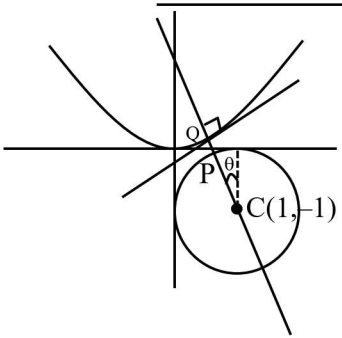




Sol.



$$Q = (t, t^2)$$

$$m_{CQ} = m_{\text{normal}}$$

$$\frac{t^2 + 1}{t - 1} = -\frac{1}{2t}$$

$$\text{Let } f(t) = 2t^3 + 3t - 1$$

$$f\left(\frac{1}{4}\right)f\left(\frac{1}{3}\right) < 0 \Rightarrow t \in \left(\frac{1}{4}, \frac{1}{3}\right)$$

$$P \equiv (1 + \cos(90 + \theta), -1 + \sin(90 + \theta))$$

$$P = (1 - \sin \theta, -1 + \cos \theta)$$

$$m_{\text{normal}} = m_{CP} \Rightarrow -\frac{1}{2t} = \frac{\cos \theta}{-\sin \theta} \Rightarrow \tan \theta = 2t$$

$$x = 1 - \sin \theta = 1 - \frac{2t}{\sqrt{1 + 4t^2}} = g(t) \quad (\text{let})$$

$$\Rightarrow g'(t) < 0$$

$g(t) \downarrow$  function

$$t \in \left(\frac{1}{4}, \frac{1}{3}\right)$$

$$\Rightarrow g(t) \in (0.44, 0.485) \in \left(\frac{1}{4}, \frac{1}{2}\right)$$

6. If the maximum value of a, for which the function  $f_a(x) = \tan^{-1} 2x - 3ax + 7$  is non-decreasing in

$\left(-\frac{\pi}{6}, \frac{\pi}{6}\right)$ , is  $\bar{a}$ , then  $f_{\bar{a}}\left(\frac{\pi}{8}\right)$  is equal to

(A)  $8 - \frac{9\pi}{4(9 + \pi^2)}$       (B)  $8 - \frac{4\pi}{9(4 + \pi^2)}$

(C)  $8 \left(\frac{1 + \pi^2}{9 + \pi^2}\right)$       (D)  $8 - \frac{\pi}{4}$

**Official Ans. by NTA (A)**

**Ans. (Bonus)**

Sol.  $f_a(x) = \tan^{-1} 2x - 3ax + 7$

$$f'_a(x) = \frac{2}{1 + 4x^2} - 3a \geq 0$$

$$a \leq \left(\frac{2}{3(1 + 4x^2)}\right)_{\text{min.}} \quad \text{at } x = \pm \frac{\pi}{6}$$

$$a_{\text{max}} = \bar{a} = \frac{6}{9 + \pi^2}$$

$$f_{\bar{a}}\left(\frac{\pi}{8}\right) = \tan^{-1} \frac{\pi}{4} - 3 \frac{6}{9 + \pi^2} \frac{\pi}{8} + 7 = \tan^{-1} \frac{\pi}{4} - \frac{9\pi}{4(\pi^2 + 9)} + 7$$

7. Let  $\beta = \lim_{x \rightarrow 0} \frac{\alpha x - (e^{3x} - 1)}{\alpha x (e^{3x} - 1)}$  for some  $\alpha \in \mathbb{R}$ . Then

the value of  $\alpha + \beta$  is :

- (A)  $\frac{14}{5}$       (B)  $\frac{3}{2}$       (C)  $\frac{5}{2}$       (D)  $\frac{7}{2}$

**Official Ans. by NTA (C)**

**Ans. (C)**

Sol.  $\beta = \lim_{x \rightarrow 0} \frac{\alpha x - (e^{3x} - 1)}{\alpha x (e^{3x} - 1)}$

$$\beta = \lim_{x \rightarrow 0} \frac{1 + \alpha x - \left[1 + 3x + \frac{9x^2}{2!} + \dots\right]}{(\alpha x) \frac{(e^{3x} - 1)}{3x} 3x}$$

$$\beta = \lim_{x \rightarrow 0} \frac{(\alpha x - 3x) - \frac{9x^2}{2!} - \dots}{3\alpha x^2}$$

