

FINAL JEE-MAIN EXAMINATION – JUNE, 2022

(Held On Sunday 26th June, 2022)

TIME : 03 : 00 PM to 06 : 00 PM

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = x-1$ and $g : \mathbb{R} - \{1, -1\} \rightarrow \mathbb{R}$ be defined as $g(x) = \frac{x^2}{x^2-1}$.

Then the function fog is :

- (A) one-one but not onto function
- (B) onto but not one-one function
- (C) both one-one and onto function
- (D) neither one-one nor onto function

Official Ans. by NTA (D)

Ans. (D)

Sol. $f(x) = x - 1 ; g(x) = \frac{x^2}{x^2-1}$

$f(g(x)) = g(x) - 1$
 $= \frac{x^2}{x^2-1} - 1 = \frac{x^2 - x^2 + 1}{x^2-1}$

$f(g(x)) = \frac{1}{x^2-1}; x \neq \pm 1$, even function
 \rightarrow Hence $f(g(x))$ is many one function

$y = \frac{1}{x^2-1}$
 $y \cdot x^2 - y = 1$
 $x^2 = \left(\frac{1+y}{y}\right)$

$\left(\frac{1+y}{y}\right) \geq 0$



Range:- $y \in (-\infty, -1] \cup (0, \infty)$

Hence, Range \neq Co-domain $\Rightarrow f(g(x))$ is into function

2. If the system of equations $\alpha x + y + z = 5, x + 2y + 3z = 4, x + 3y + 5z = \beta$, has infinitely many solutions, then the ordered pair (α, β) is equal to :
- (A) (1,-3) (B) (-1, 3)
 - (C) (1, 3) (D) (-1, -3)

Official Ans. by NTA (C)

Ans. (C)

For infinitely many solutions,

$\Delta = 0 = \Delta_x = \Delta_y = \Delta_z$

Sol. $\Delta = \begin{vmatrix} \alpha & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{vmatrix} = 0$

$\Rightarrow \alpha(10-9) - 1(5-3) + 1(3-2) = 0$

$\Rightarrow \alpha - 2 + 1 = 0$

$\Rightarrow \alpha = 1$

$\Delta_x = \begin{vmatrix} 5 & 1 & 1 \\ 4 & 2 & 3 \\ \beta & 3 & 5 \end{vmatrix} = 0$

$\Rightarrow 5(10-9) - 1(20-3\beta) + 1(12-2\beta)$

$\Rightarrow 5 - 20 + 3\beta + 12 - 2\beta$

$\Rightarrow -3 + \beta = 0$

$\Rightarrow \beta = 3$

3. If $A = \sum_{n=1}^{\infty} \frac{1}{(3+(-1)^n)^n}$ and $B = \sum_{n=1}^{\infty} \frac{(-1)^n}{(3+(-1)^n)^n}$, then

$\frac{A}{B}$ is equal to :

- (A) $\frac{11}{9}$ (B) 1 (C) $-\frac{11}{9}$ (D) $-\frac{11}{3}$

Official Ans. by NTA (C)

Ans. (C)

Sol.

$A = \left(\frac{1}{2} + \frac{1}{4^2} + \frac{1}{2^3} + \frac{1}{4^4} + \dots \infty \right)$

$A = \left(\frac{1}{2} + \frac{1}{2^3} + \dots \infty \right) + \left(\frac{1}{4^2} + \frac{1}{4^4} + \dots \infty \right)$

$A = \left(\frac{1}{2} + \frac{1}{16} \right) + \left(\frac{1}{4} + \frac{1}{16} \right)$

$\Rightarrow A = \frac{1}{2} \times \frac{4}{3} + \frac{1}{16} \times \frac{16}{15} \Rightarrow A = \frac{11}{15}$

$B = \frac{-1}{2} + \frac{1}{4^2} + \frac{-1}{2^3} + \frac{1}{4^4} + \dots \infty$

$B = \left(\frac{-1}{2} + \frac{-1}{2^3} + \dots \infty \right) + \left(\frac{1}{4^2} + \frac{1}{4^4} + \dots \infty \right)$

$$B = \frac{-\frac{1}{2}}{1 - \frac{1}{4}} + \frac{\frac{1}{16}}{1 - \frac{1}{16}}$$

$$\Rightarrow B = -\frac{1}{2} \times \frac{4}{3} + \frac{1}{16} \times \frac{16}{15}$$

$$B = -\frac{9}{15}$$

$$\frac{A}{B} = \frac{11}{15} \times \frac{15}{(-9)}$$

$$\frac{A}{B} = -\frac{11}{9}$$

4. $\lim_{x \rightarrow 0} \frac{\cos(\sin x) - \cos x}{x^4}$ is equal to :
- (A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{6}$ (D) $\frac{1}{12}$

Official Ans. by NTA (C)

Ans. (C)

Sol. $\lim_{x \rightarrow 0} \frac{\cos(\sin x) - \cos x}{x^4}; \left(\frac{0}{0}\right)$

$$\lim_{x \rightarrow 0} \left(\frac{2 \cdot \sin\left(\frac{x + \sin x}{2}\right) \cdot \sin\left(\frac{x - \sin x}{2}\right)}{x^4} \right)$$

$$\lim_{x \rightarrow 0} 2 \left(\frac{\sin\left(\frac{x + \sin x}{2}\right)}{\left(\frac{x + \sin x}{2}\right)} \right) \left(\frac{\sin\left(\frac{x - \sin x}{2}\right)}{\left(\frac{x - \sin x}{2}\right)} \right) \left(\frac{x + \sin x}{x^4} \right) \left(\frac{x - \sin x}{2} \right)$$

$$\lim_{x \rightarrow 0} \left(\frac{x^2 - \sin^2 x}{2x^4} \right); \left(\frac{0}{0}\right)$$

