



64. For the system of linear equations  
 $x + y + z = 6$   
 $\alpha x + \beta y + 7z = 3$   
 $x + 2y + 3z = 14$ ,  
 which of the following is NOT true ?
- (1) If  $\alpha = \beta = 7$ , then the system has no solution
  - (2) If  $\alpha = \beta$  and  $\alpha \neq 7$  then the system has a unique solution.
  - (3) There is a unique point  $(\alpha, \beta)$  on the line  $x + 2y + 18 = 0$  for which the system has infinitely many solutions
  - (4) For every point  $(\alpha, \beta) \neq (7, 7)$  on the line  $x - 2y + 7 = 0$ , the system has infinitely many solutions.

**Official Ans. by NTA (4)**

**Ans. (4)**

- Sol.** By equation 1 and 3  $y + 2z = 8$   
 $y = 8 - 2z$   
 And  $x = -2 + z$

Now putting in equation 2

$$\alpha(z-2) + \beta(-2z+8) + 7z = 3$$

$$\Rightarrow (\alpha - 2\beta + 7)z = 2\alpha - 8\beta + 3$$

So equations have unique solution if  
 $\alpha - 2\beta + 7 \neq 0$

And equations have no solution if  
 $\alpha - 2\beta + 7 = 0$  and  $2\alpha - 8\beta + 3 \neq 0$

And equations have infinite solution if  
 $\alpha - 2\beta + 7 = 0$  and  $2\alpha - 8\beta + 3 = 0$

65. Let the shortest distance between the lines  
 $L : \frac{x-5}{-2} = \frac{y-\lambda}{0} = \frac{z+\lambda}{1}$ ,  $\lambda \geq 0$  and  $L_1 : x + 1 = y - 1 = 4 - z$  be  $2\sqrt{6}$ . If  $(\alpha, \beta, \gamma)$  lies on  $L$ , then which of the following is NOT possible?

- (1)  $\alpha + 2\gamma = 24$
- (2)  $2\alpha + \gamma = 7$
- (3)  $2\alpha - \gamma = 9$
- (4)  $\alpha - 2\gamma = 19$

**Official Ans. by NTA (1)**

**Ans. (1)**

**Sol.**  $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix} = -\hat{i} - \hat{j} - 2\hat{k}$

$$\vec{a}_2 - \vec{a}_1 = 6\hat{i} + (\lambda - 1)\hat{j} + (-\lambda - 4)\hat{k}$$

$$2\sqrt{6} = \left| \frac{-6 - \lambda + 1 + 2\lambda + 8}{\sqrt{1+1+4}} \right|$$

$$|\lambda + 3| = 12 \Rightarrow \lambda = 9, -15$$

$$\alpha = -2k + 5, \gamma = k - \lambda \text{ where } k \in \mathbb{R}$$

$$\Rightarrow \alpha + 2\gamma = 5 - 2\lambda = -13, 35$$

66. Let  $y = f(x)$  represent a parabola with focus  $\left(-\frac{1}{2}, 0\right)$  and directrix  $y = -\frac{1}{2}$ .

Then

$$S = \left\{ x \in \mathbb{R} : \tan^{-1}(\sqrt{f(x)}) + \sin^{-1}(\sqrt{f(x)+1}) = \frac{\pi}{2} \right\} :$$

- (1) contains exactly two elements
- (2) contains exactly one element
- (3) is an infinite set
- (4) is an empty set

**Official Ans. by NTA (1)**

**Ans. (1)**

**Sol.**  $\left(x + \frac{1}{2}\right)^2 = \left(y + \frac{1}{4}\right)$

$$y = (x^2 + x)$$

$$\tan^{-1} \sqrt{x(x+1)} + \sin^{-1} \sqrt{x^2 + x + 1} = \frac{\pi}{2}$$

$$0 \leq x^2 + x + 1 \leq 1$$

$$x^2 + x \leq 0 \quad \dots(1)$$

$$\text{Also } x^2 + x \geq 0 \quad \dots(2)$$

$$\therefore x^2 + x = 0 \Rightarrow x = 0, -1$$

S contains 2 element.