For existence of limit  $\alpha - 3 = 0$   
 $\alpha = 3$

$$\text{Limit } \beta = \frac{-3}{2\alpha}$$

$$\beta = -\frac{1}{2}$$

Now,

$$\alpha + \beta = \frac{5}{2}$$

8. The value of  $\log_e 2 \frac{d}{dx} (\log_{\cos x} \operatorname{cosec} x)$  at  $x = \frac{\pi}{4}$  is

- (A)  $-2\sqrt{2}$       (B)  $2\sqrt{2}$       (C)  $-4$       (D)  $4$

**Official Ans. by NTA (D)**

**Ans. (D)**

**Sol.**  $\log_e 2 \frac{d}{dx} (\log_{\cos x} \operatorname{cosec} x)$

Let,

$$y = \log_{\cos x} \operatorname{cosec} x$$

$$y = -\frac{\ln(\sin x)}{\ln(\cos x)}$$

$$\frac{dy}{dx} = -\frac{[\cot x \cdot \ln(\cos x) + \tan x \cdot \ln(\sin x)]}{(\ln(\cos x))^2}$$

$$\left. \frac{dy}{dx} \right)_{x=\frac{\pi}{4}} = \frac{4}{\ln 2}$$

Now,

$$\Rightarrow \log_e 2 \cdot \frac{4}{\ln 2} = 4$$

9.  $\int_0^{20\pi} (|\sin x| + |\cos x|)^2 dx$  is equal to :-

- (A)  $10(\pi + 4)$                       (B)  $10(\pi + 2)$   
 (C)  $20(\pi - 2)$                       (D)  $20(\pi + 2)$

**Official Ans. by NTA (D)**

**Ans. (D)**

**Sol.**  $I = \int_0^{20\pi} (|\sin x| + |\cos x|)^2 dx$  ; (Jack property)

$$I = 40 \int_0^{\pi/2} (\sin x + \cos x)^2 dx$$

$$I = 40 \int_0^{\pi/2} (1 + \sin 2x) dx$$

$$I = 20[\pi + 2]$$

10. Let the solution curve  $y = f(x)$  of the differential

equation  $\frac{dy}{dx} + \frac{xy}{x^2 - 1} = \frac{x^4 + 2x}{\sqrt{1 - x^2}}$ ,  $x \in (-1, 1)$  pass

through the origin. Then  $\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x) dx$  is equal to

- (A)  $\frac{\pi}{3} - \frac{1}{4}$                       (B)  $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$   
 (C)  $\frac{\pi}{6} - \frac{\sqrt{3}}{4}$                       (D)  $\frac{\pi}{6} - \frac{\sqrt{3}}{2}$

**Official Ans. by NTA (B)**

**Ans. (B)**

**Sol.**  $\frac{dy}{dx} + \frac{xy}{x^2 - 1} = \frac{x^4 + 2x}{\sqrt{1 - x^2}}$

$$I.F = e^{\int \frac{x}{x^2 - 1} dx}$$

$$I.F = \sqrt{1 - x^2}$$

Solution of D.E.

$$y \cdot \sqrt{1 - x^2} = \int \frac{x^4 + 2x}{\sqrt{1 - x^2}} \cdot \sqrt{1 - x^2} dx$$

$$y \cdot \sqrt{1 - x^2} = \int (x^4 + 2x) dx$$

$$y \cdot \sqrt{1 - x^2} = \frac{x^5}{5} + x^2 + C$$

At  $x = 0$ ,  $y = 0$ , get  $C = 0$

$$y = \frac{x^5}{5\sqrt{1 - x^2}} + \frac{x^2}{\sqrt{1 - x^2}}$$

Now,

$$\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x) dx = \int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} \frac{x^5}{5\sqrt{1 - x^2}} dx + \int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} \frac{x^2}{\sqrt{1 - x^2}} dx$$

$$\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x) dx = 0 + 2 \int_0^{\frac{\sqrt{3}}{2}} \frac{x^2}{\sqrt{1 - x^2}} dx$$

$$\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x) dx = \frac{\pi}{3} - \frac{\sqrt{3}}{4}$$

11. The acute angle between the pair of tangents drawn to the ellipse  $2x^2 + 3y^2 = 5$  from the point  $(1, 3)$  is

- (A)  $\tan^{-1}\left(\frac{16}{7\sqrt{5}}\right)$                       (B)  $\tan^{-1}\left(\frac{24}{7\sqrt{5}}\right)$   
 (C)  $\tan^{-1}\left(\frac{32}{7\sqrt{5}}\right)$                       (D)  $\tan^{-1}\left(\frac{3 + 8\sqrt{5}}{35}\right)$