Apply L-Hopital Rule :

$$\lim_{x \rightarrow 0} \frac{2x - 2 \sin x \cos x}{2 \cdot 4x^3}$$

$$\lim_{x \rightarrow 0} \frac{2x - \sin 2x}{8x^3}; \frac{0}{0} : \text{Again apply L-Hopital rule}$$

$$\lim_{x \rightarrow 0} \frac{2 - 2 \cos(2x)}{8(3)x^2}$$

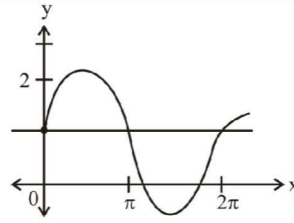
$$\lim_{x \rightarrow 0} \frac{2(1 - \cos(2x))}{24(4x^2)} \times 4 \Rightarrow \frac{2}{24} \times \frac{1}{2} \times 4 \Rightarrow \frac{1}{6}$$

5. Let $f(x) = \min \{1, 1 + x \sin x\}$, $0 \leq x \leq 2\pi$. If m is the number of points, where f is not differentiable and n is the number of points, where f is not continuous, then the ordered pair (m, n) is equal to
- (A) (2, 0) (B) (1, 0)
 (C) (1, 1) (D) (2, 1)

Official Ans. by NTA (B)

Ans. (B)

Sol.



No. of non-differentiable points = 1 (m)

No. of not continuous points = 0 (n)

$(m, n) = (1, 0)$

6. Consider a cuboid of sides $2x$, $4x$ and $5x$ and a closed hemisphere of radius r . If the sum of their surface areas is a constant k , then the ratio $x : r$, for which the sum of their volumes is maximum, is :
- (A) 2 : 5 (B) 19:45 (C) 3 : 8 (D) 19 : 15

Official Ans. by NTA (B)

Ans. (B)

Sol. Surface area = $76x^2 + 3\pi r^2 = \text{constant (K)}$

$$V = 40x^3 + \frac{2}{3}\pi r^3$$

$$[76x^2 + 3\pi r^2 = K]$$

$$r^2 = \frac{K - 76x^2}{3\pi}$$

$$r = \left(\frac{K - 76x^2}{3\pi} \right)^{\frac{1}{2}}$$

$$V = 40x^3 + \frac{2}{3}\pi \left(\frac{K - 76x^2}{3\pi} \right)^{\frac{3}{2}}$$

$$\frac{dV}{dx} = 120x^2 + \frac{2}{3}\pi \cdot \frac{3}{2} \left(\frac{K - 76x^2}{3\pi} \right)^{\frac{1}{2}} \cdot \left(\frac{-76(2x)}{3\pi} \right)$$

Put

$$\frac{dV}{dx} = 0 \Rightarrow 120x^2 + \frac{2}{3}\pi \cdot \frac{3}{2} \left(\frac{K - 76x^2}{3\pi} \right)^{\frac{1}{2}} \cdot \left(\frac{-76(2x)}{3\pi} \right) = 0$$

$$\Rightarrow 120x^2 = \frac{152x}{3} \left(\frac{k - 76x^2}{3\pi} \right)^{\frac{1}{2}}$$

$$\Rightarrow \frac{45}{19}x^2 = x \left(\frac{k - 76x^2}{3\pi} \right)^{\frac{1}{2}}; x \neq 0$$

$$\Rightarrow \frac{45}{19}x = \left(\frac{k - 76x^2}{3\pi} \right)^{\frac{1}{2}} \Rightarrow \left(\frac{45}{19} \right)^2 x^2 = \frac{k - 76x^2}{3\pi}$$

$$\Rightarrow \left(\frac{45}{19} \right)^2 x^2 = r^2 \Rightarrow \frac{x^2}{r^2} = \left(\frac{19}{45} \right)^2$$

$$\Rightarrow \frac{x}{r} = \frac{19}{45}$$

7. The area of the region bounded by $y^2 = 8x$ and $y^2 = 16(3-x)$ is equal to :-

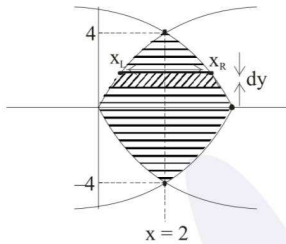
- (A) $\frac{32}{3}$ (B) $\frac{40}{3}$ (C) 16 (D) 19

Official Ans. by NTA (C)

Ans. (C)

Sol. $y^2 = 8x$; $y^2 = 16(3-x)$

$$y^2 = -16(x-3)$$



finding their intersection pts.

$$y^2 = 8x \text{ \& } y^2 = -16(x-3)$$

$$8x = -16x + 48$$

$$24x = 48$$

$$x = 2; y = \pm 4$$

$$A = 2 \cdot \int_0^4 (x_R - x_L) dy$$

Required Area

$$= 2 \cdot \int_0^4 \left(3 - \frac{y^2}{8} - \frac{y^2}{16} \right) dy$$

$$= 2 \left(3y - \frac{y^3}{3 \times 16} - \frac{y^3}{3 \times 8} \right)_0^4$$

$$= 2 \left(3 \times 4 - \frac{4 \times 4 \times 4}{3 \times 16} - \frac{4 \times 4 \times 4 \times 2}{3 \times 8 \times 2} \right)$$

$$= 2 \left(12 - \frac{4}{3} - \frac{8}{3} \right) = 2 \times 12 \left(1 - \frac{1}{3} \right) = 2 \times 12 \times \frac{2}{3} = 16$$

