FINAL JEE–MAIN EXAMINATION – JULY, 2021

(Held On Sunday 25th July, 2021) **TIME : 9 : 00 AM to 12 : 00**

MATHEMATICS — TEST PAPER WITH ANSWER

SECTION-A

1. A spherical gas balloon of radius 16 meter subtends an angle 60° at the eye of the observer A while the angle of elevation of its center from the eye of A is 75°. Then the height (in meter) of the top most point of the balloon from the level of the observer's eye is :

(1) $8(2+2\sqrt{3}+\sqrt{2})$ (2) $8(\sqrt{6}+\sqrt{2}+2)$

(3) $8(\sqrt{2}+2+\sqrt{3})$ (4) $8(\sqrt{6}-\sqrt{2}+2)$

Official Ans. by NTA (2)

2. Let $f(x) = 3\sin^4 x + 10\sin^3 x + 6\sin^2 x - 3$, $x \in \left[-\frac{\pi}{6}, \frac{\pi}{2}\right]$. Then, f is :

(1) increasing in $\left(-\frac{\pi}{6}, \frac{\pi}{2}\right)$

(2) decreasing in $\left(0, \frac{\pi}{2}\right)$

(3) increasing in $\left(-\frac{\pi}{6}, 0\right)$

(4) decreasing in $\left(-\frac{\pi}{6}, 0\right)$

Official Ans. by NTA (4)

3. Let S_n be the sum of the first n terms of an arithmetic progression. If $S_{3n} = 3S_{2n}$, then the value of $\frac{S_{4n}}{S_{2n}}$ is :

(1) 6 (2) 4 (3) 2 (4) 8

Official Ans. by NTA (1)

4. The locus of the centroid of the triangle formed by any point P on the hyperbola $16x^2 - 9y^2 + 32x + 36y - 164 = 0$, and its foci is :

(1) $16x^2 - 9y^2 + 32x + 36y - 36 = 0$

(2) $9x^2 - 16y^2 + 36x + 32y - 144 = 0$

(3) $16x^2 - 9y^2 + 32x + 36y - 144 = 0$

(4) $9x^2 - 16y^2 + 36x + 32y - 36 = 0$

Official Ans. by NTA (1)

5. Let the vectors

$(2+a+b)\hat{i}+(a+2b+c)\hat{j}-(b+c)\hat{k}$, $(1+b)\hat{i}+2b\hat{j}-b\hat{k}$

and $(2+b)\hat{i}+2b\hat{j}+(1-b)\hat{k}$ a, b, c, $\in \mathbf{R}$

be co-planar. Then which of the following is true?

(1) 2b = a + c (2) 3c = a + b

(3) a = b + 2c (4) 2a = b + c

Official Ans. by NTA (1)

6. Let $f : \mathbf{R} \to \mathbf{R}$ be defined as

$$f(x) = \begin{cases} \frac{\lambda|x^2-5x+6|}{\mu(5x-x^2-6)}, & x<2 \\ e^{\frac{\tan(x-2)}{x-[x]}} & ,x>2 \\ \mu & ,x=2 \end{cases}$$

where [x] is the greatest integer less than or equal to x. If f is continuous at x = 2, then λ + μ is equal to :

(1) e(–e + 1) (2) e(e – 2)

(3) 1 (4) 2e – 1

Official Ans. by NTA (1)

7. The value of the definite integral

$$\int_{\pi/24}^{5\pi/24} \frac{dx}{1+\sqrt[3]{\tan 2x}} \text{ is :}$$

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{12}$ (4) $\frac{\pi}{18}$

Official Ans. by NTA (3)

8. If b is very small as compared to the value of a, so that the cube and other higher powers of $\frac{b}{a}$ can be neglected in the identity

$$\frac{1}{a-b}+\frac{1}{a-2b}+\frac{1}{a-3b}+....+\frac{1}{a-nb} = \alpha n + \beta n^2 + \gamma n^3,$$

then the value of γ is :

(1) $\frac{a^2+b}{3a^3}$ (2) $\frac{a+b}{3a^2}$ (3) $\frac{b^2}{3a^3}$ (4) $\frac{a+b^2}{3a^3}$

Official Ans. by NTA (3)

