FINAL JEE–MAIN EXAMINATION – JANUARY, 2023

(Held On Monday 30th January, 2023) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS | **TEST PAPER WITH SOLUTION**

SECTION-A

61. Let $A=\begin{pmatrix} m & n \\ p & q \end{pmatrix}, d=|A|\neq 0$ $|A-d(\text{Adj}A)|=0$. Then

(1) $(1+d)^2=(m+q)^2$

(2) $1+d^2=(m+q)^2$

(3) $(1+d)^2=m^2+q^2$

(4) $1+d^2=m^2+q^2$

Official Ans. by NTA (1)

Ans. (1)

Sol. $A=\begin{bmatrix} m & n \\ p & q \end{bmatrix}$, $|A-d(\text{adj}A)|=0$

$\Rightarrow$ $|A-d(\text{adj}A)|=\left|\begin{bmatrix} m & n \\ p & q \end{bmatrix}-d\begin{bmatrix} q & -n \\ -p & m \end{bmatrix}\right|$

$=\begin{vmatrix} m-qd & n(1+d) \\ p(1+d) & q-md \end{vmatrix}=0$

$\Rightarrow$ $(m-qd)(q-md)-np(1+d)^2=0$

$\Rightarrow$ $mq-m^2d-q^2d+mqd^2-np(1+d)^2=0$

$\Rightarrow$ $(mq-np)+d^2(mq-np)-d(m^2+q^2+2np)=0$

$\Rightarrow$ $d+d^3-d((m+q)^2-2d)=0$

$\Rightarrow$ $1+d^2=(m+q)^2-2d$

$\Rightarrow$ $(1+d)^2=(m+q)^2$

$\therefore$ Option (1) is correct.

62. The line l_1 passes through the point (2,6,2) and is perpendicular to the plane $2x+y-2z=10$. Then the shortest distance between the line l_1 and the line $\frac{x+1}{2}=\frac{y+4}{-3}=\frac{z}{2}$ is :

(1) 7

(2) $\frac{19}{3}$

(3) $\frac{19}{3}$

(4) 9

Official Ans. by NTA (4)

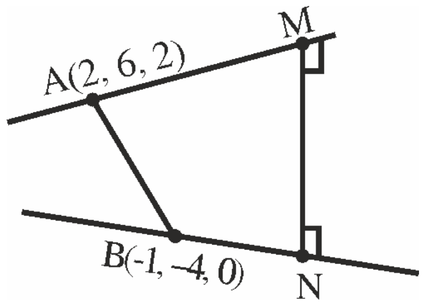
Ans. (4)

Sol. Line ℓ, is given by

$L_1:\frac{x-2}{2}=\frac{y-6}{1}=\frac{z-2}{-2}$

Given,

$L_2:\frac{x+1}{2}=\frac{y+4}{-3}=\frac{z}{2}$



Shortest distance $=\left|\frac{\overrightarrow{AB}\cdot\overrightarrow{MN}}{MN}\right|$

$\overrightarrow{AB}=3\hat{i}+10\hat{j}+2\hat{k}$

$\overrightarrow{MN}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 2 & -3 & 2 \end{vmatrix}=-4\hat{i}-8\hat{j}-8\hat{k}$

$MN=\sqrt{16+64+64}=12$

$\therefore$ Shortest distance $=\left|\frac{-12-80-16}{12}\right|=9$

$\therefore$ Option (4) is correct.

63. If an unbiased die, marked with –2, –1, 0, 1, 2, 3 on its faces, is through five times, then the probability that the product of the outcomes is positive, is :

(1) $\frac{881}{2592}$

(2) $\frac{521}{2592}$

(3) $\frac{440}{2592}$

(4) $\frac{27}{288}$

Official Ans. by NTA (2)

Ans. (2)

76. A straight line cuts off the intercepts $OA = a$ and $OB = b$ on the positive directions of x–axis and y–axis respectively. If the perpendicular from origin O to this line makes an angle of $\frac{\pi}{6}$ with positive direction of y–axis and the area of ΔOAB is $\frac{98}{3}\sqrt{3}$, then $a^2 - b^2$ is equal to:

