 1.007825 + 34 \times 1.008665 - 62.9296$

= 0.591935 u

Mass defect of all the atoms present in the coin, $\Delta m = 0.591935 \times 2.868 \times 10^{22}$

$= 1.69766958 \times 10^{22}$ u

But 1 u = 931.5 MeV/c^2

$\therefore \Delta m = 1.69766958 \times 10^{22} \times 931.5$ MeV/c^2

Hence, the binding energy of the nuclei of the coin is given as:

$E_b = \Delta mc^2$

$= 1.69766958 \times 10^{22} \times 931.5 \left(\dfrac{\text{MeV}}{c^2}\right) \times c^2$

$= 1.581 \times 10^{25}$ MeV

But 1 MeV = 1.6×10^{-13} J

$E_b = 1.581 \times 10^{25} \times 1.6 \times 10^{-13}$

$= 2.5296 \times 10^{12}$ J

This much energy is required to separate all the neutrons and protons from the given coin.

Question 4:

Obtain approximately the ratio of the nuclear radii of the gold isotope ${}^{197}_{79}\text{Au}$ and the silver isotope ${}^{107}_{47}\text{Ag}$.

Answer

Nuclear radius of the gold isotope ${}_{79}\text{Au}^{197}$ $= R_{\text{Au}}$

Nuclear radius of the silver isotope ${}_{47}\text{Ag}^{107}$ $= R_{\text{Ag}}$

Mass number of gold, $A_{\text{Au}} = 197$

Mass number of silver, $A_{\text{Ag}} = 107$

The ratio of the radii of the two nuclei is related with their mass numbers as:

$$\frac{R_{\text{Au}}}{R_{\text{Ag}}} = \left(\frac{R_{\text{Au}}}{R_{\text{Ag}}}\right)^{\frac{1}{3}}$$

$$= \left(\frac{197}{107}\right)^{\frac{1}{3}} = 1.2256$$

Hence, the ratio of the nuclear radii of the gold and silver isotopes is about 1.23.

Question 5:

The Q value of a nuclear reaction $A + b \rightarrow C + d$ is defined by

$Q = [\, m_A + m_b - m_C - m_d]c^2$ where the masses refer to the respective nuclei. Determine from the given data the Q-value of the following reactions and state whether the reactions are exothermic or endothermic.

$${}_1^1\mathrm{H} + {}_1^3\mathrm{H} \rightarrow {}_1^2\mathrm{H} + {}_1^2\mathrm{H}$$

$${}_6^{12}\mathrm{C} + {}_6^{12}\mathrm{C} \rightarrow {}_{10}^{20}\mathrm{Ne} + {}_2^4\mathrm{He}$$

Atomic masses are given to be

$$m\left({}_1^2\mathrm{H}\right) = 2.014102\ \mathrm{u}$$
$$m\left({}_1^3\mathrm{H}\right) = 3.016049\ \mathrm{u}$$
$$m\left({}_6^{12}\mathrm{C}\right) = 12.000000\,\mathrm{u}$$
$$m\left({}_{10}^{20}\mathrm{Ne}\right) = 19.992439\,\mathrm{u}$$

Answer

The given nuclear reaction is:

$${}_1\mathrm{H}^1 + {}_1^3\mathrm{H} \rightarrow {}_1^2\mathrm{H} + {}_1^2\mathrm{H}$$

It is given that:

Atomic mass $m\left({}_1^1\mathrm{H}\right) = 1.007825\ \mathrm{u}$

Atomic mass $m\left({}_1^3\mathrm{H}\right) = 3.016049\ \mathrm{u}$

Atomic mass $m\left({}_1^2\mathrm{H}\right) = 2.014102\ \mathrm{u}$

According to the question, the Q-value of the reaction can be written as:

$$Q = \left[m\left({}_1^1\text{H}\right) + m\left({}_1^3\text{H}\right) - 2m\left({}_1^2\text{H}\right) \right] c^2$$

$$= [1.007825 + 3.016049 - 2 \times 2.014102] c^2$$

$$Q = \left(-0.00433\ c^2\right)\ \text{u}$$

But $1\ \text{u} = 931.5\ \text{MeV}/c^2$

$$\therefore Q = -0.00433 \times 931.5 = -4.0334\ \text{MeV}$$

The negativeQ-value of the reaction shows that the reaction is endothermic.