67. Let  $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{pmatrix}$ . Then the sum of the

diagonal elements of the matrix  $(A + I)^{11}$  is equal to:

- (1) 6144
- (2) 4094
- (3) 4097
- (4) 2050

**Official Ans. by NTA (3)**

**Ans. (3)**

**Sol.**  $A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix} = A$$

$\Rightarrow A^3 = A^4 = \dots = A$

$(A + I)^{11} = {}^{11}C_0 A^{11} + {}^{11}C_1 A^{10} + \dots + {}^{11}C_{10} A + {}^{11}C_{11} I$

$= ({}^{11}C_0 + {}^{11}C_1 + \dots + {}^{11}C_{10}) A + I$

$= (2^{11} - 1) A + I = 2047A + I$

$\therefore$  Sum of diagonal elements  $= 2047(1 + 4 - 3) + 3$

$= 4094 + 3 = 4097$

**68.** Let R be a relation on  $N \times N$  defined by (a, b) R

(c, d) if and only if  $ad(b - c) = bc(a - d)$ . Then R is

- (1) symmetric but neither reflexive nor transitive
- (2) transitive but neither reflexive nor symmetric
- (3) reflexive and symmetric but not transitive
- (4) symmetric and transitive but not reflexive

**Official Ans. by NTA (1)**

**Ans. (1)**

**Sol.** (a, b) R (c, d)  $\Rightarrow ad(b - c) = bc(a - d)$

Symmetric:

(c, d) R (a, b)  $\Rightarrow cb(d - a) = da(c - b) \Rightarrow$

Symmetric

Reflexive:

(a, b) R (a, b)  $\Rightarrow ab(b - a) \neq ba(a - b) \Rightarrow$

Not reflexive

Transitive: (2,3) R (3,2) and (3,2) R (5,30) but

$((2,3), (5,30)) \notin R \Rightarrow$  Not transitive

**69.** Let

$$y = f(x) = \sin^3 \left( \frac{\pi}{3} \left( \cos \left( \frac{\pi}{3\sqrt{2}} (-4x^3 + 5x^2 + 1)^{\frac{3}{2}} \right) \right) \right)$$

. Then, at  $x = 1$ ,

(1)  $2y' + \sqrt{3}\pi^2 y = 0$

(2)  $2y' + 3\pi^2 y = 0$

(3)  $\sqrt{2}y' - 3\pi^2 y = 0$

(4)  $y' + 3\pi^2 y = 0$

**Official Ans. by NTA (2)**

**Ans. (2)**

**Sol.**  $y = \sin^3(\pi/3 \cos g(x))$

$$g(x) = \frac{\pi}{3\sqrt{2}} (-4x^3 + 5x^2 + 1)^{3/2}$$

$g(1) = 2\pi/3$

$$y' = 3 \sin^2 \left( \frac{\pi}{3} \cos g(x) \right) \times \cos \left( \frac{\pi}{3} \cos g(x) \right)$$

$$\times \frac{\pi}{3} (-\sin g(x)) g'(x)$$

$$y'(1) = 3 \sin^2 \left( -\frac{\pi}{6} \right) \cdot \cos \left( \frac{\pi}{6} \right) \cdot \frac{\pi}{3} \left( -\sin \frac{2\pi}{3} \right) g'(1)$$

$$g'(x) = \frac{\pi}{3\sqrt{2}} (-4x^3 + 5x^2 + 1)^{1/2} (-12x^2 + 10x)$$

$$g'(1) = \frac{\pi}{2\sqrt{2}} (\sqrt{2}) (-2) = -\pi$$

$$y'(1) = \frac{3}{4} \cdot \frac{\sqrt{3}}{2} \cdot \frac{\pi}{3} \left( \frac{-\sqrt{3}}{2} \right) (-\pi) = \frac{3\pi^2}{16}$$

$$y(1) = \sin^3 (\pi/3 \cos 2\pi/3) = -\frac{1}{8}$$

$$2y'(1) + 3\pi^2 y(1) = 0$$

**70.** If the sum and product of four positive consecutive terms of a G.P., are 126 and 1296, respectively, then the sum of common ratios of all such GPs is