8. If $\int \frac{1}{x} \sqrt{\frac{1-x}{1+x}} dx = g(x) + c, g(1) = 0$, then $g\left(\frac{1}{2}\right)$ is equal

to :

(A) $\log_e \left(\frac{\sqrt{3}-1}{\sqrt{3}+1} \right) + \frac{\pi}{3}$ (B) $\log_e \left(\frac{\sqrt{3}+1}{\sqrt{3}-1} \right) + \frac{\pi}{3}$

(C) $\log_e \left(\frac{\sqrt{3}+1}{\sqrt{3}-1} \right) - \frac{\pi}{3}$ (D) $\frac{1}{2} \log_e \left(\frac{\sqrt{3}-1}{\sqrt{3}+1} \right) - \frac{\pi}{6}$

Official Ans. by NTA (A)

Ans. (A)

Sol. $\int \frac{1}{x} \sqrt{\frac{1-x}{1+x}} dx = g(x) + c$

Put $x = \cos 2\theta$

$$dx = -2\sin 2\theta \cdot d\theta$$

$$= \int \frac{1}{\cos 2\theta} \tan \theta (-4\sin \theta \cdot \cos \theta) d\theta$$

$$= \int \frac{1}{\cos 2\theta} (-4\sin^2 \theta) d\theta$$

$$= -2 \int \frac{1 - \cos 2\theta}{\cos 2\theta} d\theta$$

$$= -\frac{2}{2} \ln |\sec 2\theta + \tan 2\theta| + 2\theta + c$$

$$= \ln |\sec 2\theta - \tan 2\theta| + 2\theta + c$$

$$= \ln \left| \frac{1 - \sin 2\theta}{\cos 2\theta} \right| + \cos^{-1} x + c$$

$$= \ln \left| \frac{1 - \sqrt{1-x^2}}{x} \right| + \cos^{-1} x + c$$

$$\therefore g(1) = 0$$

$$g(x) = \ln \left| \frac{1 - \sqrt{1-x^2}}{x} \right| + \cos^{-1} x$$

$$g\left(\frac{1}{2}\right) = \ln |2 - \sqrt{3}| + \frac{\pi}{3}$$

$$g\left(\frac{1}{2}\right) = \ln \left| \frac{\sqrt{3}-1}{\sqrt{3}+1} \right| + \frac{\pi}{3}$$

9. If $y = y(x)$ is the solution of the differential

equation $x \frac{dy}{dx} + 2y = xe^x, y(1) = 0$ then the local

maximum value of the function $z(x) = x^2 y(x) - e^x, x \in \mathbb{R}$ is :

- (A) $1 - e$ (B) 0 (C) $\frac{1}{2}$ (D) $\frac{4}{e} - e$

Official Ans. by NTA (D)

Ans. (D)

Sol. $x \frac{dy}{dx} + 2y = xe^x$

$$\frac{dy}{dx} + \frac{2y}{x} = e^x$$

$$\text{I.F.} = x^2$$

$$y \cdot x^2 = \int x^2 e^x dx$$

$$= \int e^x (x^2 + 2x - 2x - 2 + 2) dx$$

$$yx^2 = e^x (x^2 - 2x + 2) + c$$

$$y(1) = 0$$

$$0 = e(1 + 0) + c$$

$$c = -e$$

$$z(x) = x^2 y(x) - e^x$$

$$= e^x (x^2 - 2x + 2) - e - e^x$$

$$= e^x (x - 1)^2 - e$$

$$\frac{dz}{dx} = e^x \cdot 2(x - 1) + e^x (x - 1)^2 = 0$$

$$x^x (x - 1) (2 + x - 1) = 0$$

$$e^x (x - 1) (x + 1) = 0$$

$$x = -1, 1$$



$x = -1$ local maxima. Then maximum value is

$$z(-1) = \frac{4}{e} - e$$

10. If the solution of the differential equation

$$\frac{dy}{dx} + e^x (x^2 - 2)y = (x^2 - 2x)(x^2 - 2)e^{2x} \text{ satisfies}$$

$y(0) = 0$, then the value of $y(2)$ is _____ .

- (A) -1 (B) 1 (C) 0 (D) e

Official Ans. by NTA (C)

Ans. (C)

Sol. $I.F. = e^{\int e^x (x^2 - 2) dx} = e^{\int e^x (x^2 - 2x + 2x - 2) dx}$
 $= e^{e^x (x^2 - 2x)}$

$$y \cdot e^{e^x (x^2 - 2x)} = \int e^{e^x (x^2 - 2x)} e^x (x^2 - 2x)(x^2 - 2) e^x dx$$

Let $e^x (x^2 - 2x) = t$

So, $y \cdot e^{e^x (x^2 - 2x)} = \int e^t \cdot t dt$

At $x = 0, t = 0$

$$x = 2, t = 0$$

$$= t \cdot e^t - e^t + c$$

$$x = 0; 0 \cdot 1 = 0 - 1 + c \Rightarrow c = 1$$

$$\text{for } x = 2; y \cdot 1 = 0 - 1 + 1 = 0$$

$$y(2) = 0$$

11. If m is the slope of a common tangent to the curves

$$\frac{x^2}{16} + \frac{y^2}{9} = 1 \text{ and } x^2 + y^2 = 12, \text{ then } 12m^2 \text{ is equal to :}$$