9. Let y = y(x) be the solution of the differential equation $\frac{dy}{dx} = 1 + xe^{y-x}, -\sqrt{2} < x < \sqrt{2}, y(0) = 0$ then, the minimum value of y(x), $x \in (-\sqrt{2}, \sqrt{2})$ is equal to :

(1) $(2-\sqrt{3}) - \log_e 2$

(2) $(2+\sqrt{3}) + \log_e 2$

(3) $(1+\sqrt{3}) - \log_e(\sqrt{3}-1)$

(4) $(1-\sqrt{3}) - \log_e(\sqrt{3}-1)$

Official Ans. by NTA (4)

10. The Boolean expression $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to :

(1) ~q (2) q
(3) p (4) ~p

Official Ans. by NTA (4)

11. The area (in sq. units) of the region, given by the set $\{(x, y) \in \mathbf{R} \times \mathbf{R} \mid x \geq 0, 2x^2 \leq y \leq 4 - 2x\}$ is :

(1) $\frac{8}{3}$ (2) $\frac{17}{3}$

(3) $\frac{13}{3}$ (4) $\frac{7}{3}$

Official Ans. by NTA (4)

12. The sum of all values of x in [0, 2π], for which sinx + sin2x + sin3x + sin4x = 0, is equal to :

(1) 8π (2) 11 π
(3) 12 π (4) 9 π

Official Ans. by NTA (4)

13. Let $g : \mathbf{N} \to \mathbf{N}$ be defined as

g(3n + 1) = 3n + 2,

g(3n + 2) = 3n + 3,

g(3n + 3) = 3n + 1, for all n ≥ 0.

Then which of the following statements is true ?

(1) There exists an onto function $f : \mathbf{N} \to \mathbf{N}$ such that fog = f

(2) There exists a one–one function $f: \mathbf{N} \to \mathbf{N}$ such that fog = f

(3) gogog = g

(4) There exists a function $f : \mathbf{N} \to \mathbf{N}$ such that gof = f

Official Ans. by NTA (1)

14. Let $f : [0, \infty) \to [0, \infty)$ be defined as

$$f(x) = \int_0^x [y]\,dy$$

where [x] is the greatest integer less than or equal to x. Which of the following is true?

(1) f is continuous at every point in [0, ∞) and differentiable except at the integer points.

(2) f is both continuous and differentiable except at the integer points in [0, ∞).

(3) f is continuous everywhere except at the integer points in [0, ∞).

(4) f is differentiable at every point in [0, ∞).

Official Ans. by NTA (1)

15. The values of a and b, for which the system of equations

2x + 3y + 6z = 8
x + 2y + az = 5
3x + 5y + 9z = b

has no solution, are :

(1) a = 3, b ≠ 13 (2) a ≠ 3, b ≠ 13
(3) a ≠ 3, b = 3 (4) a = 3, b = 13

Official Ans. by NTA (1)

16. Let 9 distinct balls be distributed among 4 boxes, B_1, B_2, B_3 and B_4. If the probability than B_3 contains exactly 3 balls is $k\left(\frac{3}{4}\right)^9$ then k lies in the set :

(1) $\{x \in \mathbf{R} : |x - 3| < 1\}$ (2) $\{x \in \mathbf{R} : |x - 2| \leq 1\}$
(3) $\{x \in \mathbf{R} : |x - 1| < 1\}$ (4) $\{x \in \mathbf{R} : |x - 5| \leq 1\}$

Official Ans. by NTA (1)

17. Let a parabola P be such that its vertex and focus lie on the positive x–axis at a distance 2 and 4 units from the origin, respectively. If tangents are drawn from O(0, 0) to the parabola P which meet P at S and R, then the area (in sq. units) of ΔSOR is equal to :

(1) $16\sqrt{2}$ (2) 16
(3) 32 (4) $8\sqrt{2}$

Official Ans. by NTA (2)

18. The number of real roots of the equation $e^{6x} - e^{4x} - 2e^{3x} - 12e^{2x} + e^x + 1 = 0$ is :

(1) 2 (2) 4
(3) 6 (4) 1

Official Ans. by NTA (1)

19. Let an ellipse $E:\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $a^2 > b^2$, passes through $\left(\sqrt{\frac{3}{2}},1\right)$ and has eccentricity $\frac{1}{\sqrt{3}}$. If a circle, centered at focus $F(\alpha, 0)$, $\alpha > 0$, of E and radius $\frac{2}{\sqrt{3}}$, intersects E at two points P and Q, then PQ^2 is equal to :

