

FINAL JEE-MAIN EXAMINATION – JANUARY, 2023

(Held On Monday 30th January, 2023)

TIME : 3 : 00 PM to 6 : 00 PM

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

61. Consider the following statements:

P : I have fever

Q : I will not take medicine

R : I will take rest

The statement “If I have fever, then I will take medicine and I will take rest” is equivalent to:

(1) $((\sim P) \vee \sim Q) \wedge ((\sim P) \vee R)$

(2) $((\sim P) \vee \sim Q) \wedge ((\sim P) \vee \sim R)$

(3) $(P \vee Q) \wedge ((\sim P) \vee R)$

(4) $(P \vee \sim Q) \wedge (P \vee \sim R)$

Official Ans. by NTA (1)

Ans. (1)

Sol. $P \rightarrow (\sim Q \wedge R)$

$\sim P \vee (\sim Q \wedge R)$

$(\sim P \vee \sim Q) \wedge (\sim P \vee R)$

62. Let A be a point on the x-axis. Common tangents are drawn from A to the curves $x^2 + y^2 = 8$ and $y^2 = 16x$. If one of these tangents touches the two curves at Q and R, then $(QR)^2$ is equal to

(1) 64 (2) 76

(3) 81 (4) 72

Official Ans. by NTA (4)

Ans. (4)

Sol. $y = mx + \frac{4}{m}$

$\frac{\left| \frac{4}{m} \right|}{\sqrt{1+m^2}} = 2\sqrt{2} \therefore m = \pm 1$

$y = \pm x \pm 4$. Point of contact on parabola

Let $m = 1, \left(\frac{a}{m^2}, \frac{2a}{m} \right)$

R (4, 8)

Point of contact on circle Q (-2, 2)

$\therefore (QR)^2 = 36 + 36 = 72$

63. Let q be the maximum integral value of p in [0, 10] for which the roots of the equation $x^2 - px + \frac{5}{4}p = 0$ are rational. Then the area of the region $\{(x, y) : 0 \leq y \leq (x - q)^2, 0 \leq x \leq q\}$ is

(1) 243 (2) 25

(3) $\frac{125}{3}$ (4) 164

Official Ans. by NTA (1)

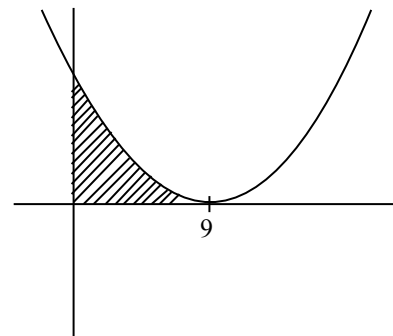
Ans. (1)

Sol. $x^2 - px + \frac{5p}{4} = 0$

$D = p^2 - 5p = p(p - 5)$

$\therefore q = 9$

$0 \leq y \leq (x - 9)^2$



$Area = \int_0^9 (x - 9)^2 dx = 243$

64. If the functions $f(x) = \frac{x^3}{3} + 2bx + \frac{ax^2}{2}$ and

$g(x) = \frac{x^3}{3} + ax + bx^2, a \neq 2b$ have a common

extreme point, then $a + 2b + 7$ is equal to

(1) 4

(2) $\frac{3}{2}$

(3) 3

(4) 6

Official Ans. by NTA (4)

Ans. (4)

72. Let $\lambda \in \mathbb{R}$, $\vec{a} = \lambda\hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{b} = \hat{i} - \lambda\hat{j} + 2\hat{k}$.
If $\left((\vec{a} + \vec{b}) \times (\vec{a} \times \vec{b}) \right) \times (\vec{a} - \vec{b}) = 8\hat{i} - 40\hat{j} - 24\hat{k}$,

then $\left| \lambda(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) \right|^2$ is equal to

- (1) 140 (2) 132
(3) 144 (4) 136

Official Ans. by NTA (1)

Ans. (1)