- (A) 6 (B) 9
(C) 10 (D) 12

Official Ans. by NTA (B)

Ans. (B)

Sol. $\frac{x^2}{16} + \frac{y^2}{9} = 1$

equation of tangent to the ellipse is

$$y = mx \pm \sqrt{a^2 m^2 + b^2}$$

$$y = mx \pm \sqrt{16m^2 + 9} \quad \dots(i)$$

$$x^2 + y^2 = 12$$

equation of tangent to the circle is

$$y = mx \pm \sqrt{12} \sqrt{1 + m^2} \quad \dots(ii)$$

for common tangent equate eq. (i) and (ii)

$$\Rightarrow 16m^2 + 9 = 12(1 + m^2)$$

$$16m^2 - 12m^2 = 3$$

$$4m^2 = 3$$

$$12m^2 = 9$$

12. The locus of the mid point of the line segment joining the point (4, 3) and the points on the ellipse $x^2 + 2y^2 = 4$ is an ellipse with eccentricity :

- (A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2\sqrt{2}}$
(C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{2}$

Official Ans. by NTA (C)

Ans. (C)

Sol. $\frac{x^2}{4} + \frac{y^2}{2} = 1$



Coordinate of D is

$$\left(\frac{2 \cos \theta + 4}{2}, \frac{\sqrt{2} \sin \theta + 3}{2} \right) \equiv (h, k)$$

$$\frac{2h - 4}{2} = \cos \theta \quad \dots(i)$$

$$\frac{2k - 3}{\sqrt{2}} = \sin \theta \quad \dots(ii)$$

(i)² + (ii)², then we get

$$\left(\frac{2h-4}{2}\right)^2 + \left(\frac{2k-3}{\sqrt{2}}\right)^2 = 1 \Rightarrow \frac{(x-2)^2}{1} + \frac{\left(y-\frac{3}{2}\right)^2}{\left(\frac{1}{2}\right)} = 1$$

∴ Required eccentricity is

$$e = \sqrt{1 - \frac{1}{2}} = \frac{1}{\sqrt{2}}$$

13. The normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{9} = 1$ at the point $(8, 3\sqrt{3})$ on it passes through the point :

- (A) $(15, -2\sqrt{3})$ (B) $(9, 2\sqrt{3})$
 (C) $(-1, 9\sqrt{3})$ (D) $(-1, 6\sqrt{3})$

Official Ans. by NTA (C)

Ans. (C)

Sol. $\frac{x^2}{a^2} - \frac{y^2}{9} = 1$: $(8, 3\sqrt{3})$ lie on Hyperbola then

$$\frac{64}{a^2} - \frac{27}{9} = 1 \Rightarrow a^2 = \frac{64}{4} = 16$$

equation of normal at $(8, 3\sqrt{3})$:

$$\frac{16x}{8} + \frac{9y}{3\sqrt{3}} = 16 + 9$$

$$2x + \sqrt{3}y = 25$$

Check options.

14. If the plane $2x + y - 5z = 0$ is rotated about its line of intersection with the plane $3x - y + 4z - 7 = 0$ by an angle of $\frac{\pi}{2}$, then the plane after the rotation passes through the point :

- (A) $(2, -2, 0)$ (B) $(-2, 2, 0)$
 (C) $(1, 0, 2)$ (D) $(-1, 0, -2)$

Official Ans. by NTA (C)

Ans. (C)

Sol. $(2x + y - 5z) + \lambda(3x - y + 4z - 7) = 0$

Rotated by $\pi/2$

$$(2 + 3\lambda)x + (1 - \lambda)y + (-5 + 4\lambda)z - 7\lambda = 0$$

$$2x + y - 5z = 0$$

$$2(2 + 3\lambda) + (1 - \lambda) - 5(-5 + 4\lambda) = 0$$

$$\Rightarrow 4 + 6\lambda + 1 - \lambda + 25 - 20\lambda = 0$$

$$30 = 15\lambda$$

$$\lambda = 2$$

$$\text{Required plane :- } 8x - y + 3z - 14 = 0$$

Check options

15. If the lines $\vec{r} = (\hat{i} - \hat{j} + \hat{k}) + \lambda(3\hat{j} - \hat{k})$ and $\vec{r} = (\alpha\hat{i} - \hat{j}) + \mu(2\hat{i} - 3\hat{k})$ are co-planar, then distance of the plane containing these two lines from the point $(\bullet, 0, 0)$ is :

- (A) $\frac{2}{9}$ (B) $\frac{2}{11}$
 (C) $\frac{4}{11}$ (D) 2

Official Ans. by NTA (B)

Allen Ans. (B)

Sol. $\vec{r} = (\hat{i} - \hat{j} + \hat{k}) + \lambda(3\hat{j} - \hat{k})$ L1

$\vec{r} = (\alpha\hat{i} - \hat{j}) + \mu(2\hat{i} - 3\hat{k})$ L2

• L1 and L2 are coplanar

$$\therefore \begin{vmatrix} 0 & 3 & -1 \\ 2 & 0 & -3 \\ (1-\alpha) & 0 & 1 \end{vmatrix} = 0$$

$$-3(2 + 3(1 - \bullet)) = 0$$

$$2 + 3 - 3\bullet = 0$$

$$\bullet \cdot 3\bullet = 5$$

$$\Rightarrow \alpha = \frac{5}{3}$$

Now,

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & -1 \\ 2 & 0 & -3 \end{vmatrix} = \hat{i}(-9) - \hat{j}(2) + \hat{k}(-6)$$

$$= (9, 2, 6)$$

Equation of plane :

$$9(x - 1) + 2(y + 1) + 6(z - 1) = 0$$

$$9x + 2y + 6z - 13 = 0$$

Perpendicular distance from $(\bullet, 0, 0)$

$$= \frac{\left| \left(9 \cdot \frac{5}{3} + 0 + 0 - 13 \right) \right|}{\sqrt{81 + 36 + 4}} = \frac{2}{\sqrt{121}} = \frac{2}{11}$$