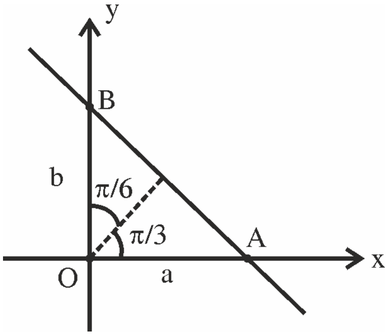
(1) $\frac{392}{3}$ (2) 196

(3) $\frac{196}{3}$ (4) 98

Official Ans. by NTA (1)

Ans. (1)

Sol.



Equation of straight line : $\frac{x}{a} + \frac{y}{b} = 1$

Or $x\cos\frac{\pi}{3} + y\sin\frac{\pi}{3} = p$

$\frac{x}{2} + \frac{y\sqrt{3}}{2} = p$

$\frac{x}{3p} + \frac{y}{2p} = 1$

Comparing both : $a = 2p, b = \frac{2p}{\sqrt{3}}$

Now area of $\Delta OAB = \frac{1}{2}\cdot ab = \frac{98}{3}\cdot\sqrt{3}$

$\frac{1}{2}\cdot 2p\cdot\frac{2p}{\sqrt{3}} = \frac{98}{3}\cdot\sqrt{3}$

$p^2 = 49$

$a^2 - b^2 = 4p^2 - \frac{4p^2}{3} = \frac{2}{3}4p^2$

$= \frac{8}{3}\cdot 49 = \frac{392}{3}$

77. The coefficient of x^{301} in $(1+x)^{500} + x(1+x)^{499} + x^2(1+x)^{498} + ... + x^{500}$ is:

(1) ${}^{501}C_{302}$ (2) ${}^{500}C_{301}$

(3) ${}^{500}C_{300}$ (4) ${}^{501}C_{200}$

Official Ans. by NTA (4)

Ans. (4)

Sol. $(1+x)^{500} + x(1+x)^{499} + x^2(1+x)^{498} + ... + x^{500}$

$$= (1+x)^{500}.\left\{\frac{1-\left(\frac{x}{1+x}\right)^{501}}{1-\frac{x}{1+x}}\right\}$$

$$= (1+x)^{500}\frac{\left((1+x)^{501} - x^{501}\right)}{(1+x)^{501}}\cdot(1+x)$$

$$= (1+x)^{501} - x^{501}$$

Coefficient of x^{301} in $(1+x)^{501} - x^{501}$ is given by ${}^{501}C_{301} = {}^{501}C_{200}$

78. Among the statements:

(S1) $((p \vee q) \Rightarrow r) \Leftrightarrow (p \Rightarrow r)$

(S2) $((p \vee q) \Rightarrow r) \Leftrightarrow ((p \Rightarrow r) \vee (q \Rightarrow r))$

(1) Only (S1) is a tautology

(2) Neither (S1) nor (S2) is a tautology

(3) Only (S2) is a tautology

(4) Both (S1) and (S2) are tautologies

Official Ans. by NTA (2)

Ans. (2)

Sol. $S_1 \equiv ((p \vee q) \Rightarrow r) \Leftrightarrow (p \Rightarrow r)$

p	q	r	$p \vee q$	$(p \vee q) \Rightarrow r$	$p \Rightarrow r$	$((p \vee q) \Rightarrow r) \Leftrightarrow (p \Rightarrow r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	T	T	T	T
F	T	T	T	T	T	T
T	F	F	T	F	F	T
F	T	F	T	F	T	T
F	F	F	T	T	T	T
F	F	F	F	T	T	T

$S_2 \equiv (p\vee q)\Rightarrow r \Leftrightarrow ((p\Rightarrow r)\vee(q\Rightarrow r))$

p	q	r	$(p\vee q)\Rightarrow r$	$p\Rightarrow r$	$q\Rightarrow r$	$(p\Rightarrow r\vee(q\Rightarrow r))$	S2
T	T	T	T	T	T	T	T
T	T	F	F	F	F	T	T
T	F	T	T	T	T	T	T
F	T	T	T	T	T	T	T
T	F	F	F	T	T	T	F
F	T	F	F	F	F	T	F
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