Sol. $\vec{a} = \lambda\hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{b} = \hat{i} - \lambda\hat{j} + 2\hat{k}$

$$\Rightarrow (\vec{b} - \vec{a}) \times \left((\vec{a} + \vec{b}) \times (\vec{a} \times \vec{b}) \right) = 8\hat{i} - 40\hat{j} - 24\hat{k}$$

$$\Rightarrow \left((\vec{a} - \vec{b}) \cdot (\vec{a} + \vec{b}) \right) (\vec{a} \times \vec{b}) = 8\hat{i} - 40\hat{j} - 24\hat{k}$$

$$\Rightarrow 8(\vec{a} \times \vec{b}) = 8\hat{i} - 40\hat{j} - 24\hat{k}$$

$$\text{Now, } \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \lambda & 2 & -3 \\ 1 & -\lambda & 2 \end{vmatrix}$$

$$= (4 - 3\lambda)\hat{i} - (2\lambda + 3)\hat{j} + (-\lambda^2 - 2)\hat{k}$$

$$\Rightarrow \lambda = 1$$

$$\therefore \vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$$

$$\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$$

$$\Rightarrow \vec{a} + \vec{b} = 2\hat{i} + \hat{j} - \hat{k}, \vec{a} - \vec{b} = 3\hat{j} - 5\hat{k}$$

$$\Rightarrow (\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 0 & 3 & -5 \end{vmatrix} = 2\hat{i} + 10\hat{j} + 6\hat{k}$$

$$\therefore \text{required answer} = 4 + 100 + 36 = 140$$

73. Let \vec{a} and \vec{b} be two vectors. Let $|\vec{a}| = 1$, $|\vec{b}| = 4$ and

$\vec{a} \cdot \vec{b} = 2$. If $\vec{c} = (2\vec{a} \times \vec{b}) - 3\vec{b}$, then the value of

$\vec{b} \cdot \vec{c}$ is

- (1) -24
(2) -48
(3) -84
(4) -60

Official Ans. by NTA (2)

Ans. (2)

Sol. $\vec{c} = (2\vec{a} \times \vec{b}) - 3\vec{b}$

$$\vec{b} \cdot \vec{c} = \vec{b} \cdot (2\vec{a} \times \vec{b}) - 3\vec{b} \cdot \vec{b}$$

$$= -3|\vec{b}|^2$$

$$= -48$$

74. Let $a_1 = 1, a_2, a_3, a_4, \dots$ be consecutive natural numbers. Then $\tan^{-1}\left(\frac{1}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{1}{1+a_2a_3}\right)$

$+ \dots + \tan^{-1}\left(\frac{1}{1+a_{2021}a_{2022}}\right)$ is equal to

(1) $\frac{\pi}{4} - \cot^{-1}(2022)$ (2) $\cot^{-1}(2022) - \frac{\pi}{4}$

(3) $\tan^{-1}(2022) - \frac{\pi}{4}$ (4) $\frac{\pi}{4} - \tan^{-1}(2022)$

Official Ans. by NTA (1,3)

Ans. (1,3)

Sol. $a_2 - a_1 = a_3 - a_2 = \dots = a_{2022} - a_{2021} = 1$.

$$\therefore \tan^{-1}\left(\frac{a_2 - a_1}{1 + a_1a_2}\right) + \tan^{-1}\left(\frac{a_3 - a_2}{1 + a_2a_3}\right) + \dots + \tan^{-1}\left(\frac{a_{2022} - a_{2021}}{1 + a_{2021}a_{2022}}\right)$$

$$= \left[(\tan^{-1} a_2) - \tan^{-1} a_1 \right] + \left[\tan^{-1} a_3 - \tan^{-1} a_2 \right] + \dots + \left[\tan^{-1} a_{2022} - \tan^{-1} a_{2021} \right]$$

$$= \tan^{-1} a_{2022} - \tan^{-1} a_1$$

$$= \tan^{-1}(2022) - \tan^{-1} 1 = \tan^{-1} 2022 - \frac{\pi}{4} \text{ (option 3)}$$

$$= \left(\frac{\pi}{2} - \cot^{-1}(2022) \right) - \frac{\pi}{4}$$

$$= \frac{\pi}{4} - \cot^{-1}(2022) \text{ (option 1)}$$

75. The parabolas : $ax^2 + 2bx + cy = 0$ and $dx^2 + 2ex + fy = 0$ intersect on the line $y = 1$. If a, b, c, d, e, f are positive real numbers and a, b, c are in G.P., then

(1) d, e, f are in A.P. (2) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.