(1) $\frac{8}{3}$ (2) $\frac{4}{3}$ (3) $\frac{16}{3}$ (4) 3

Official Ans. by NTA (3)

20. Let the foot of perpendicular from a point P(1, 2, –1) to the straight line $L:\frac{x}{1}=\frac{y}{0}=\frac{z}{-1}$ be N. Let a line be drawn from P parallel to the plane $x + y + 2z = 0$ which meets L at point Q. If α is the acute angle between the lines PN and PQ, then $\cos\alpha$ is equal to ________.

(1) $\frac{1}{\sqrt{5}}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\frac{1}{\sqrt{3}}$ (4) $\frac{1}{2\sqrt{3}}$

Official Ans. by NTA (3)

SECTION-B

1. Let y = y(x) be solution of the following differential equation

$$e^y\frac{dy}{dx}-2e^y\sin x+\sin x\cos^2 x=0,\quad y\left(\frac{\pi}{2}\right)=0$$

If $y(0) = \log_e(\alpha + \beta e^{-2})$, then $4(\alpha + \beta)$ is equal to ________.

Official Ans. by NTA (4)

2. If the value of

$$\left(1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+......\text{upto }\infty\right)^{\log_{(0.25)}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+.......\text{upto }\infty\right)}$$

is l, then l^2 is equal to ________.

Official Ans. by NTA (3)

3. Consider the following frequency distribution :

class :	10–20	20–30	30–40	40–50	50–60
Frequency :	α	110	54	30	β

If the sum of all frequencies is 584 and median is 45, then $|\alpha - \beta|$ is equal to ________.

Official Ans. by NTA (164)

4. Let $\vec{p}=2\hat{i}+3\hat{j}+\hat{k}$ and $\vec{q}=\hat{i}+2\hat{j}+\hat{k}$ be two vectors. If a vector $\vec{r}=(\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k})$ is perpendicular to each of the vectors $(\vec{p}+\vec{q})$ and $(\vec{p}-\vec{q})$, and $|\vec{r}|=\sqrt{3}$, then $|\alpha| + |\beta| + |\gamma|$ is equal to ________.

Official Ans. by NTA (3)

5. The ratio of the coefficient of the middle term in the expansion of $(1 + x)^{20}$ and the sum of the coefficients of two middle terms in expansion of $(1 + x)^{19}$ is ________.

Official Ans. by NTA (1)

6. Let $M=\left\{A=\begin{pmatrix}a & b\\ c & d\end{pmatrix}: a,b,c,d\in\{\pm3,\pm2,\pm1,0\}\right\}$.

Define $f : M \to \mathbf{Z}$, as $f(A) = \det(A)$, for all $A \in M$, where **Z** is set of all integers. Then the number of $A \in M$ such that $f(A) = 15$ is equal to ________.

Official Ans. by NTA (16)

7. There are 5 students in class 10, 6 students in class 11 and 8 students in class 12. If the number of ways, in which 10 students can be selected from them so as to include at least 2 students from each class and at most 5 students from the total 11 students of class 10 and 11 is 100 k, then k is equal to ________.

Official Ans. by NTA (238)

8. If α, β are roots of the equation $x^2+5(\sqrt{2})x+10=0$, $\alpha>\beta$ and $P_n = \alpha^n - \beta^n$ for each positive integer n, then the value of $\left(\frac{P_{17}P_{20}+5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19}+5\sqrt{2}P_{18}^2}\right)$ is equal to ________.

Official Ans. by NTA (1)

9. The term independent of 'x' in the expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$, where $x\neq 0, 1$ is equal to ________.

Official Ans. by NTA (210)

10. Let

$$S=\left\{n\in\mathbf{N}\left|\begin{pmatrix}0 & i\\ 1 & 0\end{pmatrix}^n\begin{pmatrix}a & b\\ c & d\end{pmatrix}=\begin{pmatrix}a & b\\ c & d\end{pmatrix}\forall a,b,c,d\in\mathbf{R}\right.\right\},$$

where $i=\sqrt{-1}$. Then the number of 2–digit numbers in the set S is ______.

Official Ans. by NTA (11)