FINAL JEE–MAIN EXAMINATION – JULY, 2021

(Held On Tuesday 27th July, 2021) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS — TEST PAPER WITH ANSWER

SECTION-A

1. The point P (a,b) undergoes the following three transformations successively :

(a) reflection about the line y = x.

(b) translation through 2 units along the positive direction of x-axis.

(c) rotation through angle $\frac{\pi}{4}$ about the origin in the anti-clockwise direction.

If the co-ordinates of the final position of the point P are $\left(-\frac{1}{\sqrt{2}},\frac{7}{\sqrt{2}}\right)$, then the value of 2a + b is equal to :

(1) 13 (2) 9 (3) 5 (4) 7

Official Ans. by NTA (2)

2. A possible value of 'x', for which the ninth term in the expansion of $\left\{3^{\log_3\sqrt{25^{x-1}+7}}+3^{\left(-\frac{1}{8}\right)\log_3\left(5^{x-1}+1\right)}\right\}^{10}$ in the increasing powers of $3^{\left(-\frac{1}{8}\right)\log_3\left(5^{x-1}+1\right)}$ is equal to 180, is :

(1) 0 (2) –1 (3) 2 (4) 1

Official Ans. by NTA (4)

3. For real numbers α and $\beta \neq 0$, if the point of intersection of the straight lines

$\frac{x-\alpha}{1}=\frac{y-1}{2}=\frac{z-1}{3}$ and $\frac{x-4}{\beta}=\frac{y-6}{3}=\frac{z-7}{3}$,

lies on the plane x + 2y – z = 8, then $\alpha-\beta$ is equal to :

(1) 5 (2) 9 (3) 3 (4) 7

Official Ans. by NTA (4)

4. Let f : **R** $\rightarrow$ **R** be defined as

f(x + y) + f(x – y) = 2 f(x) f(y), $f\left(\frac{1}{2}\right)=-1$. Then, the value of $\sum_{k=1}^{20}\frac{1}{\sin(k)\sin(k+f(k))}$ is equal to :

(1) $\text{cosec}^2(21)\cos(20)\cos(2)$

(2) $\sec^2(1)\sec(21)\cos(20)$

(3) $\text{cosec}^2(1)\,\text{cosec}(21)\sin(20)$

(4) $\sec^2(21)\sin(20)\sin(2)$

Official Ans. by NTA (3)

5. Let $\mathbb{C}$ be the set of all complex numbers. Let

$S_1=\{z\in\mathbb{C}:|z-2|\le 1\}$ and

$S_2=\{z\in\mathbb{C}:z(1+i)+\bar{z}(1-i)\ge 4\}$.

Then, the maximum value of $\left|z-\frac{5}{2}\right|^2$ for $z\in S_1\cap S_2$ is equal to :

(1) $\frac{3+2\sqrt{2}}{4}$ (2) $\frac{5+2\sqrt{2}}{2}$

(3) $\frac{3+2\sqrt{2}}{2}$ (4) $\frac{5+2\sqrt{2}}{4}$

Official Ans. by NTA (4)

6. A student appeared in an examination consisting of 8 true–false type questions. The student guesses the answers with equal probability. The smallest value of n, so that the probability of guessing at least 'n' correct answers is less than $\frac{1}{2}$, is :

(1) 5 (2) 6 (3) 3 (4) 4

Official Ans. by NTA (1)

7. If $\tan\left(\frac{\pi}{9}\right),x,\tan\left(\frac{7\pi}{18}\right)$ are in arithmetic progression and $\tan\left(\frac{\pi}{9}\right),y,\tan\left(\frac{5\pi}{18}\right)$ are also in arithmetic progression, then |x – 2y| is equal to :

(1) 4 (2) 3 (3) 0 (4) 1

Official Ans. by NTA (3)

8. Let the mean and variance of the frequency distribution

x :	$x_1=2$	$x_2=6$	$x_3=8$	$x_4=9$
f :	4	4	α	β

be 6 and 6.8 respectively. If x_3 is changed from 8 to 7, then the mean for the new data will be:

(1) 4 (2) 5

(3) $\frac{17}{3}$ (4) $\frac{16}{3}$

Official Ans. by NTA (3)

9. The area of the region bounded by $y - x = 2$ and $x^2 = y$ is equal to :-

(1) $\frac{16}{3}$ (2) $\frac{2}{3}$ (3) $\frac{9}{2}$ (4) $\frac{4}{3}$

Official Ans. by NTA (3)

10. Let $y = y(x)$ be the solution of the differential equation $(x - x^3)dy = (y + yx^2 - 3x^4)dx$, $x > 2$. If $y(3) = 3$, then $y(4)$ is equal to :

(1) 4 (2) 12 (3) 8 (4) 16

Official Ans. by NTA (2)

11. The value of $\lim\limits_{x \to 0}\left(\frac{x}{\sqrt[8]{1-\sin x}-\sqrt[8]{1+\sin x}}\right)$ is equal to :

(1) 0 (2) 4 (3) -4 (4) -1

Official Ans. by NTA (3)

12. Two sides of a parallelogram are along the lines $4x + 5y = 0$ and $7x + 2y = 0$. If the equation of one of the diagonals of the parallelogram is $11x + 7y = 9$, then other diagonal passes through the point :

(1) (1,2) (2) (2,2) (3) (2,1) (4) (1,3)

Official Ans. by NTA (2)

13. Let $\alpha = \max\limits_{x \in \mathbf{R}}\{8^{2\sin 3x} \cdot 4^{4\cos 3x}\}$ and $\beta = \min\limits_{x \in \mathbf{R}}\{8^{2\sin 3x} \cdot 4^{4\cos 3x}\}$. If $8x^2 + bx + c = 0$ is a quadratic equation whose roots are $\alpha^{1/5}$ and $\beta^{1/5}$, then the value of $c - b$ is equal to :

(1) 42 (2) 47 (3) 43 (4) 50

Official Ans. by NTA (1)

14. Let $f : [0, \infty) \to [0, 3]$ be a function defined by

$$f(x) = \begin{cases} \max\{\sin t : 0 \le t \le x\}, & 0 \le x \le \pi \\ 2 + \cos x, & x > \pi \end{cases}$$

Then which of the following is true ?

(1) f is continuous everywhere but not differentiable exactly at one point in $(0, \infty)$

(2) f is differentiable everywhere in $(0, \infty)$

(3) f is not continuous exactly at two points in $(0, \infty)$

(4) f is continuous everywhere but not differentiable exactly at two points in $(0, \infty)$

Official Ans. by NTA (2)

15. Let **N** be the set of natural numbers and a relation R on **N** be defined by

$R = \{(x,y) \in \mathbf{N} \times \mathbf{N} : x^3 - 3x^2y - xy^2 + 3y^3 = 0\}$. Then the relation R is :

(1) symmetric but neither reflexive nor transitive

(2) reflexive but neither symmetric nor transitive

(3) reflexive and symmetric, but not transitive

(4) an equivalence relation

Official Ans. by NTA (2)

16. Which of the following is the negation of the statement "for all $M > 0$, there exists $x \in S$ such that $x \ge M$" ?

(1) there exists $M > 0$, such that $x < M$ for all $x \in S$

(2) there exists $M > 0$, there exists $x \in S$ such that $x \ge M$

(3) there exists $M > 0$, there exists $x \in S$ such that $x < M$

(4) there exists $M > 0$, such that $x \ge M$ for all $x \in S$

Official Ans. by NTA (1)

17. Consider a circle C which touches the y-axis at (0, 6) and cuts off an intercept $6\sqrt{5}$ on the x-axis. Then the radius of the circle C is equal to :

(1) $\sqrt{53}$ (2) 9 (3) 8 (4) $\sqrt{82}$

Official Ans. by NTA (2)

18. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a} = \vec{b} \times (\vec{b} \times \vec{c})$. If magnitudes of the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are $\sqrt{2}, 1$ and 2 respectively and the angle between $\vec{b}$ and $\vec{c}$ is θ $\left(0 < \theta < \frac{\pi}{2}\right)$, then the value of $1 + \tan\theta$ is equal to :