(1) 7 (2)  $\frac{9}{2}$

(3) 3 (4) 14

**Official Ans. by NTA (1)**

**Ans. (1)**

**Sol.**  $a, ar, ar^2, ar^3$  ( $a, r > 0$ )

$$a^4 r^6 = 1296$$

$$a^2 r^3 = 36$$

$$a = \frac{6}{r^{3/2}}$$

$$a + ar + ar^2 + ar^3 = 126$$

$$\frac{1}{r^{3/2}} + \frac{r}{r^{3/2}} + \frac{r^2}{r^{3/2}} + \frac{r^3}{r^{3/2}} = \frac{126}{6} = 21$$

$$(r^{-3/2} + r^{3/2}) + (r^{1/2} + r^{-1/2}) = 21$$

$$r^{1/2} + r^{-1/2} = A$$

$$r^{-3/2} + r^{3/2} + 3A = A^3$$

$$A^3 - 3A + A = 21$$

$$A^3 - 2A = 21$$

$$A = 3$$

$$\sqrt{r} + \frac{1}{\sqrt{r}} = 3$$

$$r + 1 = 3\sqrt{r}$$

$$r^2 + 2r + 1 = 9r$$

$$r^2 - 7r + 1 = 0$$

**71.** The number of real roots of the equation

$$\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 6}, \text{ is:}$$

(1) 0

(2) 1

(3) 3

(4) 2

**Official Ans. by NTA (2)**

**Ans. (2)**

**Sol.**  $\sqrt{(x-1)(x-3)} + \sqrt{(x-3)(x+3)}$

$$= \sqrt{4\left(x - \frac{12}{4}\right)\left(x - \frac{2}{4}\right)}$$

$$\Rightarrow \sqrt{x-3} = 0 \Rightarrow x = 3 \text{ which is in domain}$$

or

$$\sqrt{x-1} + \sqrt{x+3} = \sqrt{4x-2}$$

$$2\sqrt{(x-1)(x+3)} = 2x-4$$

$$x^2 + 2x - 3 = x^2 - 4x + 4$$

$$6x = 7$$

$$x = 7/6 \text{ (rejected)}$$

**72.** Let a differentiable function  $f$  satisfy

$$f(x) + \int_3^x \frac{f(t)}{t} dt = \sqrt{x+1}, x \geq 3. \text{ Then } 12f(8) \text{ is}$$

equal to:

(1) 34

(2) 19

(3) 17

(4) 1

**Official Ans. by NTA (3)**

**Ans. (3)**

**Sol.** Differentiate w.r.t.  $x$

$$f'(x) + \frac{f(x)}{x} = \frac{1}{2\sqrt{x+1}}$$

$$\text{I.F.} = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$$

$$xf(x) = \int \frac{x}{2\sqrt{x+1}} dx$$

$$x + 1 = t^2$$

$$= \int \frac{t^2 - 1}{2t} 2t dt$$

$$xf(x) = \frac{t^3}{3} - t + c$$

$$xf(x) = \frac{(x+1)^{3/2}}{3} - \sqrt{x+1} + c$$

Also putting  $x = 3$  in given equation  $f(3) + 0 = \sqrt{4}$

$$f(3) = 2$$

$$\Rightarrow C = 8 - \frac{8}{3} = \frac{16}{3}$$

$$f(x) = \frac{(x+1)^{3/2}}{3} - \sqrt{x+1} + \frac{16}{3}$$

$$f(8) = \frac{9-3 + \frac{16}{3}}{8} = \frac{34}{24}$$

$$\Rightarrow 12f(8) = 17$$

73. If the domain of the function  $f(x) = \frac{[x]}{1+x^2}$ , where  $[x]$  is greatest integer  $\leq x$ , is  $[2, 6)$ , then its range is

(1)  $\left(\frac{5}{26}, \frac{2}{5}\right] - \left\{\frac{9}{29}, \frac{27}{109}, \frac{18}{89}, \frac{9}{53}\right\}$