**Official Ans. by NTA (B)**

**Ans. (B)**

**Sol.** Equation of tangent to the ellipse  $2x^2 + 3y^2 = 5$  is

$$y = mx \pm \sqrt{\frac{5}{2}m^2 + \frac{5}{3}}$$

It pass through  $(1, 3)$

$$3 = m \pm \sqrt{\frac{5}{2}m^2 + \frac{5}{3}}$$

$$3m^2 + 12m - \frac{44}{3} = 0$$

Let  $\theta$  be the angle between the tangents

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$\tan \theta = \left| \frac{3\sqrt{320}}{-35} \right|$$

$$\theta = \tan^{-1} \left( \frac{24}{7\sqrt{5}} \right)$$

12. The equation of a common tangent to the parabolas  $y = x^2$  and  $y = -(x-2)^2$  is  
 (A)  $y = 4(x-2)$  (B)  $y = 4(x-1)$   
 (C)  $y = 4(x+1)$  (D)  $y = 4(x+2)$

**Official Ans. by NTA (B)**  
**Ans. (B)**

**Sol.** Equation of tangent of  $y = x^2$  be  
 $tx = y + at^2$  .....(1)

$$y = tx - \frac{t^2}{4}$$

Solve with  $y = -(x-2)^2$

$$tx - \frac{t^2}{4} = -(x-2)^2$$

$$x^2 + x(t-4) - \frac{t^2}{4} + 4 = 0$$

$$D = 0$$

$$(t-4)^2 - 4 \cdot \left( 4 - \frac{t^2}{4} \right) = 0$$

$$t^2 - 4t = 0$$

$$t = 0 \text{ or } t = 4$$

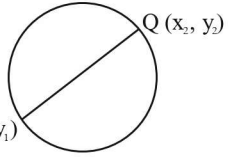
From eq. (1), required common tangent is

$$y = 4(x-1)$$

13. Let the abscissae of the two points P and Q on a circle be the roots of  $x^2 - 4x - 6 = 0$  and the ordinates of P and Q be the roots of  $y^2 + 2y - 7 = 0$ . If PQ is a diameter of the circle  $x^2 + y^2 + 2ax + 2by + c = 0$ , then the value of  $(a+b-c)$  is  
 (A) 12 (B) 13 (C) 14 (D) 16

**Official Ans. by NTA (A)**

**Ans. (A)**



**Sol.**

Equation of circle diameter form

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

(where  $x_1, x_2$  are the roots of  $x^2 - 4x - 6 = 0$  and  $y_1, y_2$  are the roots of  $y^2 + 2y - 7 = 0$ )

$$x^2 + y^2 - 4x + 2y - 13 = 0$$

Now,

Compare it with the given equation, we get

$$a = -2, b = 1, c = -13$$

Now

$$a + b - c = 12$$

14. If the line  $x-1 = 0$ , is a directrix of the hyperbola  $kx^2 - y^2 = 6$ , then the hyperbola passes through the point

- (A)  $(-2\sqrt{5}, 6)$  (B)  $(-\sqrt{5}, 3)$   
 (C)  $(\sqrt{5}, -2)$  (D)  $(2\sqrt{5}, 3\sqrt{6})$

**Official Ans. by NTA (C)**

**Ans. (C)**

**Sol.**  $\frac{x^2}{6/k} - \frac{y^2}{6} = 1$  .....(1)

$$e^2 = 1 + \frac{6}{6/k}$$

$$e = \sqrt{1+k}$$

$$a = \sqrt{\frac{6}{k}}$$

Eq. of directrix  $x = \frac{a}{e} \Rightarrow x = \sqrt{\frac{6}{k(k+1)}}$

$$\frac{6}{k(k+1)} = 1$$

$$k = 2$$

From eq. (1), we get  $2x^2 - y^2 = 6$

Check options



15. A vector  $\vec{a}$  is parallel to the line of intersection of the plane determined by the vectors  $\hat{i}, \hat{i} + \hat{j}$  and the plane determined by the vectors  $\hat{i} - \hat{j}, \hat{i} + \hat{k}$ . The obtuse angle between  $\vec{a}$  and the vector  $\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$  is