S2 $\rightarrow$ not a tautology

79. The minimum number of elements that must be added to the relation $R=\{(a,b),(b,c)\}$ on the set $\{a,b,c\}$ so that it becomes symmetric and transitive is:

(1) 4 (2) 7
(3) 5 (4) 3

Official Ans. by NTA (2)

Ans. (2)

Sol. *For Symmetric* $(a,b),(b,c)\in R$

$\Rightarrow (b,a),(c,b)\in R$

For Transitive $(a,b),(b,c)\in R$

$\Rightarrow (a,c)\in R$

Now

1. *Symmetric*

$\therefore (a,c)\in R \Rightarrow (c,a)\in R$

2. *Transitive*

$\therefore (a,b),(b,a)\in R$

$\Rightarrow (a,a)\in R \ \&\ (b,c),(c,b)\in R$

$\Rightarrow (b,b)\ \&\ (c,c)\in R$

$\therefore$ *Elements to be added*

$$\left\{\begin{matrix}(b,a),(c,b),(a,c),(c,a)\\ ,(a,a),(b,b),(c,c)\end{matrix}\right\}$$

Number of elements to be added $=7$

80. If the solution of the equation $\log_{\cos x}\cot x + 4\log_{\sin x}\tan x = 1, x\in\left(0,\frac{\pi}{2}\right)$, is $\sin^{-1}\left(\frac{\alpha+\sqrt{\beta}}{2}\right)$, where α, β are integers, then $\alpha+\beta$ is equal to:

(1) 3
(2) 5
(3) 6
(4) 4

Official Ans. by NTA (4)

Ans. (4)

Sol.

$\log_{\cos x}\cot x + 4\log_{\sin x}\tan x = 1$

$\Rightarrow \frac{\ln\cos x-\ln\sin x}{\ln\cos x}+4\frac{\ln\sin x-\ln\cos x}{\ln\sin x}=1$

$\Rightarrow (\ln\sin x)^2-4(\ln\sin x)(\ln\cos x)+4(\ln\cos x)^2=1$

$\Rightarrow \ln\sin x = 2\ln\cos x$

$\Rightarrow \sin^2 x+\sin x-1=0 \Rightarrow \sin x=\frac{-1+\sqrt{5}}{2}$

$\therefore \alpha+\beta=4$

Correct option (4)

SECTION-B

81. Let $S=\{1,2,3,4,5,6\}$. Then the number of one–one functions $f:S\rightarrow P(S)$, where $P(S)$ denote the power set of S, such that $f(n)\subset f(m)$ where $n<m$ is ______.

Official Ans. by NTA (3240)

Ans. (3240)

Sol. Let S={1,2,3,4,5,6}, then the number of one-one functions, f:S•P(S), where P(S) denotes the power set of S, such that f(n)<f(m) where n<m is

$n(S)=6$

$$P(S)=\left\{\begin{matrix}\phi,\{1\},...\{6\},\{1,2\},...,\\ \{5,6\},...,\{1,2,3,4,5,6\}\end{matrix}\right\}$$

