FINAL JEE–MAIN EXAMINATION – JULY, 2021

(Held On Tuesday 27th July, 2021) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS

TEST PAPER WITH ANSWER

SECTION-A

1. If the mean and variance of the following data:
6, 10, 7, 13, a, 12, b, 12
are 9 and $\frac{37}{4}$ respectively, then $(a-b)^2$ is equal to:

(1) 24 (2) 12 (3) 32 (4) 16

Official Ans. by NTA (4)

2. The value of $\lim\limits_{n\to\infty}\frac{1}{n}\sum\limits_{j=1}^{n}\frac{(2j-1)+8n}{(2j-1)+4n}$ is equal to :

(1) $5+\log_e\left(\frac{3}{2}\right)$ (2) $2-\log_e\left(\frac{2}{3}\right)$

(3) $3+2\log_e\left(\frac{2}{3}\right)$ (4) $1+2\log_e\left(\frac{3}{2}\right)$

Official Ans. by NTA (4)

3. Let $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=-\hat{i}+2\hat{j}+3\hat{k}$. Then the vector product $\left(\vec{a}+\vec{b}\right)\times\left(\left(\vec{a}\times\left(\left(\vec{a}-\vec{b}\right)\times\vec{b}\right)\right)\times\vec{b}\right)$ is equal to :

(1) $5\left(34\hat{i}-5\hat{j}+3\hat{k}\right)$ (2) $7\left(34\hat{i}-5\hat{j}+3\hat{k}\right)$

(3) $7\left(30\hat{i}-5\hat{j}+7\hat{k}\right)$ (4) $5\left(30\hat{i}-5\hat{j}+7\hat{k}\right)$

Official Ans. by NTA (2)

4. The value of the definite integral

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{dx}{(1+e^{x\cos x})(\sin^4 x+\cos^4 x)}$$

is equal to :

(1) $-\frac{\pi}{2}$ (2) $\frac{\pi}{2\sqrt{2}}$ (3) $-\frac{\pi}{4}$ (4) $\frac{\pi}{\sqrt{2}}$

Official Ans. by NTA (2)

5. Let C be the set of all complex numbers. Let
$S_1=\{z\in C : |z-3-2i|^2=8\}$,
$S_2=\{z\in C : \text{Re}(z)\geq 5\}$ and
$S_3=\{z\in C : |z-\bar{z}|\geq 8\}$.
Then the number of elements in $S_1\cap S_2\cap S_3$ is equal to

(1) 1 (2) 0 (3) 2 (4) Infinite

Official Ans. by NTA (1)

6. If the area of the bounded region

$$R=\left\{(x,y):\max\{0,\log_e x\}\leq y\leq 2^x,\frac{1}{2}\leq x\leq 2\right\}$$

is, $\alpha(\log_e 2)^{-1}+\beta(\log_e 2)+\gamma$, then the value of $(\alpha+\beta-2\gamma)^2$ is equal to :

(1) 8 (2) 2 (3) 4 (4) 1

Official Ans. by NTA (2)

7. A ray of light through (2,1) is reflected at a point P on the y-axis and then passes through the point (5, 3). If this reflected ray is the directrix of an ellipse with eccentricity $\frac{1}{3}$ and the distance of the nearer focus from this directrix is $\frac{8}{\sqrt{53}}$, then the equation of the other directrix can be:

(1) $11x + 7y + 8 = 0$ or $11x + 7y - 15 = 0$
(2) $11x - 7y - 8 = 0$ or $11x + 7y + 15 = 0$
(3) $2x - 7y + 29 = 0$ or $2x - 7y - 7 = 0$
(4) $2x - 7y - 39 = 0$ or $2x - 7y - 7 = 0$

Official Ans. by NTA (3)

8. If the coefficients of x^7 in $\left(x^2+\frac{1}{bx}\right)^{11}$ and x^{-7} in $\left(x-\frac{1}{bx^2}\right)^{11}$, $b\neq 0$, are equal, then the value of b is equal to:

(1) 2 (2) –1 (3) 1 (4) –2

Official Ans. by NTA (3)