- (A)  $\frac{3\pi}{4}$  (B)  $\frac{2\pi}{3}$   
 (C)  $\frac{4\pi}{5}$  (D)  $\frac{5\pi}{6}$

**Official Ans. by NTA (A)**

**Ans. (A)**

**Sol.**  $\vec{n}_1 = \hat{i} \times (\hat{i} + \hat{j}) = \hat{k}$

$$\begin{aligned} \vec{n}_2 &= (\hat{i} + \hat{k}) \times (\hat{i} - \hat{j}) \\ &= \hat{i} + \hat{j} - \hat{k} \end{aligned}$$

Line of intersection along  $\vec{n}_1 \times \vec{n}_2$

$$= \hat{k} \times (\hat{i} + \hat{j} - \hat{k}) = -\hat{i} + \hat{j}$$

D.R of  $\vec{a} = -\hat{i} + \hat{j}$

D.R of  $\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$

$$\vec{a} \cdot \vec{b} = -3 \text{ and } (\vec{a} \wedge \vec{b}) = 0$$

$$\cos \theta = \frac{-3}{\sqrt{2} \times 3}$$

$$\theta = \frac{3\pi}{4}$$

16. If  $0 < x < \frac{1}{\sqrt{2}}$  and  $\frac{\sin^{-1} x}{\alpha} = \frac{\cos^{-1} x}{\beta}$ , then a value

of  $\sin\left(\frac{2\pi\alpha}{\alpha + \beta}\right)$  is

- (A)  $4\sqrt{(1-x^2)}(1-2x^2)$   
 (B)  $4x\sqrt{(1-x^2)}(1-2x^2)$   
 (C)  $2x\sqrt{(1-x^2)}(1-4x^2)$   
 (D)  $4\sqrt{(1-x^2)}(1-4x^2)$

**Official Ans. by NTA (B)**

**Ans. (B)**

**Sol.**  $\frac{\sin^{-1} x}{\alpha} = \frac{\cos^{-1} x}{\beta} = k$

$$\sin^{-1} x = k\alpha$$

$$\cos^{-1} x = k\beta$$

$$k = \frac{\pi}{2(\alpha + \beta)} \quad \dots(i)$$

$$\begin{aligned} \sin\left(\frac{2\pi\alpha}{\alpha + \beta}\right) &= \sin(4\sin^{-1} x) \\ &= 2\sin(2\sin^{-1} x) \cos(2\sin^{-1} x) \\ &= 4x\sqrt{1-x^2}(1-2x^2) \end{aligned}$$

17. Negation of the Boolean expression  $p \leftrightarrow (q \Rightarrow p)$  is

- (A)  $(\sim p) \wedge q$  (B)  $p \wedge (\sim q)$   
 (C)  $(\sim p) \vee (\sim q)$  (D)  $(\sim p) \wedge (\sim q)$

**Official Ans. by NTA (D)**

**Ans. (D)**

**Sol.**  $\sim(p \leftrightarrow (q \rightarrow p))$

$$\sim(p \leftrightarrow q) = (p \wedge \sim q) \vee (q \wedge \sim p)$$

$$\sim(p \leftrightarrow (q \rightarrow p)) = (p \wedge \sim(q \rightarrow p)) \vee ((q \rightarrow p) \wedge \sim p)$$

$$(p \wedge \sim(q \rightarrow p)) = p \wedge (q \wedge \sim p) = (p \wedge \sim p) \wedge q = c$$

$$\begin{aligned} (q \rightarrow p) \wedge \sim p &= (\sim q \vee p) \wedge \sim p = \sim p \wedge (\sim q \vee p) \\ &= (\sim p \wedge \sim q) \vee (\sim p \wedge p) = \sim p \wedge \sim q \end{aligned}$$

$$\sim(p \leftrightarrow (q \rightarrow p)) = c \vee (\sim p \wedge \sim q) = \sim p \wedge \sim q$$

18. Let X be a binomially distributed random variable with mean 4 and variance  $\frac{4}{3}$ . Then  $54 P(X \leq 2)$  is equal to

- (A)  $\frac{73}{27}$  (B)  $\frac{146}{27}$   
 (C)  $\frac{146}{81}$  (D)  $\frac{126}{81}$