– 64 *elements*

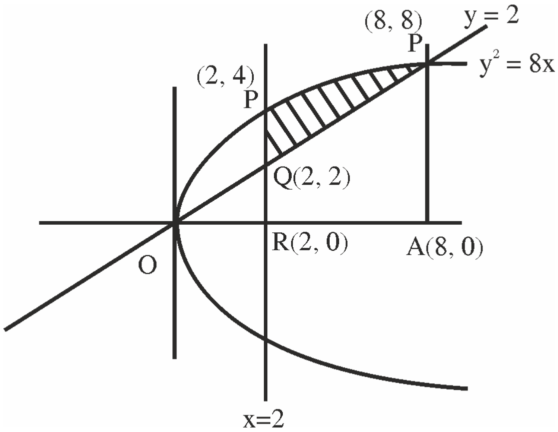
case – 1
f (6) = S i.e. 1 option,
f (5) = any 5 element subset A of S i.e. 6 options,
f (4) = any 4 element subset B of A i.e. 5 options,
f (3) = any 3 element subset C of B i.e. 4 options,
f (2) = any 2 element subset D of C i.e. 3 options,
f (1) = any 1 element subset E of D or empty subset i.e. 3 options,
Total functions = 1080
Case – 2
f (6) = any 5 element subset A of S i.e. 6 options,
f (5) = any 4 element subset B of A i.e. 5 options,
f' (4) = any 3 element subset C of B i.e. 4 options,
f (3) = any 2 element subset D of C i.e. 3 options,
f' (2) = any 1 element subset E of D i.e. 2 options,
f (1) = empty subset i.e. 1 option
Total functions = 720
Case – 3
f(6)= S
f(5) = any 4 element subset A of' S i.e. 15 options,
f(4) = any 3 element subset B of A i.e. 4 options,
f(3) = any 2 element subset C of B i.e. 3 options,
f(2) = any 1 element subset D of C i.e. 2 options,
f(1) = empty subset i.e. 1 option
Total functions = 360
Case – 4
f(6) = S
f(5) = any 5 element subset A of S i.e. 6 options,
f(4) = any 3 element subset B of A i.e. 10 options,
f(3) = any 2 element subset C of B i.e. 3 options,
f(2) = any 1 element subset D of C i.e. 2 options,
f(1) = empty subset i.e. 1 option
Total functions = 360
Case – 5
f(6) = S
f(5) = any 5 element subset A of S i.e. 6 options,
f(4) = any 4 element subset B of A i.e. 5 options,
f(3) = any 2 element subset C of B i.e. 6 options,
f(2) = any 1 element subset D of C i.e. 2 options,
f(1) = empty subset i.e. 1 option
Total functions = 360
Case – 6
f(6) = S
f(5) = any 5 element subset A of S i.e. 6 options,
f(4) = any 4 element subset B of A i.e. 5 options,
f(3) = any 3 element subset C of B i.e. 4 options,
f(2) = any 1 element subset D of C i.e. 3 options,
f(1) = empty subset i.e. 1 option
Total functions = 360
$\therefore$ Number of such functions = 3240

82. Let α be the area of the larger region bounded by the curve $y^2 = 8x$ and the lines $y = x$ and $x = 2$, which lies in the first quadrant. Then the value of 3α is equal to ___________.

Official Ans. by NTA (22)

Ans. (22)

Sol.



$y = x$

$\& \ y^2 = 8x$

Solving it

$x^2 = 8x$

$\therefore$ $x = 0, 8$

$\therefore$ $y = 0, 8$

$x = 2$ will intersect occur at

$y^2 = 16 \Rightarrow \quad y = \pm 4$

$\therefore$ Area of shaded

$$= \int_2^8 \left(\sqrt{8x} - x\right)dx = \int_2^8 \left(2\sqrt{2}\sqrt{x} - x\right)dx$$

$$= \left[2\sqrt{2} \cdot \frac{x^{3/2}}{3/2} - \frac{x^2}{2}\right]_0^8$$

$$= \left(\frac{4\sqrt{2}}{3} \cdot 2^{9/2} - 32\right) - \left(\frac{4\sqrt{2}}{3} \cdot 2^{9/2} - 2\right)$$

$$= \frac{128}{3} - 32 - \frac{16}{3} + 2 = \frac{112 - 90}{3} = \frac{22}{3} = A$$

$\therefore 3A = 22$

83. If $\lambda_1 < \lambda_2$ are two values of λ such that the angle between the planes $P_1 : \vec{r}(3\hat{i}-5\hat{j}+\hat{k})=7$ and $P_2 : \vec{r}\cdot(\lambda\hat{i}+\hat{j}-3\hat{k})=9$ is $\sin^{-1}\left(\frac{2\sqrt{6}}{5}\right)$, then the square of the length of perpendicular from the point $(38\lambda_1, 10\lambda_2, 2)$ to the plane P_1 is _____.