(3) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P. (4) d, e, f are in G.P.

Official Ans. by NTA (3)

Ans. (3)

Sol. $ax^2 + 2bx + c = 0$

$$\Rightarrow ax^2 + 2\sqrt{ac}x + c = 0 (\because b^2 = ac)$$

$$\Rightarrow (x\sqrt{a} + \sqrt{c})^2 = 0$$

$$x^2 - \frac{\sqrt{c}}{\sqrt{a}} \dots (1)$$

Now, $dx^2 + 2ex + f = 0$

$$\Rightarrow d\left(\frac{c}{a}\right) + 2e\left[-\frac{\sqrt{c}}{\sqrt{a}}\right] + f = 0$$

$$\Rightarrow \frac{dc}{a} + f = 2e\sqrt{\frac{c}{a}}$$

$$\Rightarrow \frac{d}{a} + \frac{f}{c} = 2e\sqrt{\frac{1}{ac}}$$

$$\Rightarrow \frac{d}{a} + \frac{f}{c} = \frac{2e}{b} \left[\text{as } b = \sqrt{ae} \right]$$

$$\therefore \frac{d}{a}, \frac{e}{b}, \frac{f}{c} \text{ are in A.P.}$$

76. If a plane passes through the points $(-1, k, 0)$, $(2, k, -1)$, $(1, 1, 2)$ and is parallel to the line $\frac{x-1}{1} = \frac{2y+1}{2}$

$$= \frac{z+1}{-1}, \text{ then the value of } \frac{k^2+1}{(k-1)(k-2)} \text{ is}$$

- (1) $\frac{17}{5}$ (2) $\frac{5}{17}$
 (3) $\frac{6}{13}$ (4) $\frac{13}{6}$

Official Ans. by NTA (4)

Ans. (4)

Sol. $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$

$$\frac{x-1}{1} = \frac{y+\frac{1}{2}}{1} = \frac{z+1}{-1}$$

Points : $A(-1, k, 0)$, $B(2, k, -1)$, $C(1, 1, 2)$

$$\overrightarrow{CA} = -2\hat{i} + (k-1)\hat{j} - 2\hat{k}$$

$$\overrightarrow{CB} = \hat{i} + (k-1)\hat{j} - 3\hat{k}$$

$$\overrightarrow{CA} \times \overrightarrow{CB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & k-1 & -2 \\ 1 & k-1 & -3 \end{vmatrix}$$

$$= \hat{i}(-3k+3+2k-2) - \hat{j}(6+2) + \hat{k}(-2k+2-k+1)$$

$$= (1-k)\hat{i} - 8\hat{j} + (3-3k)\hat{k}$$

The line $\frac{x-1}{1} = \frac{y+\frac{1}{2}}{1} = \frac{z+1}{-1}$ is perpendicular to normal vector.

$$\therefore 1 \cdot (1-k) + 1(-8) + (-1)(3-3k) = 0$$

$$\Rightarrow 1-k-8-3+3k = 0$$

$$\Rightarrow 2k = 10 \Rightarrow \boxed{k=5}$$

$$\therefore \frac{k^2+1}{(k-1)(k-2)} = \frac{26}{4 \cdot 3} = \frac{13}{6}$$

77. Let $a, b, c > 1$, a^3, b^3 and c^3 be in A.P., and $\log_a b, \log_c a$ and $\log_b c$ be in G.P. If the sum of first 20 terms of an A.P., whose first term is $\frac{a+4b+c}{3}$

and the common difference is $\frac{a-8b+c}{10}$ is -444 ,

then abc is equal to

- (1) 343 (2) 216
 (3) $\frac{343}{8}$ (4) $\frac{125}{8}$

Official Ans. by NTA (2)

Ans. (2)

Sol. As a^3, b^3, c^3 be in A.P. $\rightarrow \boxed{a^3 + c^3 = 2b^3}$ (1)