**Official Ans. by NTA (B)**

**Ans. (B)**

**Sol.**  $np = 4$

$$npq = 4/3$$

$$n = 6, p = 2/3, q = 1/3$$

$$54(P(X = 2) + P(X = 1) + P(X = 0))$$

$$\begin{aligned} &54\left[{}^6C_2\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^4 + {}^6C_1\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^5 + {}^6C_0\left(\frac{2}{3}\right)^0\left(\frac{1}{3}\right)^6\right] \\ &= \frac{146}{27} \end{aligned}$$

19. The integral  $\int \frac{\left(1 - \frac{1}{\sqrt{3}}\right)(\cos x - \sin x)}{\left(1 + \frac{2}{\sqrt{3}} \sin 2x\right)} dx$  is equal to

(A)  $\frac{1}{2} \log_e \left| \frac{\tan\left(\frac{x}{2} + \frac{\pi}{12}\right)}{\left(\frac{x}{2} + \frac{\pi}{6}\right)} \right| + C$

(B)  $\frac{1}{2} \log_e \left| \frac{\tan\left(\frac{x}{2} + \frac{\pi}{6}\right)}{\left(\frac{x}{2} + \frac{\pi}{3}\right)} \right| + C$

(C)  $\log_e \left| \frac{\tan\left(\frac{x}{2} + \frac{\pi}{6}\right)}{\tan\left(\frac{x}{2} + \frac{\pi}{12}\right)} \right| + C$

(D)  $\frac{1}{2} \log_e \left| \frac{\tan\left(\frac{x}{2} - \frac{\pi}{12}\right)}{\tan\left(\frac{x}{2} - \frac{\pi}{6}\right)} \right| + C$

**Official Ans. by NTA (A)**

**Ans. (A)**

**Sol.**  $I = \int \frac{\left(1 - \frac{1}{\sqrt{3}}\right)(\cos x - \sin x)}{\left(1 + \frac{2}{\sqrt{3}} \sin 2x\right)} dx$

$$\frac{\sqrt{3}}{2} \int \frac{\left(1 - \frac{1}{\sqrt{3}}\right)(\cos x - \sin x)}{\left(\frac{\sqrt{3}}{2} + \sin 2x\right)} dx$$

$$\int \frac{\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)(\cos x - \sin x)}{\sin 60^\circ + \sin 2x} dx$$

$$\int \frac{\left(\frac{\sqrt{3}}{2} \cos x - \frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \sin x\right)}{2 \sin\left(x + \frac{\pi}{6}\right) \cos\left(x - \frac{\pi}{6}\right)} dx$$

$$\int \frac{\left(\cos\left(x - \frac{\pi}{6}\right) - \sin\left(x + \frac{\pi}{6}\right)\right)}{2 \sin\left(x + \frac{\pi}{6}\right) \cos\left(x - \frac{\pi}{6}\right)} dx$$

$$\frac{1}{2} \left( \int \frac{dx}{\sin\left(x + \frac{\pi}{6}\right)} - \int \frac{dx}{\cos\left(x - \frac{\pi}{6}\right)} \right)$$

$$\frac{1}{2} \ln \left| \frac{\tan\left(\frac{x}{2} + \frac{\pi}{12}\right)}{\tan\left(\frac{x}{2} + \frac{\pi}{6}\right)} \right|$$

20. The area bounded by the curves  $y = |x^2 - 1|$  and  $y = 1$  is

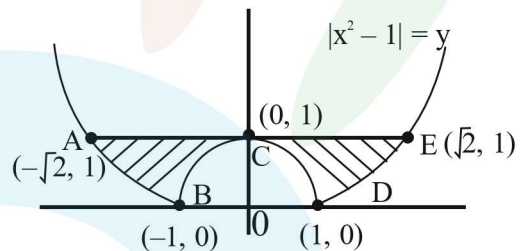
(A)  $\frac{2}{3}(\sqrt{2} + 1)$  (B)  $\frac{4}{3}(\sqrt{2} - 1)$

(C)  $2(\sqrt{2} - 1)$  (D)  $\frac{8}{3}(\sqrt{2} - 1)$

**Official Ans. by NTA (D)**

**Ans. (D)**

**Sol.**  $y = |x^2 - 1|$



Area = ABCDEA

$$= 2 \left( \int_0^1 (1 - (1 - x^2)) dx + \int_1^{\sqrt{2}} (1 - (x^2 - 1)) dx \right)$$

$$= \frac{8}{3}(\sqrt{2} - 1)$$

**SECTION-B**

1. Let  $A = \{1, 2, 3, 4, 5, 6, 7\}$  and  $B = \{3, 6, 7, 9\}$ . Then the number of elements in the set  $\{C \subseteq A : C \cap B \neq \phi\}$  is \_\_\_\_\_