(2)  $\left(\frac{5}{26}, \frac{2}{5}\right]$

(3)  $\left(\frac{5}{37}, \frac{2}{5}\right] - \left\{\frac{9}{29}, \frac{27}{109}, \frac{18}{89}, \frac{9}{53}\right\}$

(4)  $\left(\frac{5}{37}, \frac{2}{5}\right]$

Official Ans. by NTA (4)

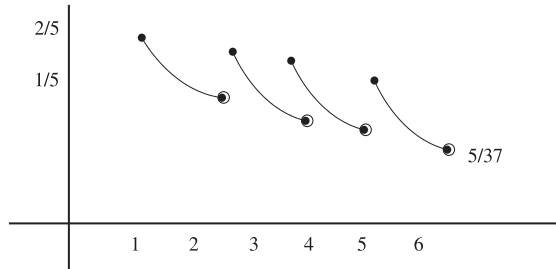
Ans. (4)

Sol.  $f(x) = \frac{2}{1+x^2} \quad x \in [2, 3)$

$f(x) = \frac{3}{1+x^2} \quad x \in [3, 4)$

$f(x) = \frac{4}{1+x^2} \quad x \in [4, 5)$

$f(x) = \frac{5}{1+x^2} \quad x \in [5, 6)$



$\left(\frac{5}{37}, \frac{2}{5}\right]$

74. Let  $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$ , and  $\vec{b}$  and  $\vec{c}$  be two nonzero vectors such that  $|\vec{a} + \vec{b} + \vec{c}| = |\vec{a} + \vec{b} - \vec{c}|$  and  $\vec{b} \cdot \vec{c} = 0$ . Consider the following two statements:

(A)  $|\vec{a} + \lambda\vec{c}| \geq |\vec{a}|$  for all  $\lambda \in \mathbb{R}$ .

(B)  $\vec{a}$  and  $\vec{c}$  are always parallel

(1) only (B) is correct

(2) neither (A) nor (B) is correct

(3) only (A) is correct

(4) both (A) and (B) are correct.

Official Ans. by NTA (3)

Ans. (3)

Sol.  $|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a} + \vec{b} - \vec{c}|^2$

$2\vec{a} \cdot \vec{b} + 2\vec{b} \cdot \vec{c} + 2\vec{c} \cdot \vec{a} = 2\vec{a} \cdot \vec{b} - 2\vec{b} \cdot \vec{c} - 2\vec{c} \cdot \vec{a}$

$4\vec{a} \cdot \vec{c} = 0$

B is incorrect

$|\vec{a} + \lambda\vec{c}|^2 \geq |\vec{a}|^2$

$\lambda^2 c^2 \geq 0$

True  $\forall \lambda \in \mathbb{R}$  (A) is correct.

75. Let  $\alpha \in (0, 1)$  and  $\beta = \log_e(1 - \alpha)$ . Let

$P_n(x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \dots + \frac{x^n}{n}, x \in (0, 1)$ .

Then the integral  $\int_0^\alpha \frac{t^{50}}{1-t} dt$  is equal to

(1)  $\beta - P_{50}(\alpha)$

(2)  $-(\beta + P_{50}(\alpha))$

(3)  $P_{50}(\alpha) - \beta$

(4)  $\beta + P_{50}(\alpha)$

Official Ans. by NTA (2)

Ans. (2)

Sol.  $\int_0^\alpha \frac{t^{50} - 1 + 1}{1-t} dt = -\int_0^\alpha (1+t+\dots+t^{49}) dt + \int_0^\alpha \frac{1}{1-t} dt$

$= -\left(\frac{\alpha^{50}}{50} + \frac{\alpha^{49}}{49} + \dots + \frac{\alpha^1}{1}\right) + \left(\frac{\ln(1-f)}{-1}\right)_0^\alpha$

$= -P_{50}(\alpha) - \ln(1-\alpha)$

$= -P_{50}(\alpha) - \beta$

76. If  $\sin^{-1} \frac{\alpha}{17} + \cos^{-1} \frac{4}{5} - \tan^{-1} \frac{77}{36} = 0, 0 < \alpha < 13$ ,

then  $\sin^{-1}(\sin \alpha) + \cos^{-1}(\cos \alpha)$  is equal to

(1)  $\pi$

(2) 16

(3) 0

(4)  $16 - 5\pi$

Official Ans. by NTA (1)