Official Ans. by NTA (315)

Ans. (315)

Sol. $P_1 = \vec{r}\cdot(3\hat{i}-5\hat{j}+\hat{k})=7$

$$P_2 = \vec{r}\cdot(\lambda\hat{i}+\hat{j}-3\hat{k})=9$$

$$\theta = \sin^{-1}\left(\frac{2\sqrt{6}}{5}\right)$$

$$\Rightarrow \sin\theta = \frac{2\sqrt{6}}{5}$$

$$\therefore \cos\theta = \frac{1}{5}\cdot$$

$$\cos\theta = \frac{\vec{r}\cdot\vec{r}}{|\vec{r}_1||\vec{r}_2|}$$

$$= \frac{(3i-5j+K)(\lambda i+j-3K)}{\sqrt{35}\cdot\sqrt{\lambda^2+10}}$$

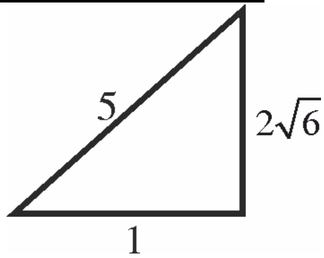
$$\frac{1}{5} = \left|\frac{3\lambda-8}{\sqrt{35}\cdot\sqrt{\lambda^2+10}}\right|$$

$$\text{Square} \Rightarrow \frac{1}{25} = \frac{9\lambda^2+64-48\lambda}{35(\lambda^2+10)}$$

$$\Rightarrow 19\lambda^2-120\lambda+125=0$$

$$\Rightarrow 19\lambda^2-95\lambda-25\lambda+125=0$$

$$\Rightarrow x = 5, \frac{25}{19}$$



Perpendicular distance of point

$(38\lambda_1, 10\lambda_2, 2) \equiv (50, 50, 2)$ from plane P_1

$$= \frac{|3\times 50-5\times 50+2-7|}{\sqrt{35}} = \frac{105}{\sqrt{35}}$$

$$\text{Square} = \frac{105\times 105}{35} = 315$$

84. Let $z = 1+i$ and $z_1 = \frac{1+i\bar{z}}{\bar{z}(1-z)+\frac{1}{z}}$. Then $\frac{12}{\pi}$ $\arg(z_1)$ is equal to _______ .

Official Ans. by NTA (9)

Ans. (9)

Sol. $z = 1+i$

$$z_1 = \frac{1+i\bar{z}}{\bar{z}(1-z)+\frac{1}{z}}$$

$$z_1 = \frac{1+i(1-i)}{(1-i)(1-1-i)+\frac{1}{1+i}}$$

$$= \frac{1+i-i^2}{(1-i)(-i)+\frac{1-i}{2}}$$

$$= \frac{2+i}{\frac{-3i-1}{2}} = \frac{4+2i}{-3i-1}$$

$$= \frac{-(4+2i)(3i-1)}{(3i)^2-(1)^2}$$

$$\text{Arg}(z_1) = \frac{3\pi}{4}$$

$$\therefore \frac{12}{\pi}\arg(z_1) = \frac{12}{\pi}\times\frac{3\pi}{4} = 9$$

85. $\lim_{x\to 0}\frac{48}{x^4}\int_0^x \frac{t^3}{t^6+1}dt$ is equal to ______.

Official Ans. by NTA (12)

Ans. (12)

Sol. $48\lim_{x\to 0}\frac{\int_0^x \frac{t^3}{t^6+1}dt}{x^4}\left(\frac{0}{0}\right)$

Applying L' Hospitals Rule

$48\lim_{x\to 0}\frac{x^3}{x^6+1}\times\frac{1}{4x^3}$

$= 12$

86. The mean and variance of 7 observations are 8 and 16 respectively. If one observation 14 is omitted a and b are respectively mean and variance of remaining 6 observation, then $a+3b-5$ is equal to ______.