16. Let $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + \hat{k}$ be three given vectors. Let \vec{v} be a vector in the plane of \vec{a} and \vec{b} whose projection on \vec{c} is $\frac{2}{\sqrt{3}}$. If $\vec{v} \cdot \hat{j} = 7$,

then $\vec{v} \cdot (\hat{i} + \hat{k})$ is equal to :

- (A) 6 (B) 7 (C) 8 (D) 9

Official Ans. by NTA (D)

Ans. (D)

Sol. $\vec{v} = \lambda\vec{a} + \mu\vec{b}$

$$\vec{v} = \lambda(1, 1, 2) + \mu(2, -3, 1)$$

$$\vec{v} = (\lambda + 2\mu, \lambda - 3\mu, 2\lambda + \mu)$$

$$\vec{v} \cdot \hat{j} = 7 \quad \vec{v} \cdot \frac{\vec{c}}{|\vec{c}|} = \frac{2}{\sqrt{3}}$$

$$\lambda - 3\mu = 7 \quad \vec{v} \cdot \vec{c} = 2$$

$$\lambda + 2\mu - \lambda + 3\mu + 2\lambda + \mu = 2$$

$$2\lambda + 6\mu = 2$$

$$\lambda + 3\mu = 1$$

$$\lambda - 3\mu = 7$$

$$2\lambda = 8$$

$$\lambda = 4$$

$$\mu = -1$$

We get $\vec{v} = (2, 7, 7)$

17. The mean and standard deviation of 50 observations are 15 and 2 respectively. It was found that one incorrect observation was taken such that the sum of correct and incorrect observations is 70. If the correct mean is 16, then the correct variance is equal to :

- (A) 10 (B) 36 (C) 43 (D) 60

Official Ans. by NTA (C)

Ans. (C)

Sol. No. of observations: - 50

$$\text{mean}(\bar{x}) = 15$$

$$\text{Standard deviation} (\sigma) = 2$$

Let incorrect observation is x_1 & correct observation is (x')

$$\text{Given } x_1 + x' = 70$$

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_{50}}{50} = 15 \text{ (given)}$$

$$\Rightarrow x_1 + x_2 + \dots + x_{50} = 750 \quad \dots \text{(i)}$$

Now

Mean of correct observation is 16

$$\frac{x'_1 + x'_2 + \dots + x'_{50}}{50} = 16$$

$$x'_1 + x'_2 + x'_3 + \dots + x'_{50} = 16 \times 50 \quad \dots \text{(ii)}$$

eq. (ii) - eq. (i)

$$\Rightarrow x'_1 - x_1 = 16 \times 50 - 15 \times 50$$

$$x'_1 - x_1 = 50 \text{ \& } x_1 + x'_1 = 70$$

$$x'_1 = 60$$

$$x_1 = 10$$

$$\Rightarrow 4 = \frac{x_1^2 + x_2^2 + \dots + x_{50}^2}{50} - 15^2 \quad \dots \text{(iii)}$$

$$\Rightarrow \sigma^2 = \frac{x_1'^2 + x_2'^2 + \dots + x_{50}'^2}{50} - 16^2 \quad \dots \text{(iv)}$$

from (iii)

$$\Rightarrow 4 = \frac{(10)^2}{50} + \frac{x_2^2 + x_3^2 + \dots + x_{50}^2}{50} - 225$$

$$\Rightarrow 4 = 2 - 225 + \frac{(x_2^2 + x_3^2 + \dots + x_{50}^2)}{50}$$

$$\Rightarrow 227 = \frac{(x_2^2 + x_3^2 + \dots + x_{50}^2)}{50}$$

From (iv)

$$\sigma^2 = \frac{(60)^2}{50} + \left(\frac{x_2^2 + x_3^2 + \dots + x_{50}^2}{50} \right) - (16)^2$$

$$\sigma^2 = \frac{60 \times 60}{50} + 227 - 256$$

$$\sigma^2 = 72 + 227 - 256$$

$$\sigma^2 = 43$$

18. $16\sin(20^\circ) \sin(40^\circ) \sin(80^\circ)$ is equal to :

- (A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) 3 (D) $4\sqrt{3}$

Official Ans. by NTA (B)

Ans. (B)

Sol. $16 \sin 20^\circ \sin 40^\circ \sin 80^\circ$

$$= 16 \sin 40^\circ \sin 20^\circ \sin 80^\circ$$

$$= 4(4 \sin(60 - 20) \sin(20) \sin(60 + 20))$$

$$= 4 \times \sin(3 \times 20^\circ)$$

$$[\because \sin 3\theta = 4 \sin(60 - \theta) \times \sin \theta \times \sin(60 + \theta)]$$

$$= 4 \times \sin 60^\circ$$

$$= 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$$

19. If the inverse trigonometric functions take principal values, then

$$\cos^{-1}\left(\frac{3}{10}\cos\left(\tan^{-1}\left(\frac{4}{3}\right)\right)+\frac{2}{5}\sin\left(\tan^{-1}\left(\frac{4}{3}\right)\right)\right)$$

is equal to :