Ans. (1)

**Sol.**  $\cos^{-1} \frac{4}{5} = \tan^{-1} \frac{3}{4}$

$$\therefore \sin^{-1} \frac{\alpha}{17} = \tan^{-1} \frac{77}{36} - \tan^{-1} \frac{3}{4} = \tan^{-1} \left( \frac{\frac{77}{36} - \frac{3}{4}}{1 + \frac{77}{36} \cdot \frac{3}{4}} \right)$$

$$\sin^{-1} \frac{\alpha}{17} = \tan^{-1} \frac{8}{15} = \sin^{-1} \frac{8}{17}$$

$$\Rightarrow \frac{\alpha}{17} = \frac{8}{17} \Rightarrow \alpha = 8$$

$$\therefore \sin^{-1}(\sin 8) + \cos^{-1}(\cos 8)$$

$$= 3\pi - 8 + 8 - 2\pi$$

$$= \pi$$

77. Let a circle  $C_1$  be obtained on rolling the circle  $x^2 + y^2 - 4x - 6y + 11 = 0$  upwards 4 units on the tangent  $T$  to it at the point  $(3, 2)$ . Let  $C_2$  be the image of  $C_1$  in  $T$ . Let  $A$  and  $B$  be the centers of circles  $C_1$  and  $C_2$  respectively, and  $M$  and  $N$  be respectively the feet of perpendiculars drawn from  $A$  and  $B$  on the  $x$ -axis. Then the area of the trapezium  $AMNB$  is :

(1)  $2(2 + \sqrt{2})$

(2)  $4(1 + \sqrt{2})$

(3)  $3 + 2\sqrt{2}$

(4)  $2(1 + \sqrt{2})$

**Official Ans. by NTA (2)**

**Ans. (2)**

**Sol.**  $C = (2, 3), r = \sqrt{2}$

$$\text{Centre of } G = A = 2 + 4 \frac{1}{\sqrt{2}},$$

$$3 + \frac{4}{\sqrt{2}} = (2 + 2\sqrt{2}, 3 + 2\sqrt{2})$$

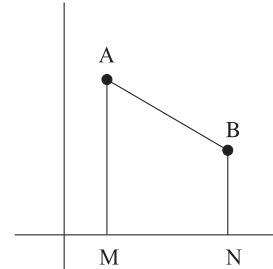
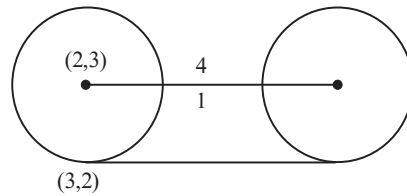
$$A(2 + 2\sqrt{2}, 3 + 2\sqrt{2})$$

$$B(4 + 2\sqrt{2}, 1 + 2\sqrt{2})$$

$$\frac{x - (2 + 2\sqrt{2})}{1} = \frac{y - (3 + 2\sqrt{2})}{-1} = 2$$

$\therefore$  area of trapezium:

$$\frac{1}{2}(4 + 4\sqrt{2})2 = 4(1 + \sqrt{2})$$



78. (S1)  $(p \Rightarrow q) \vee (p \wedge (\sim q))$  is a tautology

(S2)  $((\sim p) \Rightarrow (\sim q)) \wedge ((\sim p) \vee q)$  is a

Contradiction. Then

(1) only (S2) is correct

(2) both (S1) and (S2) are correct

(3) both (S1) and (S2) are wrong

(4) only (S1) is correct

**Official Ans. by NTA (4)**

**Ans. (4)**

**Sol.**

p	q	$p \Rightarrow q$	$\sim q$	$p \wedge \sim q$	$(p \Rightarrow q) \vee (p \wedge \sim q)$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	T	F	T

$\sim p$	$\sim q$	$\sim p \Rightarrow \sim q$	$\sim p \vee \sim q$	$((\sim p) \Rightarrow (\sim q)) \wedge (\sim p) \vee q$
F	F	T	T	T
F	T	T	F	F
T	F	F	T	F
T	T	T	T	T