Official Ans. by NTA (37)

Ans. (37)

Sol. $\frac{x_1+x_2+....+x_7}{7}=8$

$\frac{x_1+x_2+x_3....+x_6+14}{7}=8$

$\Rightarrow x_1+x_2+....+x_6=42$

$\therefore \frac{x_1+x_2....+x_6}{6}=\frac{42}{6}=7=a$

$\frac{\Sigma x_i^2}{7}-8^2=16$

$\Sigma xi^2=560$

$\Rightarrow x_1^2+x_2^2+...+x_6^2=364$

$b=\frac{x_1^2+x_2^2+....+x_6^2}{6}-7^2$

$=\frac{364}{6}-49$

$b=\frac{70}{6}$

$a+3b-5=7+3\times\frac{70}{6}-5$

$= 37$

87. If the equation of the plane passing through the point $(1,1,2)$ and perpendicular to the line $x-3y+2z-1=0$ $4x-y+z$ is $Ax+By+Cz=1$, then $140(C-B+A)$ is equal to _____.

Official Ans. by NTA (15)

Ans. (15)

Sol. $x-3y+2z-1=0$

$4x-y+z=0$

$$\vec{n}_1\times\vec{n}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -3 & 2\\ 4 & -1 & 1\end{vmatrix}$$

$=-\hat{i}+7\hat{j}+11\hat{k}$

Dr^s of normal to the plane is –1, 7, 11

Equation of plane :

$-1(x-1)+7(y-1)+11(z-2)=0$

$-x+7y+11z=28$

$\frac{-1}{28}x+\frac{7y}{28}+\frac{11z}{28}=1$

$Ax+By+Cz=1$

$140(C-B+A)=140\left(\frac{11}{28}-\frac{7}{28}-\frac{1}{28}\right)$

$=140\times\frac{3}{28}=15$

88. Let $\sum_{n=0}^{\infty}\frac{n^3((2n)!)+(2n-1)(n!)}{(n!)((2n)!)}=ae+\frac{b}{e}+c$, where $a,b,c\in\mathbb{Z}$ and $e=\sum_{n=0}^{\infty}\frac{1}{n!}$ Then a^2-b+c is equal to ________.

Official Ans. by NTA (26)

Ans. (26)

Sol. $$\sum_{n=0}^{\infty}\frac{n^3((2n)!)+(2n-1)(n!)}{(n!)((2n)!)}$$

$$=\sum_{n=0}^{\infty}\frac{1}{(n-3)!}+\sum_{n=0}^{\infty}\frac{3}{(n-2)!}$$

$$+\sum_{n=0}^{\infty}\frac{1}{(n-1)!}+\sum_{n=0}^{\infty}\frac{1}{(2n-1)!}-\sum_{n=0}^{\infty}\frac{1}{(2n)!}$$

$$=e+3e+e+\frac{1}{2}\left(e-\frac{1}{e}\right)-\frac{1}{2}\left(e+\frac{1}{e}\right)$$

$$=5e-\frac{1}{e}$$

$\therefore a^2-b+c=26$

89. Number of 4–digit numbers (the repetition of digits is allowed) which are made using the digits 1, 2, 3 and 5, and are divisible by 15, is equal to ______

Official Ans. by NTA (21)

Ans. (21)

Sol. For number to be divisible by 15, last digit should be 5 and sum of digits must be divisible by 3.

Possible combinations are

1	2	1	5

Numbers = 3

2	2	3	5

Numbers = 3

3	3	1	5

Numbers = 3

1	1	5	5

Numbers = 3

2	3	5	5

Numbers = 6

3	5	5	5

Numbers = 3

Total Numbers = 21

90. Let $f^1(x)=\frac{3x+2}{2x+3}, x\in R-\left\{\frac{-3}{2}\right\}$

For $n\geq 2$, define $f^n(x)=f^1 0 f^{n-1}(x)$.

If $f^5(x)=\frac{ax+b}{bx+a}, \gcd(a,b)=1$, then $a+b$ is equal to __________.

Official Ans. by NTA (3125)

Ans. (3125)

Sol. $f^1(x)=\frac{3x+2}{2x+3}$

$\Rightarrow f^2(x)=\frac{13x+12}{12x+13}$

$\Rightarrow f^3(x)=\frac{63x+62}{62x+63}$

$\therefore f^5(x)=\frac{1563x+1562}{1562x+1563}$

$a+b=3125$