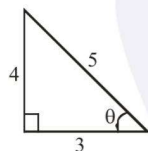
- (A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

Official Ans. by NTA (C)

Ans. (C)

Sol. Let

$$\tan^{-1}\frac{4}{3} = \theta \Rightarrow \tan \theta = \frac{4}{3}$$



$$E = \cos^{-1}\left(\frac{3}{10}\cos\theta + \frac{2}{5}\sin\theta\right)$$

$$= \cos^{-1}\left(\frac{3}{10} \times \frac{3}{5} + \frac{2}{5} \cdot \frac{4}{5}\right)$$

$$= \cos^{-1}\left(\frac{9}{50} + \frac{8}{25}\right) = \cos^{-1}\left(\frac{25}{50}\right) = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

20. Let $r \in \{p, q, \sim p, \sim q\}$ be such that the logical statement $r \vee (\sim p) \Rightarrow (p \wedge q) \vee r$ is a tautology.

Then 'r' is equal to :

- (A) p (B) q (C) $\sim p$ (D) $\sim q$

Official Ans. by NTA (C)

Ans. (C)

Sol. By options

(1)

$p=r$	q	$\sim p$	$r \vee (\sim p)$	$(p \wedge q)$	$(p \wedge q) \vee r$	$r \vee (\sim p) \Rightarrow (p \wedge q) \vee r$
T	F	F	T	F	T	T
F	T	T	T	F	F	F
T	T	F	T	T	T	T
F	F	T	T	F	F	T

(2)

p	$\sim p$	$r \vee (\sim p)$	$q=r$	$(p \wedge q)$	$(p \wedge q) \vee r$	$r \vee (\sim p) \Rightarrow (p \wedge q) \vee r$
T	F	T	T	T	T	T
F	T	T	T	F	T	T
T	F	F	F	F	F	T
F	T	T	F	F	F	F

(3)

p	q	$r = \sim p$	$r \vee (\sim p)$	$(p \wedge q)$	$(p \wedge q) \vee r$	$r \vee (\sim p) \Rightarrow (p \wedge q) \vee r$
T	T	F	F	T	T	T
F	T	T	T	F	T	T
T	F	F	F	F	F	T
F	F	T	T	F	T	T

(4)

$\sim p$	p	q	$r \vee (\sim p)$	$r = \sim q$	$(p \wedge q)$	$(p \wedge q) \vee r$	$r \vee (\sim p) \Rightarrow (p \wedge q) \vee r$
F	T	T	F	F	T	T	T
F	T	F	T	T	F	T	T
T	F	T	T	F	F	F	F
T	F	F	T	T	F	T	T

Now final answer is option no. 3.

SECTION-B

1. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfy $f(x + y) = 2^x f(y) + 4^y f(x), \forall x, y \in \mathbb{R}$. If $f(2) = 3$, then $14 \cdot \frac{f'(4)}{f'(2)}$ is equal to _____.

Official Ans. by NTA (248)

Ans. (248)

Sol. Put $y = 2$

$$f(x + 2) = 2^x f(2) + 4^2 f(x)$$

$$f(x + 2) = 2^x \cdot 3 + 16f(x)$$

$$f'(x + 2) = 16f'(x) + 3 \cdot 2^x \ln 2$$

$$f'(4) = 16f'(2) + 12 \ln 2 \quad \dots(i)$$

$$f(y + 2) = 4f(y) + 3 \cdot 4^y$$

$$f'(4) = 4f''(y) + 3 \cdot 4^2 \ln 4$$

$$f'(4) = 4f''(2) + 96 \ln 2 \quad \dots(ii)$$

solving eq. (i) and (ii), we get

$$f''(2) = 7 \ln 2$$

from equation (i), we get

$$f'(4) = 124 \ln 2$$

$$\text{Now, } \Rightarrow 14 \cdot \frac{f'(4)}{f''(2)}$$

$$14 \times \frac{124 \ln 2}{7 \ln 2}$$

$$= 248.$$

2. Let p and q be two real numbers such that $p + q =$

$$3 \text{ and } p^4 + q^4 = 369. \text{ Then } \left(\frac{1}{p} + \frac{1}{q}\right)^{-2} \text{ is equal to}$$

_____.

Official Ans. by NTA (4)

Ans. (4)

$$\text{Sol. } p + q = 3 \quad p^4 + q^4 = 369$$

$$\left(\frac{1}{p} + \frac{1}{q}\right)^{-2}$$

$$(p + q)^2 = 9$$

$$p^2 + q^2 = 9 - 2pq$$

$$\frac{1}{\left(\frac{1}{p} + \frac{1}{q}\right)^2} = \frac{(qp)^2}{(q + p)^2} = \frac{(qp)^2}{9}$$

$$p^4 + q^4 = (p^2 + q^2)^2 - 2p^2q^2$$

$$369 = (9 - 2pq)^2 - 2(pq)^2$$

$$369 = 81 + 4p^2q^2 - 36pq - 2p^2q^2$$

$$288 = 2p^2q^2 - 36pq$$

$$144 = p^2q^2 - 18pq$$

$$(pq)^2 - 2 \times 9 \times pq + 9^2 = 144 + 9^2$$

$$(pq - 9)^2 = 225$$

$$pq - 9 = \pm 15$$

$$pq = \pm 15 + 9$$

$$pq = 24, -6$$

(24 is rejected because $p^2 + q^2 = 9 - 2pq$ is negative)

$$\frac{(qp)^2}{9} = \frac{16 \times 16}{9} = 4$$

3. If $z^2 + z + 1 = 0$, $z \in \mathbb{C}$, then $\left| \sum_{n=1}^{15} \left(z^n + (-1)^n \frac{1}{z^n} \right) \right|^2$ is equal to _____.