$\log_a^b, \log_c^a, \log_b^c$ are in G.P.

$$\therefore \frac{\log b}{\log a} \cdot \frac{\log c}{\log b} = \left(\frac{\log a}{\log c} \right)^2$$

$$\therefore (\log a)^3 = (\log c)^3 \Rightarrow \boxed{a=c}$$
 (2)

From (1) and (2)

$$\boxed{a=b=c}$$

$$T_1 = \frac{a+4b+c}{3} = 2a; d = \frac{a-8b+c}{10} = \frac{-6a}{10} = \frac{-3}{5}a$$

$$\therefore S_{20} = \frac{20}{2} \left[4a + 19 \left(-\frac{3}{5}a \right) \right]$$

$$= 10 \left[\frac{20a - 57a}{5} \right]$$

$$= -74a$$

$$\therefore -74a = -444 \Rightarrow \boxed{a=6}$$

$$\therefore abc = 6^3 = 216$$

78. Let S be the set of all values of a_1 for which the mean deviation about the mean of 100 consecutive positive integers $a_1, a_2, a_3, \dots, a_{100}$ is 25. Then S is

- (1) ϕ (2) $\{99\}$
 (3) \mathbb{N} (4) $\{9\}$

Official Ans. by NTA (3)

Ans. (3)

Sol. let a_1 be any natural number

$a_1, a_1+1, a_1+2, \dots, a_1+99$ are values of a_i 'S

$$\bar{x} = \frac{a_1 + (a_1+1) + (a_1+2) + \dots + a_1+99}{100}$$

$$= \frac{100a_1 + (1+2+\dots+99)}{100} = a_1 + \frac{99 \times 100}{2 \times 100}$$

$$= a_1 + \frac{99}{2}$$

$$\text{Mean deviation about mean} = \frac{\sum_{i=1}^{100} |x_i - \bar{x}|}{100}$$

$$= \frac{2\left(\frac{99}{2} + \frac{97}{2} + \frac{95}{2} + \dots + \frac{1}{2}\right)}{100}$$

$$= \frac{1+3+\dots+99}{100}$$

$$= \frac{\frac{50}{2}[1+99]}{100}$$

$$= 25$$

So, it is true for every natural no. 'a₁'

79. $\lim_{n \rightarrow \infty} \frac{3}{n} \left\{ 4 + \left(2 + \frac{1}{n}\right)^2 + \left(2 + \frac{2}{n}\right)^2 + \dots + \left(3 - \frac{1}{n}\right)^2 \right\}$

is equal to

(1) 12 (2) $\frac{19}{3}$

(3) 0 (4) 19

Official Ans. by NTA (4)

Ans. (4)

Sol. $\lim_{n \rightarrow \infty} \frac{3}{n} \sum_{r=0}^{n-1} \left(2 + \frac{r}{n}\right)^2$

$$= 3 \int_0^1 (2+x)^2 dx = 27 - 8 = 19$$

80. For $\alpha, \beta \in \mathbb{R}$, suppose the system of linear equations

$$x - y + z = 5$$

$$2x + 2y + \alpha z = 8$$

$$3x - y + 4z = \beta$$

has infinitely many solutions. Then α and β are the roots of

(1) $x^2 - 10x + 16 = 0$ (2) $x^2 + 18x + 56 = 0$

(3) $x^2 - 18x + 56 = 0$ (4) $x^2 + 14x + 24 = 0$

Official Ans. by NTA (3)

Ans. (3)

Sol. $\begin{vmatrix} 1 & -1 & 1 \\ 2 & 2 & \alpha \\ 3 & -1 & 4 \end{vmatrix} = 0; 8 + \alpha - 2(-4 + 1) + 3(-\alpha - 2) = 0$

$$8 + \alpha + 6 - 3\alpha - 6 = 0$$

$$\alpha = 4$$

SECTION-B

81. 50th root of a number x is 12 and 50th root of another number y is 18. Then the remainder obtained on dividing (x + y) by 25 is _____.

