Sol. Ω = sample space
A = be an event

$A = \left\{\frac{1}{2}\right\}, \Omega = [0, 1]$

If P (A) = 0 $\Rightarrow A \neq \phi$

If $P(\bar{A}) = 1 \Rightarrow \bar{A} \neq \Omega$

Then both statement are false

SECTION-B

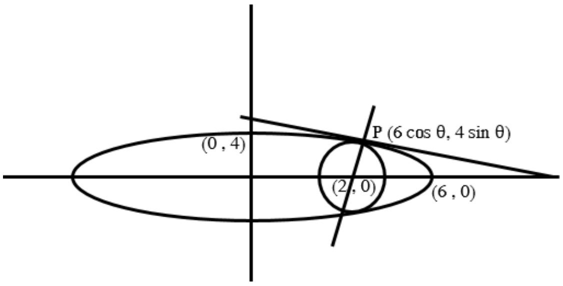
81. Let C be the largest circle centred at (2, 0) and inscribed in the ellipse = $\frac{x^2}{36}+\frac{y^2}{16}=1$.

If (1, α) lies on C, then 10 α^2 is equal to________

Official Ans. by NTA (118)

Ans. (118)

Sol.



Equation of normal of ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$ at any point P (6 cos θ, 4 sin θ) is

3 sec θx –2 cosec θy = 10 this normal is also the normal of the circle passing through the point (2, 0) So,

6 secθ = 10 or sin θ = 0 (Not possible)

$\cos\theta = \frac{3}{5}$ and $\sin\theta = \frac{4}{5}$ so point $P = \left(\frac{18}{5}, \frac{16}{5}\right)$

So the largest radius of circle

$r = \frac{\sqrt{320}}{5}$

So the equation of circle $(x-2)^2 + y^2 = \frac{64}{5}$

Passing it through $(1, \alpha)$

Then $\alpha^2 = \frac{59}{5}$

$10\alpha^2 = 118$

82. Suppose $\sum_{r=0}^{2023} r^2\ {}^{2023}C_r = 2023 \times \alpha \times 2^{2022}$. Then the value of α is __________

Official Ans. by NTA (1012)

Ans. (1012)

Sol. using result

$\sum_{r=0}^{n} r^2\ {}^{n}C_r = n(n+1).2^{n-2}$

Then $\sum_{r=0}^{2023} r^2\ {}^{2023}C_r = 2023 \times 2024 \times 2^{2021}$

$= 2023 \times \alpha \times 2^{2022}$ So,

$\Rightarrow \alpha = 1012$

83. The value of $12\int_0^3 |x^2 - 3x + 2|dx$ is _______

Official Ans. by NTA (22)

Ans. (22)

Sol. $12\int_0^3 |x^2 - 3x + 2|dx$

$= 12\int_0^3 \left|\left(x-\frac{3}{2}\right)^2 - \frac{1}{4}\right|dx$

If $x - \frac{3}{2} = t$

dx = dt

$= 24\int_0^{3/2} \left|t^2 - \frac{1}{4}\right|dt$

$= 24\left[-\int_0^{1/2}\left(t^2-\frac{1}{4}\right)dt + \int_{1/2}^{3/2}\left(t^2-\frac{1}{4}\right)dt\right] = 22$

84. The number of 9 digit numbers, that can be formed using all the digits of the number 123412341 so that the even digits occupy only even places, is___

Official Ans. by NTA (60)

Ans. (60)

Sol. **Even digits occupy at even places**

$$\frac{4!}{2!2!}\times\frac{5!}{2!3!}=\frac{24\times 120}{4\times 12}=60$$

85. Let $\lambda \in \mathbb{R}$ and let the equation E be $|x|^2-2|x|+|\lambda-3|=0$. Then the largest element in the set S = $\{x+\lambda : x$ is an integer solution of E $\}$ is_______

Official Ans. by NTA (5)

Ans. (5)