Official Ans. by NTA (2)

Ans. (2)

$$\text{Sol. } z^2 + z + 1 = 0 \Rightarrow z = w, w^2$$

$$\left| \sum_{n=1}^{15} \left(z^n + (-1)^n \frac{1}{z^n} \right) \right|^2 = \left| \sum_{n=1}^{15} \left(z^{2n} + \frac{1}{z^{2n}} + 2(-1)^n \right) \right|^2$$

$$= \left| \sum_{n=1}^{15} w^{2n} + \frac{1}{w^{2n}} + 2(-1)^n \right|^2$$

$$= \left| \frac{w^2(1-w^{30})}{1-w^2} + \frac{1}{w^2} \left(1 - \frac{1}{w^{30}} \right) + 2(-1) \right|^2$$

$$= \left| \frac{w^2(1-1)}{1-w^2} + \frac{1}{w^2} \frac{(1-1)}{1-\frac{1}{w^2}} - 2 \right|^2$$

$$= |0 + 0 - 2| = 2$$

4. Let $X = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, $Y = \alpha I + \beta X + \gamma X^2$ and

$$Z = \alpha^2 I - \alpha \beta X + (\beta^2 - \alpha \gamma) X^2, \alpha, \beta, \gamma \in \mathbb{R}. \text{ If } Y^{-1} =$$

$$\begin{bmatrix} \frac{1}{5} & \frac{-2}{5} & \frac{1}{5} \\ 0 & \frac{1}{5} & \frac{-2}{5} \\ 0 & 0 & \frac{1}{5} \end{bmatrix}, \text{ then } (\alpha - \beta + \gamma)^2 \text{ is equal to } \underline{\hspace{2cm}}.$$

Official Ans. by NTA (100)

Ans. (100)

$$\text{Sol. } X = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, X^2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$Y = \begin{bmatrix} \alpha & \beta & \gamma \\ 0 & \alpha & \beta \\ 0 & 0 & \alpha \end{bmatrix}, Z = \begin{bmatrix} \alpha^2 & -\alpha\beta & \beta^2 - \alpha\gamma \\ 0 & \alpha^2 & -\alpha\beta \\ 0 & 0 & \alpha^2 \end{bmatrix}$$

$$Y \cdot Y^{-1} = I$$

$$\begin{bmatrix} \alpha & \beta & \gamma \\ 0 & \alpha & \beta \\ 0 & 0 & \alpha \end{bmatrix} \begin{bmatrix} \frac{1}{5} & \frac{-2}{5} & \frac{1}{5} \\ 0 & \frac{1}{5} & \frac{-2}{5} \\ 0 & 0 & \frac{1}{5} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\frac{\alpha}{5} = 1 \Rightarrow \alpha = 5$$

$$-\frac{2}{5}\alpha + \frac{\beta}{5} = 0 \Rightarrow \beta = 10$$

$$\frac{\alpha}{5} - \frac{2\beta}{5} + \frac{\gamma}{5} = 0 \Rightarrow \gamma = 15$$

$$\Rightarrow (\alpha - \beta + \gamma)^2 = (5 - 10 + 15)^2 = 100$$

5. The total number of 3-digit numbers, whose greatest common divisor with 36 is 2, is _____ .

Official Ans. by NTA (150)

Ans. (150)

Sol. $36 = 2 \times 2 \times 3 \times 3$

Number should be odd multiple of 2 and does not having factor 3 and 9

Odd multiple of 2 are

102, 106, 110, 114 998 (225 no.)

No. of multiples of 3 are

102, 114, 126 990 (75 no.)

Which are also included multiple of 9

Hence,

Required = $225 - 75 = 150$

6. If $({}^{40}C_0) + ({}^{41}C_1) + ({}^{42}C_2) + \dots + ({}^{60}C_{20}) = \frac{m}{n} {}^{60}C_{20}$, m and n are coprime, then m + n is equal to _____ .

Official Ans. by NTA (102)

Ans. (102)

Sol. ${}^{40}C_0 + {}^{41}C_1 + {}^{42}C_2 + \dots + {}^{59}C_{19} + {}^{60}C_{20}$

$$\left(\frac{1}{41} + 1\right)^{41} C_1 + {}^{42}C_2 + \dots$$

$$\left[\frac{42}{41} \left(\frac{2}{42}\right) + 1\right] {}^{42}C_2 + {}^{43}C_3 + \dots$$

$$\left(\frac{2}{41} + 1\right)^{42} C_2 + {}^{43}C_3 + \dots$$

$$\left(\frac{43}{41} \times \frac{3}{43} + 1\right)^{43} C_3 + {}^{44}C_4 + \dots$$

$$\frac{3+41}{41} \cdot {}^{43}C_3 + \dots$$

Similarly :

$$\frac{20+41}{41}$$

$$\Rightarrow m = 61 ; n = 41$$

$$m + n = 102$$

7. If $a_1 (> 0)$, a_2, a_3, a_4, a_5 are in a G.P., $a_2 + a_4 = 2a_3 + 1$ and $3a_2 + a_3 = 2a_4$, then $a_2 + a_4 + 2a_5$ is equal to _____ .