**Official Ans. by NTA (112)**

**Ans. (112)**

**Sol.**  $A = \{1, 2, 3, 4, 5, 6, 7\}$  and

$$B = \{3, 6, 7, 9\}$$

Total subset of  $A = 2^7 = 128$

$C \cap B = \phi$  when set  $C$  contains the element 1, 2, 4, 5

$$\therefore S = \{C \subseteq A; C \cap B \neq \phi\}$$

$$= \text{Total} - (C \cap B = \phi)$$

$$= 128 - 2^4 = 112$$

2. The largest value of  $a$ , for which the perpendicular distance of the plane containing the lines  $\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + a\hat{j} - \hat{k})$  and  $\vec{r} = (\hat{i} + \hat{j}) + \mu(-\hat{i} + \hat{j} - a\hat{k})$  from the point  $(2, 1, 4)$  is  $\sqrt{3}$ , is \_\_\_\_\_.

**Official Ans. by NTA (20)**

**Ans. (2)**

**Sol.**  $\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + a\hat{j} - \hat{k})$

$$\vec{r} = (\hat{i} + \hat{j}) + \mu(-\hat{i} + \hat{j} - a\hat{k})$$

D.R's of plane containing these lines is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & a & -1 \\ -1 & 1 & -a \end{vmatrix} = \hat{i}(1 - a^2) - \hat{j}(-a - 1) + \hat{k}(1 + a)$$

$$\vec{n} = (1 - a)\hat{i} + \hat{j} + \hat{k}$$

One point in plane :  $(1, 1, 0)$

$\therefore$  equation of plane is

$$(1 - a)(x - 1) + (y - 1) + (z - 0) = 0$$

$$(1 - a)x + y + z + a - 2 = 0$$

$$\therefore D = \frac{|(1 - a)2 + 1 + 4 + a - 2|}{\sqrt{(1 - a)^2 + 1 + 1}}$$

$$\Rightarrow |5 - a| = \sqrt{3} \cdot \sqrt{a^2 - 2a + 3}$$

$$\Rightarrow a^2 + 2a - 8 = 0$$

$$\Rightarrow a = 2, -4$$

$\therefore$  largest value of  $a = 2$

3. Numbers are to be formed between 1000 and 3000, which are divisible by 4, using the digits 1, 2, 3, 4, 5 and 6 without repetition of digits. Then the total number of such numbers is \_\_\_\_\_.

**Official Ans. by NTA (30)**

**Ans. (30)**

**Sol.** Here 1<sup>st</sup> digit is 1 or 2 only

**Case-I**

If first digit is 1

Then last two digits can be 24, 32, 36, 52, 56, 64

$$\begin{array}{|c|c|c|c|} \hline 1 & & & \\ \hline \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & \times & 3 & \times & 6 & = 18 \text{ ways} \end{array}$$

**Case - II**

If first digit is 2 then last two digit can be 16, 36, 56, 64

$$\begin{array}{|c|c|c|c|} \hline 2 & & & \\ \hline \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & \times & 3 & \times & 4 & = 12 \text{ ways} \end{array}$$

Total ways =  $12 + 18 = 30$  ways

4. If  $\sum_{k=1}^{10} \frac{k}{k^4 + k^2 + 1} = \frac{m}{n}$ , where  $m$  and  $n$  are co-prime, then  $m + n$  is equal to

**Official Ans. by NTA (166)**

**Ans. (166)**

**Sol.**  $\sum_{k=1}^{10} \frac{k}{k^4 + k^2 + 1} = \frac{m}{n}$

$$\Rightarrow \frac{1}{2} \sum_{k=1}^{10} \frac{(k^2 + k + 1) - (k^2 - k + 1)}{(k^2 + k + 1)(k^2 - k + 1)}$$

$$\Rightarrow \frac{1}{2} \left( \sum_{k=1}^{10} \left( \frac{1}{(k^2 - k + 1)} - \frac{1}{k^2 + k + 1} \right) \right)$$

$$\Rightarrow \frac{55}{111} = \frac{m}{n}$$

$$m + n = 166$$

5. If the sum of solutions of the system of equations  $2\sin^2 \theta - \cos 2\theta = 0$  and  $2\cos^2 \theta + 3\sin \theta = 0$  in the interval  $[0, 2\pi]$  is  $k\pi$ , then  $k$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (3)**