Sol. $|x|^2-2|x|+|\lambda-3|=0$

$|x|^2-2|x|+|\lambda-3|-1=0$

$(|x|-1)^2+|\lambda-3|=1$

At $\lambda=3$, $x=0$ and 2,

at $\lambda = 4$ or 2, then

$x = 1$ or -1

So maximum value of $x+\lambda=5$

86. A boy needs to select five courses from 12 available courses, out of which 5 courses are language courses. If he can choose at most two language courses, then the number of ways he can choose five courses is

Official Ans. by NTA (546)

Ans. (546)

Sol. For at most two language courses

$= {}^5C_2\times{}^7C_3+{}^5C_1\times{}^7C_4+{}^7C_5=546$

87. Let a tangent to the Curve $9x^2+16y^2=144$ intersect the coordinate axes at the points A and B. Then, the minimum length of the line segment AB is_________

Official Ans. by NTA (7)

Ans. (7)

Sol. Equation of tangent at point $P(4\cos\theta,3\sin\theta)$ is

$\frac{x\cos\theta}{4}+\frac{y\sin\theta}{3}=1$ So A is $(4\sec\theta,0)$ and point B is $(0, 3\operatorname{cosec}\theta)$

Length AB = $\sqrt{16\sec^2\theta+9\operatorname{cosec}^2\theta}$

$=\sqrt{25+16\tan^2\theta+9\cot^2\theta}\ge 7$

88. The value of $\frac{8}{\pi}\int_0^{\frac{\pi}{2}}\frac{(\cos x)^{2023}}{(\sin x)^{2023}+(\cos x)^{2023}}dx$ is ________.

Official Ans. by NTA (2)

Ans. (2)

Sol. $I=\frac{8}{\pi}\int_0^{\frac{\pi}{2}}\frac{(\cos x)^{2023}}{(\sin x)^{2023}+(\cos x)^{2023}}dx$ ....(1)

Using $\int_0^a f(x)dx=\int_0^a f(a-x)dx$

$I=\frac{8}{\pi}\int_0^{\frac{\pi}{2}}\frac{(\sin x)^{2023}}{(\sin x)^{2023}+(\cos x)^{2023}}dx$ ....(2)

Adding (1) & (2)

$2I=\frac{8}{\pi}\int_0^{\frac{\pi}{2}}1\,dx$

$I=2$

89. The shortest distance between the lines $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-6}{2}$ and $\frac{x-6}{3}=\frac{1-y}{2}=\frac{z+8}{0}$ is equal to ________

Official Ans. by NTA (14)

Ans. (14)

Sol. **Shortest distance between the lines**

$$=\frac{\begin{vmatrix}4 & 2 & -14\\ 3 & 2 & 2\\ 3 & -2 & 0\end{vmatrix}}{\left|\begin{vmatrix}\hat{i} & \hat{j} & k\\ 3 & 2 & 2\\ 3 & -2 & 0\end{vmatrix}\right|}$$

$$=\frac{16+12+168}{|-4\hat{i}+6\hat{j}-12k|}=\frac{196}{14}=14$$

90. The 4^{th} term of GP is 500 and its common ratio is $\frac{1}{m}$, $m \in N$. Let S_n denote the sum of the first n terms of this GP. If $S_6 > S_5 + 1$ and $S_7 < S_6 + \frac{1}{2}$, then the number of possible values of m is ______

Official Ans. by NTA (12)

Ans. (12)

Sol. $T_4 = 500$ where a = first term,

$r = \text{common ratio} = \frac{1}{m}, \ m \in N$

$ar^3 = 500$

$\frac{a}{m^3} = 500$

$S_n - S_{n-1} = ar^{n-1}$

$S_6 > S_5 + 1$ and $S_7 - S_6 < \frac{1}{2}$

$S_6 - S_5 > 1$ $\frac{a}{m^6} < \frac{1}{2}$

$ar^5 > 1$ $m^3 > 10^3$

$\frac{500}{m^2} > 1$ $m > 10$ ……(2)

$m^2 < 500$ …….(1)

From (1) and (2)

m = 11, 12, 13…………..,22

So number of possible values of m is 12