Official Ans. by NTA (40)

Ans. (40)

Sol. $a_1 > 0$, $a_2, a_3, a_4, a_5 \rightarrow$ G.P.

$$3a_2 + a_3 = 2a_4$$

$$3ar + ar^2 = 2ar^3$$

$$3 + r = 2r^2$$

$$2r^2 - r - 3 = 0$$

$$r = -1 \text{ \& } r = \frac{3}{2}$$

$$a_2 + a_4 = 2a_3 + 1$$

$$ar + ar^3 = 2ar^2 + 1$$

$$a(r + r^3 - 2r^2) = 1$$

$$a\left(\frac{3}{2} + \frac{27}{8} - \frac{18}{4}\right) = 1$$

$$a = \frac{8}{3}$$

When $r = -1$, $a = -\frac{1}{4}$ (rejected, $a_1 > 0$)

$$r = \frac{2}{3}, a = \frac{8}{3} \text{ (selected)}$$

Now

$$a_2 + a_4 + 2a_5$$

$$= \frac{8}{3} \times \frac{2}{3} + \frac{8}{3} \times \frac{27}{8} + 2 \times \frac{8}{3} \times \frac{81}{16}$$

$$= 4 + 9 + 27 = 40$$

8. The integral $\frac{24}{\pi} \int_0^{\sqrt{2}} \frac{(2-x^2)dx}{(2+x^2)\sqrt{4+x^4}}$ is equal to _____ .

Official Ans. by NTA (3)

Ans. (3)

Sol. $\frac{24}{\pi} \int_0^{\sqrt{2}} \frac{(2-x^2)}{(x^2+2)\sqrt{4+x^4}} dx$

$$\frac{24}{\pi} \int_0^{\sqrt{2}} \frac{x^2 \left(\frac{2}{x^2} - 1\right) dx}{x \left(x + \frac{2}{x}\right) \times x \sqrt{\frac{4}{x^2} + x^2}}$$

$$\frac{24}{\pi} \int_0^{\sqrt{2}} \frac{\left(\frac{2}{x^2}-1\right) dx}{\left(x+\frac{2}{x}\right) \sqrt{\left(x+\frac{2}{x}\right)^2-4}}$$

$$x + \frac{2}{x} = t$$

$$dt = \left(1 - \frac{2}{x^2}\right) dx$$

$$I = -\frac{24}{\pi} \int_{t\sqrt{t^2-4}} \frac{dt}{t\sqrt{t^2-4}}$$

$$= -\frac{24}{\pi} \times \frac{1}{2} \sec^{-1} \left(\frac{x + \frac{2}{x}}{2} \right) \Bigg|_0^{\sqrt{2}}$$

$$= -\frac{12}{\pi} \left[\sec^{-1} \left(\frac{2\sqrt{2}}{2} \right) - \sec^{-1}(\infty) \right]$$

$$= -\frac{12}{\pi} \left[\frac{\pi}{4} - \frac{2\pi}{2 \times 2} \right] = -\frac{12}{\pi} \left[-\frac{\pi}{4} \right]$$

$$= 3$$

9. Let a line L_1 be tangent to the hyperbola

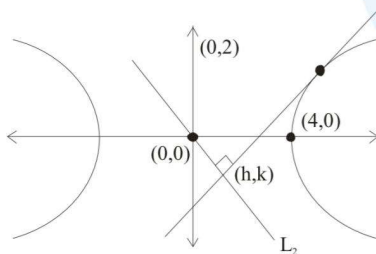
$$\frac{x^2}{16} - \frac{y^2}{4} = 1$$

and let L_2 be the line passing through the origin and perpendicular to L_1 . If the locus of the point of intersection of L_1 and L_2 is $(x^2 + y^2)^2 = \alpha x^2 + \beta y^2$, then $\alpha + \beta$ is equal to _____ .

Official Ans. by NTA (12)

Ans. (12)

Sol.



$$\frac{x \sec \theta}{4} - \frac{y \tan \theta}{2} = 1$$

$$m_1 = \frac{\sec \theta \times 2}{4(\tan \theta)} = \frac{\sec \theta}{2 \tan \theta}$$

$$m_2 = \frac{k}{h}$$

$$m_1 m_2 = -1$$

$$\frac{k \sec \theta}{h 2 \tan \theta} = -1$$

$$\frac{k}{2h \sin \theta} = -1$$

$$\sin \theta = \frac{-k}{2h} \quad \cos \theta = \frac{\sqrt{4h^2 - k^2}}{2h}$$

also

$$\frac{h \sec \theta}{4} - \frac{k \tan \theta}{2} = 1$$

$$\frac{h}{4} \frac{2h}{\sqrt{4h^2 - k^2}} - \frac{k}{2} \left(\frac{-k}{\sqrt{4h^2 - k^2}} \right) = 1$$

$$h^2 + k^2 = 2\sqrt{4h^2 - k^2}$$

$$(x^2 + y^2)^2 = 4(4x^2 - y^2)$$

$$(x^2 + y^2)^2 = 16x^2 - 4y^2$$

$$\alpha = 16, \beta = -4$$

$$\alpha + \beta = 16 - 4 = 12$$

10. If the probability that a randomly chosen 6-digit number formed by using digits 1 and 8 only is a multiple of 21 is p , then $96p$ is equal to _____ .

Official Ans. by NTA (33)

Ans. (33)

Sol. $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

Divisible by 21 when divided by 3.

Case - I : All 1 \rightarrow (1)

Case - II : All 8 \rightarrow (1)

Case - III : 3 ones & 3 eights

$$\frac{6!}{3! \times 3!} = 20$$

Required probability $\therefore p = \frac{22}{64}$

$$96p = 96 \times \frac{22}{64} = 33$$