Official Ans. by NTA (23)

Ans. (23)

Sol. $x + y = 12^{50} + 18^{50} = (150 - 6)^{25} + (325 - 1)^{25}$
 $= 25K - (6^{25} + 1) = 25K - ((5 + 1)^{25} + 1)$
 $= 25K_1 - 2$ Remainder = 23

82. Let $A = \{1, 2, 3, 5, 8, 9\}$. Then the number of possible functions $f: A \rightarrow A$ such that $f(m \cdot n) = f(m) \cdot f(n)$ for every $m, n \in A$ with $m \cdot n \in A$ is equal to _____.

Official Ans. by NTA (432)

Ans. (432)

Sol. $f(1) = 1; f(9) = f(3) \times f(3)$

i.e., $f(3) = 1$ or 3

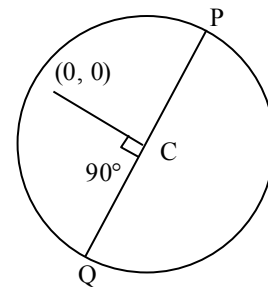
Total function = $1 \times 6 \times 2 \times 6 \times 6 \times 1 = 432$

83. Let $P(a_1, b_1)$ and $Q(a_2, b_2)$ be two distinct points on a circle with center $C(\sqrt{2}, \sqrt{3})$. Let O be the origin and OC be perpendicular to both CP and CQ. If the area of the triangle OCP is $\frac{\sqrt{35}}{2}$, then $a_1^2 + a_2^2 + b_1^2 + b_2^2$ is equal to _____.

Official Ans. by NTA (24)

Ans. (24)

Sol. $\frac{1}{2} \times PC \times \sqrt{5} = \frac{\sqrt{35}}{2}; PC = \sqrt{7}$



$$a_1^2 + b_1^2 + a_2^2 + b_2^2 = OP^2 + OQ^2$$

$$= 2(5 + 7) = 24$$

84. The 8th common term of the series

$$S_1 = 3 + 7 + 11 + 15 + 19 + \dots,$$

$$S_2 = 1 + 6 + 11 + 16 + 21 + \dots$$

is _____.

Official Ans. by NTA (151)

Ans. (151)

Sol. $T_8 = 11 + (8 - 1) \times 20$
 $= 11 + 140 = 151$

85. Let a line L pass through the point P(2, 3, 1) and be parallel to the line $x + 3y - 2z - 2 = 0 = x - y + 2z$. If the distance of L from the point (5, 3, 8) is α , then $3\alpha^2$ is equal to _____.

Official Ans. by NTA (158)

Ans. (158)

Sol.
$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -2 \\ 1 & -1 & 2 \end{vmatrix} = 4\hat{i} - 4\hat{j} - 4\hat{k}$$

\therefore Equation of line is $\frac{x-2}{1} = \frac{y-3}{-1} = \frac{z-1}{-1}$

Let Q be (5, 3, 8) and foot of \perp from Q on this line be R.

Now, $R \equiv (k+2, -k+3, -k+1)$

DR of QR are $(k-3, -k, -k-7)$

$\therefore (1)(k-3) + (-1)(-k) + (-1)(-k-7) = 0$

$\Rightarrow k = -\frac{4}{3}$

$\therefore \alpha^2 = \left(\frac{13}{3}\right)^2 + \left(\frac{4}{3}\right)^2 + \left(\frac{17}{3}\right)^2 = \frac{474}{9}$

$\therefore 3\alpha^2 = 158$

86. If $\int \sqrt{\sec 2x - 1} dx = \alpha \log_e \left| \cos 2x + \beta + \sqrt{\cos 2x \left(1 + \cos \frac{1}{\beta} x\right)} \right|$

+ constant, then $\beta - \alpha$ is equal to _____.