(1) $\sqrt{3} + 1$ (2) 2

(3) 1 (4) $\frac{\sqrt{3}+1}{\sqrt{3}}$

Official Ans. by NTA (2)

19. Let A and B be two 3×3 real matrices such that $(A^2 - B^2)$ is invertible matrix. If $A^5 = B^5$ and $A^3B^2 = A^2B^3$, then the value of the determinant of the matrix $A^3 + B^3$ is equal to :

(1) 2 (2) 4

(3) 1 (4) 0

Official Ans. by NTA (4)

20. Let $f : (a,b) \to \mathbf{R}$ be twice differentiable function such that $f(x) = \int_a^x g(t)\,dt$ for a differentiable function g(x). If $f(x) = 0$ has exactly five distinct roots in (a, b), then $g(x)g'(x) = 0$ has at least :

(1) twelve roots in (a, b)

(2) five roots in (a, b)

(3) seven roots in (a, b)

(4) three roots in (a, b)

Official Ans. by NTA (3)

SECTION-B

1. Let $\vec{a} = \hat{i} - \alpha\hat{j} + \beta\hat{k}$, $\vec{b} = 3\hat{i} + \beta\hat{j} - \alpha\hat{k}$ and $\vec{c} = -\alpha\hat{i} - 2\hat{j} + \hat{k}$, where α and β are integers. If $\vec{a} \cdot \vec{b} = -1$ and $\vec{b} \cdot \vec{c} = 10$, then $(\vec{a} \times \vec{b}) \cdot \vec{c}$ is equal to____ .

Official Ans. by NTA (9)

2. The distance of the point P(3, 4, 4) from the point of intersection of the line joining the points. Q(3, –4, –5) and R(2, –3, 1) and the plane 2x + y + z = 7, is equal to_____.

Official Ans. by NTA (7)

3. If the real part of the complex number $z = \dfrac{3 + 2i\cos\theta}{1 - 3i\cos\theta}$, $\theta \in \left(0, \dfrac{\pi}{2}\right)$ is zero, then the value of $\sin^2 3\theta + \cos^2\theta$ is equal to_____.

Official Ans. by NTA (1)

4. Let E be an ellipse whose axes are parallel to the co-ordinates axes, having its center at (3, – 4), one focus at (4, – 4) and one vertex at (5, – 4). If mx – y = 4, m > 0 is a tangent to the ellipse E, then the value of $5m^2$ is equal to_____.

Official Ans. by NTA (3)

5. If $\int_0^{\pi} (\sin^3 x) e^{-\sin^2 x}\,dx = \alpha - \dfrac{\beta}{e}\int_0^1 \sqrt{t}\, e^t dt$, then $\alpha + \beta$ is equal to _____ .

Official Ans. by NTA (5)

6. The number of real roots of the equation $e^{4x} - e^{3x} - 4e^{2x} - e^x + 1 = 0$ is equal to____ .

Official Ans. by NTA (2)

7. Let y= y(x) be the solution of the differential equation $dy = e^{\alpha x+y}\,dx$; $\alpha \in \mathbf{N}$. If $y(\log_e 2) = \log_e 2$ and $y(0) = \log_e\left(\dfrac{1}{2}\right)$, then the value of α is equal to____.

Official Ans. by NTA (2)

8. Let n be a non-negative integer. Then the number of divisors of the form "4n + 1" of the number $(10)^{10}.(11)^{11}.(13)^{13}$ is equal to_____ .

Official Ans. by NTA (924)

9. Let $A = \{n \in \mathbf{N} \mid n^2 \le n + 10{,}000\}$, $B = \{3k + 1 \mid k \in \mathbf{N}\}$ and $C = \{2k \mid k \in N\}$, then the sum of all the elements of the set $A \cap (B - C)$ is equal to_____ .

Official Ans. by NTA (832)

10. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and $M = A + A^2 + A^3 + \ldots + A^{20}$, then the sum of all the elements of the matrix M is equal to_____.

Official Ans. by NTA (2020)