**Ans. (3)**

**Sol.**  $2\sin^2 \theta - \cos 2\theta = 0$

$$2\sin^2 \theta - (1 - 2\sin^2 \theta) = 0$$

$$\Rightarrow \sin^2 \theta = \left(\frac{1}{2}\right)^2$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$2\cos^2\theta + 3\sin\theta = 0$$

$$\Rightarrow 2\sin^2\theta - 3\sin\theta - 2 = 0$$

$$\therefore \sin\theta = -\frac{1}{2}$$

$$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$$

So, the common solution is

$$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\text{Sum} = \frac{7\pi + 11\pi}{6} = 3\pi = k\pi$$

$$K = 3$$

6. The mean and standard deviation of 40 observations are 30 and 5 respectively. It was noticed that two of these observations 12 and 10 were wrongly recorded. If  $\sigma$  is the standard deviation of the data after omitting the two wrong observations from the data, then  $38\sigma^2$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (238)**

**Ans. (238)**

**Sol.** Wrong mean =  $\mu_1 = 30$

Wrong S.D =  $\sigma_1 = 5$

$$\frac{\sum x_i}{40} = 30$$

$$\Rightarrow \sum x_i = 1200$$

$$\sigma_1^2 = 25$$

$$\Rightarrow \frac{\sum x_i^2}{40} - 30^2 = 25$$

$$\Rightarrow \sum x_i^2 = 925 \times 40 = 37000$$

$$\text{New sum} = \sum x'_i = 1200 - 10 - 12 = 1178$$

$$\text{New mean} = \mu'_1 = \frac{1178}{38} = 31$$

$$\text{New } \sum x_i^2 = 37000 - (10)^2 - (12)^2 = 36756$$

$$\text{New S.D, } \sigma'_1 = \sqrt{\frac{36756}{38} - (31)^2} = \sigma$$

$$36756 - (31)^2 \times 38 = 38\sigma^2$$

$$\Rightarrow 38\sigma^2 = 238$$

7. The plane passing through the line L:  $\ell x - y + 3(1 - \ell)z = 1$ ,  $x + 2y - z = 2$  and perpendicular to the plane  $3x + 2y + z = 6$  is  $3x - 8y + 7z = 4$ . If  $\theta$  is the acute angle between the line L and the y-axis, then  $415 \cos^2\theta$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (125)**

**Ans. (125)**

**Sol.**  $\vec{n}_1 = \ell\hat{i} - \hat{j} + 3(1 - \ell)\hat{k}$

$$\vec{n}_2 = \hat{i} + 2\hat{j} - \hat{k}$$

$$\text{Direction ratio of line} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \ell & -1 & 3(1 - \ell) \\ 1 & 2 & -1 \end{vmatrix}$$

$$= (6\ell - 5)\hat{i} + (3 - 2\ell)\hat{j} + (2\ell + 1)\hat{k}$$

$3x - 8y + 7z = 4$  will contain the line

$$(6\ell - 5)\hat{i} + (3 - 2\ell)\hat{j} + (2\ell + 1)\hat{k}$$

Normal of  $3x - 8y + 7z = 4$  will be perpendicular to the line

$$= 3(6\ell - 5) + (3 - 2\ell)(-8) + 7(2\ell + 1) = 0$$

$$\Rightarrow \ell = \frac{2}{3}$$

$$\therefore \text{direction ratio of line} \left( -1, \frac{5}{3}, \frac{7}{3} \right)$$

Angle with y axis

$$\cos\theta = \frac{5/3}{\sqrt{1 + \frac{25}{9} + \frac{49}{9}}}$$

$$\cos\theta = \frac{5}{\sqrt{83}}$$

$$\therefore 415 \cos^2\theta = \frac{25}{83} \times 415 = 125$$



8. Suppose  $y = y(x)$  be the solution curve to the differential equation  $\frac{dy}{dx} - y = 2 - e^{-x}$  such that  $\lim_{x \rightarrow \infty} y(x)$  is finite. If  $a$  and  $b$  are respectively the  $x$ - and  $y$ - intercepts of the tangent to the curve at  $x=0$ , then the value of  $a-4b$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (3)**