Official Ans. by NTA (1)

Ans. (1)

Sol.
$$\int \sqrt{\sec 2x - 1} dx = \int \sqrt{\frac{1 - \cos 2x}{\cos 2x}} dx$$

$$= \sqrt{2} \int \frac{\sin x}{\sqrt{2 \cos^2 x - 1}} dx$$

put $\cos x = t \Rightarrow -\sin x dx = dt$

$$= -\sqrt{2} \int \frac{dt}{\sqrt{2t^2 - 1}}$$

$$= -\ln \left| \sqrt{2} \cos x + \sqrt{\cos 2x} \right| + c$$

$$= -\frac{1}{2} \ln \left| 2 \cos^2 x + \cos 2x + 2\sqrt{\cos 2x} \cdot \sqrt{2} \cos x \right| + c$$

$$= -\frac{1}{2} \ln \left| \cos 2x + \frac{1}{2} + \sqrt{\cos 2x} \cdot \sqrt{1 + \cos 2x} \right| + c$$

$\therefore \beta = \frac{1}{2}, \alpha = -\frac{1}{2} \Rightarrow \beta - \alpha = 1$

87. If the value of real number $a > 0$ for which $x^2 - 5ax + 1 = 0$ and $x^2 - ax - 5 = 0$ have a common real roots is $\frac{3}{\sqrt{2\beta}}$ then β is equal to _____.

Official Ans. by NTA (13)

Ans. (13)

Sol. Two equations have common root

$\therefore (4a)(26a) = (-6)^2 = 36$

$\Rightarrow a^2 = \frac{9}{26} \therefore a = \frac{3}{\sqrt{26}} \Rightarrow \beta = 13$

88. The number of seven digits odd numbers, that can be formed using all the seven digits 1, 2, 2, 2, 3, 3, 5 is _____.

Official Ans. by NTA (240)

Ans. (240)

Sol. Digits are 1, 2, 2, 2, 3, 3, 5

If unit digit 5, then total numbers = $\frac{6!}{3!2!}$

If unit digit 3, then total numbers = $\frac{6!}{3!}$

If unit digit 1, then total numbers = $\frac{6!}{3!2!}$

\therefore total numbers = $60 + 60 + 120 = 240$

89. A bag contains six balls of different colours. Two balls are drawn in succession with replacement. The probability that both the balls are of the same colour is p . Next four balls are drawn in succession with replacement and the probability that exactly three balls are of the same colours is q . If $p : q = m : n$, where m and n are coprime, then $m + n$ is equal to _____.

Official Ans. by NTA (14)

Ans. (14)

Sol. $p = \frac{{}^6C_1 \times {}^6C_1}{6 \times 6} = \frac{1}{6}$

$$q = \frac{{}^6C_1 \times {}^5C_1 \times 4}{6 \times 6 \times 6 \times 6} = \frac{5}{54}$$

$$\therefore p : q = 9 : 5 \Rightarrow m + n = 14$$

90. Let A be the area of the region

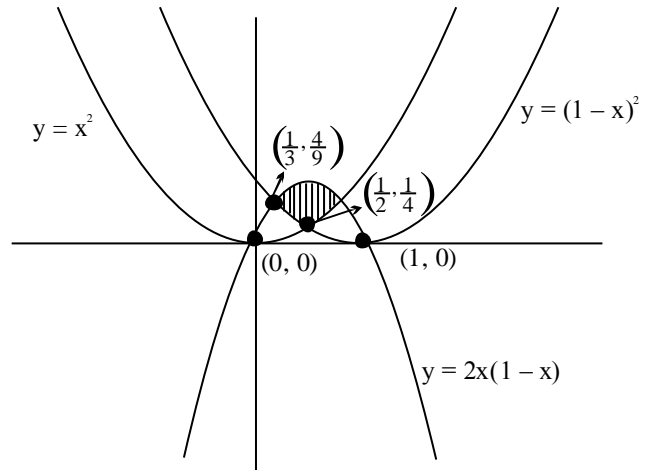
$$\{(x, y) : y \geq x^2, y \geq (1-x)^2, y \leq 2x(1-x)\}$$

Then $540A$ is equal to

Official Ans. by NTA (25)

Ans. (25)

Sol.



$$A = 2 \int_{\frac{1}{3}}^{\frac{1}{2}} (2x - 2x^2 - (1-x)^2) dx$$

$$= 2 \left[2x^2 - x^3 - x \right]_{1/3}^{1/2}$$

$$\therefore A = \frac{5}{108} \Rightarrow 540A = \frac{5}{108} \times 540 = 25$$