9. The compound statement $(P\vee Q)\wedge(\sim P)\Rightarrow Q$ is equivalent to:

(1) $P\vee Q$ (2) $P\wedge\sim Q$

(3) $\sim(P\Rightarrow Q)$ (4) $\sim(P\Rightarrow Q)\Leftrightarrow P\wedge\sim Q$

Official Ans. by NTA (4)

10. If $\sin\theta+\cos\theta=\frac{1}{2}$, then

$16(\sin(2\theta)+\cos(4\theta)+\sin(6\theta))$ is equal to:

(1) 23 (2) –27 (3) –23 (4) 27

Official Ans. by NTA (3)

11. Let $A=\begin{bmatrix}1 & 2\\ -1 & 4\end{bmatrix}$. If $A^{-1}=\alpha I+\beta A$, $\alpha, \beta \in \mathbf{R}$, I is a 2×2 identity matrix, then $4(\alpha-\beta)$ is equal to :

(1) 5 (2) $\frac{8}{3}$ (3) 2 (4) 4

Official Ans. by NTA (4)

12. Let $f:\left(-\frac{\pi}{4},\frac{\pi}{4}\right)\to \mathbf{R}$ be defined as

$$f(x)=\begin{cases}(1+|\sin x|)^{\frac{3a}{|\sin x|}} & , \quad -\frac{\pi}{4}<x<0\\ b & , \quad x=0\\ e^{\cot 4x/\cot 2x} & , \quad 0<x<\frac{\pi}{4}\end{cases}$$

If f is continuous at x = 0, then the value of $6a+b^2$ is equal to:

(1) $1-e$ (2) $e-1$ (3) $1+e$ (4) e

Official Ans. by NTA (3)

13. Let y = y(x) be solution of the differential equation $\log_e\left(\frac{dy}{dx}\right)=3x+4y$, with y(0) = 0.

If $y\left(-\frac{2}{3}\log_e 2\right)=\alpha\log_e 2$, then the value of α is equal to:

(1) $-\frac{1}{4}$ (2) $\frac{1}{4}$ (3) 2 (4) $-\frac{1}{2}$

Official Ans. by NTA (1)

14. Let the plane passing through the point (–1, 0, –2) and perpendicular to each of the planes $2x+y-z=2$ and $x-y-z=3$ be $ax+by+cz+8=0$. Then the value of a + b + c is equal to:

(1) 3 (2) 8 (3) 5 (4) 4

Official Ans. by NTA (4)

15. Two tangents are drawn from the point P(–1, 1) to the circle $x^2+y^2-2x-6y+6=0$. If these tangents touch the circle at points A and B, and if D is a point on the circle such that length of the segments AB and AD are equal, then the area of the triangle ABD is equal to:

(1) 2 (2) $(3\sqrt{2}+2)$

(3) 4 (4) $3(\sqrt{2}-1)$

Official Ans. by NTA (3)

16. Let $f:\mathbf{R}\to\mathbf{R}$ be a function such that f(2) = 4 and $f'(2)=1$. Then, the value of $\lim\limits_{x\to 2}\frac{x^2f(2)-4f(x)}{x-2}$ is equal to :

(1) 4 (2) 8 (3) 16 (4) 12

Official Ans. by NTA (4)

17. Let P and Q be two distinct points on a circle which has center at C(2, 3) and which passes through origin O. If OC is perpendicular to both the line segments CP and CQ, then the set {P, Q} is equal to

(1) $\{(4,0),(0,6)\}$

(2) $\left\{\left(2+2\sqrt{2},3-\sqrt{5}\right),\left(2-2\sqrt{2},3+\sqrt{5}\right)\right\}$

(3) $\left\{\left(2+2\sqrt{2},3+\sqrt{5}\right),\left(2-2\sqrt{2},3-\sqrt{5}\right)\right\}$

(4) $\{(-1,5),(5,1)\}$

Official Ans. by NTA (4)

18. Let α, β be two roots of the equation $x^2+(20)^{1/4}x+(5)^{1/2}=0$. Then $\alpha^8+\beta^8$ is equal to

(1) 10 (2) 100 (3) 50 (4) 160

Official Ans. by NTA (3)

19. The probability that a randomly selected 2-digit number belongs to the set$\{n \in N : (2^n - 2)$ is a multiple of 3$\}$ is equal to