**Ans. (3)**

**Sol.**  $\frac{dy}{dx} - y = 2 - e^{-x}$

I.F. =  $e^{-\int dx} = e^{-x}$

$\therefore$  solution of D.E

$$y \cdot e^{-x} = \int (2e^{-x} - e^{-2x}) dx$$

$$\Rightarrow y = -2 + \frac{e^{-x}}{2} + C \cdot e^x$$

$\therefore \lim_{x \rightarrow \infty} y$  is finite

$$\therefore \lim_{x \rightarrow \infty} \left( -2 + \frac{e^{-x}}{2} + C \cdot e^x \right) \rightarrow \text{finite}$$

This is possible only when  $C = 0$

$$\therefore y = y(x) = -2 + \frac{e^{-x}}{2}$$

$$\frac{dy}{dx} = -\frac{1}{2} e^{-x}$$

$$\left. \frac{dy}{dx} \right|_{x=0} = -\frac{1}{2} = m, \quad y(0) = -2 + \frac{1}{2} = \frac{-3}{2}$$

$\therefore$  equation of tangent

$$y + \frac{3}{2} = -\frac{1}{2}(x - 0)$$

$$\Rightarrow x + 2y = -3$$

$$a = -3, \quad b = \frac{-3}{2}$$

$$a - 4b = -3 + 6 = 3$$

9. Different A.P.'s are constructed with the first term 100, the last term 199, And integral common differences. The sum of the common differences of all such, A.P's having at least 3 terms and at most 33 terms is.

**Official Ans. by NTA (53)**

**Ans. (53)**

**Sol.** 1<sup>st</sup> term = 100 =  $a$

Last term = 199 =  $\ell$

If 3 term

$$a, a + d, a + 2d$$

$$a_n = \ell = a + (n - 1)d$$

$$d_1 = \frac{\ell - a}{n - 1}$$

$n \rightarrow$  number of terms

$$n=3, \quad d_1 = \frac{199 - 100}{2}$$

$$= \frac{99}{2} \notin I$$

$$n = 4, \quad d_2 = \frac{99}{3} = 33 \in I$$

$$n = 10, \quad d_3 = \frac{99}{9} = 11 \in I$$

$$n = 12, \quad d_4 = \frac{99}{11} = 9 \in I$$

$$\therefore \sum d_i = 33 + 11 + 9 = 53$$

10. The number of matrices  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ , where  $a, b, c, d \in \{-1, 0, 1, 2, 3, \dots, 10\}$ , such that  $A = A^{-1}$ , is \_\_\_\_\_.

**Official Ans. by NTA (50)**

**Ans. (50)**

**Sol.**  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Given  $A = A^{-1}$

$$\therefore A^2 = A \cdot A^{-1} = I$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore a^2 + bc = 1 \quad \dots(1)$$

$$ab + bd = 0 \quad \dots(2)$$

$$ac + cd = 0 \quad \dots(3)$$

$$bc + d^2 = 1 \quad \dots(4)$$

(1)  $- (4)$  gives

$$a^2 - d^2 = 0$$

$$\Rightarrow (a + d) = 0 \text{ or } a - d = 0$$

**Case – I**

$$a + d = 0 \Rightarrow (a, d) = (-1, 1), (0, 0), (1, -1)$$

(a)  $(a, d) = (-1, 1)$

$\therefore$  from equation (1)

$$1 + bc = 1 \Rightarrow bc = 0$$

$b = 0$   $C = 12$  possibilities

$c = 0$   $b = 12$  possibilities

but  $(0, 0)$  is repeated

$$\therefore 2 \times 12 = 24$$

$$24 - 1 \text{ (repeated)} = 23 \text{ pairs}$$

(b)  $(a, d) = (1, -1) \Rightarrow bc = 0 \rightarrow 23 \text{ pairs}$

(c)  $(a, d) = (0, 0) \Rightarrow bc = 1$

$$\Rightarrow (b, c) = (1, 1) \text{ \& } (-1, -1), 2 \text{ pairs}$$

**Case – II**

$$a = d$$

from (2) and (3)

$$a \neq 0 \text{ then } b = c = 0$$

$$a^2 = 1$$

$$a = \pm 1 = d$$

$$(a, d) = (1, 1), (-1, -1) \rightarrow 2 \text{ pairs}$$

$$\therefore \text{Total} = 23 + 23 + 2 + 2$$

$$= 50 \text{ pairs}$$