The given nuclear reaction is:

$${}_6^{12}\text{C} + {}_6^{12}\text{C} \rightarrow {}_{10}^{20}\text{Ne} + {}_2^4\text{He}$$

It is given that:

Atomic mass of $m\left({}_6^{12}\text{C}\right) = 12.0\ \text{u}$

Atomic mass of $m\left({}_{10}^{20}\text{Ne}\right) = 19.992439\ \text{u}$

Atomic mass of $m\left({}_2^4\text{He}\right) = 4.002603\ \text{u}$

The Q-value of this reaction is given as:

$$Q = \left[2m\left({}_6^{12}\text{C}\right) - m\left({}_{10}^{20}\text{Ne}\right) - m\left({}_2^4\text{He}\right) \right] c^2$$

$$= [2 \times 12.0 - 19.992439 - 4.002603] c^2$$
$$= \left(0.004958\ c^2\right)\ \text{u}$$
$$= 0.004958 \times 931.5 = 4.618377\ \text{MeV}$$

The positive Q-value of the reaction shows that the reaction is exothermic.

Question 9:

Calculate the height of the potential barrier for a head on collision of two deuterons. (Hint: The height of the potential barrier is given by the Coulomb repulsion between the two deuterons when they just touch each other. Assume that they can be taken as hard spheres of radius 2.0 fm.)

Answer

When two deuterons collide head-on, the distance between their centres, *d* is given as:

Radius of 1st deuteron + Radius of 2nd deuteron

Radius of a deuteron nucleus = 2 fm = 2×10^{-15} m

$\therefore d = 2 \times 10^{-15} + 2 \times 10^{-15} = 4 \times 10^{-15}$ m

Charge on a deuteron nucleus = Charge on an electron = $e = 1.6 \times 10^{-19}$ C

Potential energy of the two-deuteron system:

$$V = \frac{e^2}{4\pi \epsilon_0 d}$$

Where,

ϵ_0 = Permittivity of free space

$$\frac{1}{4\pi \epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\therefore V = \frac{9 \times 10^9 \times \left(1.6 \times 10^{-19}\right)^2}{4 \times 10^{-15}} \text{ J}$$

$$= \frac{9 \times 10^9 \times \left(1.6 \times 10^{-19}\right)^2}{4 \times 10^{-15} \times \left(1.6 \times 10^{-19}\right)} \text{ eV}$$

$$= 360 \text{ keV}$$

Hence, the height of the potential barrier of the two-deuteron system is

360 keV.

Question 10:

From the relation $R = R_0A^{1}/^{3}$, where R_0 is a constant and A is the mass number of a nucleus, show that the nuclear matter density is nearly constant (i.e. independent of A).

Answer

We have the expression for nuclear radius as:

$R = R_0A^{1}/^{3}$

Where,

R_0 = Constant.

A = Mass number of the nucleus

Nuclear matter density, $\rho = \dfrac{\text{Mass of the nucleus}}{\text{Volume of the nucleus}}$

Let m be the average mass of the nucleus.

Hence, mass of the nucleus = mA

$$\therefore \rho = \frac{mA}{\frac{4}{3}\pi R^3} = \frac{3mA}{4\pi\left(R_0 A^{\frac{1}{3}}\right)^3} = \frac{3mA}{4\pi R_0^3 A} = \frac{3m}{4\pi R_0^3}$$

Hence, the nuclear matter density is independent of A. It is nearly constant.