(1) $\frac{1}{6}$ (2) $\frac{2}{3}$ (3) $\frac{1}{2}$ (4) $\frac{1}{3}$

Official Ans. by NTA (3)

20. Let

$A=\{(x,y)\in\mathbf{R}\times\mathbf{R}\,|\,2x^2+2y^2-2x-2y=1\}$,

$B=\{(x,y)\in\mathbf{R}\times\mathbf{R}\,|\,4x^2+4y^2-16y+7=0\}$ and

$C=\{(x,y)\in\mathbf{R}\times\mathbf{R}\,|\,x^2+y^2-4x-2y+5\le r^2\}$.

Then the minimum value of $|r|$ such that $A\cup B\subseteq C$ is equal to

(1) $\frac{3+\sqrt{10}}{2}$ (2) $\frac{2+\sqrt{10}}{2}$

(3) $\frac{3+2\sqrt{5}}{2}$ (4) $1+\sqrt{5}$

Official Ans. by NTA (3)

SECTION-B

1. For real numbers α and β, consider the following system of linear equations :

$x + y - z = 2, x + 2y + \alpha z = 1, 2x - y + z = \beta.$

If the system has infinite solutions, then $\alpha + \beta$ is equal to _____

Official Ans. by NTA (5)

2. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b}$ and $\vec{c} = \hat{j} - \hat{k}$ be three vectors such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \cdot \vec{b} = 1$. If the length of projection vector of the vector $\vec{b}$ on the vector $\vec{a} \times \vec{c}$ is l, then the value of $3l^2$ is equal to _____.

Official Ans. by NTA (2)

3. If $\log_3 2, \log_3(2^x - 5), \log_3\left(2^x - \frac{7}{2}\right)$ are in an arithmetic progression, then the value of x is equal to _____.

Official Ans. by NTA (3)

4. Let the domain of the function

$f(x) = \log_4\left(\log_5\left(\log_3(18x - x^2 - 77)\right)\right)$ be (a, b). Then the value of the integral

$\int_a^b \frac{\sin^3 x}{(\sin^3 x + \sin^3(a + b - x))} dx$ is equal to _____.

Official Ans. by NTA (1)

5. Let

$$f(x) = \begin{vmatrix} \sin^2 x & -2 + \cos^2 x & \cos 2x \\ 2 + \sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & 1 + \cos 2x \end{vmatrix}, x \in [0, \pi]$$

Then the maximum value of f(x) is equal to _____.

Official Ans. by NTA (6)

6. Let F : [3, 5] $\to$ **R** be a twice differentiable function on (3, 5) such that

$F(x) = e^{-x} \int_3^x (3t^2 + 2t + 4F'(t))\, dt.$

If $F'(4) = \frac{\alpha e^\beta - 224}{(e^\beta - 4)^2}$, then $\alpha + \beta$ is equal to _____.

Official Ans. by NTA (16)

7. Let a plane P pass through the point (3, 7, –7) and contain the line, $\frac{x-2}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$. If distance of the plane P from the origin is d, then d^2 is equal to _____.

Official Ans. by NTA (3)

8. Let S = {1, 2, 3, 4, 5, 6, 7}. Then the number of possible functions f : S $\to$ S such that f(m·n) = f(m)·f(n) for every m, n $\in$ S and m · n $\in$ S is equal to_____.

Official Ans. by NTA (490)

9. If y = y(x), $y \in \left[0, \frac{\pi}{2}\right)$ is the solution of the differential equation

$\sec y \frac{dy}{dx} - \sin(x + y) - \sin(x - y) = 0$, with y(0) = 0, then $5y'\left(\frac{\pi}{2}\right)$ is equal to _____.

Official Ans. by NTA (2)

10. Let f : [0, 3] $\to$ **R** be defined by

f(x) = min {x – [x], 1 + [x] – x}

where [x] is the greatest integer less than or equal to x. Let P denote the set containing all x $\in$ [0, 3] where f is discontinuous, and Q denote the set containing all x $\in$ (0, 3) where f is not differentiable. Then the sum of number of elements in P and Q is equal to _____.

Official Ans. by NTA (5)