79. The value of  $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{(2+3\sin x)}{\sin x(1+\cos x)} dx$  is equal to

- (1)  $\frac{7}{2} - \sqrt{3} - \log_e \sqrt{3}$   
 (2)  $-2 + 3\sqrt{3} + \log_e \sqrt{3}$   
 (3)  $\frac{10}{3} - \sqrt{3} + \log_e \sqrt{3}$   
 (4)  $\frac{10}{3} - \sqrt{3} - \log_e \sqrt{3}$

**Official Ans. by NTA (3)**

**Ans. (3)**

**Sol.** 
$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \left( \frac{2+3\sin x}{\sin x(1+\cos x)} \right) dx = 2 \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{dx}{\sin x + \sin x \cos x} + 3$$

$$3 \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{dx}{1+\cos x}$$

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{dx}{1+\cos x} = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1-\cos x}{\sin^2 x} dx$$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (\operatorname{cosec}^2 x - \cot x \operatorname{cosec} x) dx$$

$$= (\operatorname{cosec} x - \cot x) \Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}} = (1) - \left( \frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}} \right) = 1 - \frac{1}{\sqrt{3}}$$

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{dx}{\sin x(1+\cos x)} =$$

$$\int \frac{dx}{(2 \tan x / 2) (1 + 1 - \tan^2 x / 2)}$$

$$= \int \frac{(1 + \tan^2 x / 2) \sec^2 x / 2 dx}{2 \tan x / 2}$$

$\tan x / 2 = t \quad \sec^2 x / 2 dx = dt$

$$\frac{1}{2} \int \left( \frac{1+t^2}{t} \right) dt = \frac{1}{2} \left[ \ell n t + \frac{t^2}{2} \right]_{\frac{1}{\sqrt{3}}}^1$$

$$= \frac{1}{2} \left[ \left( 0 + \frac{1}{2} \right) - \left( \ell n \frac{1}{\sqrt{3}} + \frac{1}{6} \right) \right] = \left( \frac{1}{3} + \ell n \sqrt{3} \right) \frac{1}{2}$$

$$= \left( \frac{1}{6} + \frac{1}{2} \ell n \sqrt{3} \right)$$

$$2 \left( \frac{1}{6} + \frac{1}{2} \ell n \sqrt{3} \right) + 3 \left( 1 - \frac{1}{\sqrt{3}} \right)$$

$$= \frac{1}{3} + \ell n \sqrt{3} + 3 - \sqrt{3} = \frac{10}{3} + \ell n \sqrt{3} - \sqrt{3}$$

80. A bag contains 6 balls. Two balls are drawn from it at random and both are found to be black. The probability that the bag contains at least 5 black balls is

- (1)  $\frac{5}{7}$  (2)  $\frac{2}{7}$   
 (3)  $\frac{3}{7}$  (4)  $\frac{5}{6}$

**Official Ans. by NTA (1)**

**Ans. (1)**

**Sol.** 
$$\frac{{}^5C_2 + {}^6C_2}{{}^2C_2 + {}^3C_2 + {}^4C_2 + {}^5C_2 + {}^6C_2} = \frac{10+15}{1+3+6+10+15}$$

$$= \frac{25}{35} = \frac{5}{7}$$

**SECTION-B**

81. Let 5 digit numbers be constructed using the digits 0, 2, 3, 4, 7, 9 with repetition allowed, and are arranged in ascending order with serial numbers. Then the serial number of the number 42923 is \_\_\_\_\_.

**Official Ans. by NTA (2997)**

**Ans. (2997)**

**Sol.**  $2 \frac{++++}{6666} = 1296$

 $3 \frac{++++}{6666} = 1296$ 
 $40 \frac{+++}{666} = 216$ 
 $420 \frac{++}{66} = 36$ 
 $422 \frac{+}{66} = 36$ 
 $423 \frac{+}{66} = 36$ 
 $424 \frac{+}{66} = 36$ 
 $427 \frac{++}{66} = 36$ 
 $429 \frac{0+}{66} = 6$ 
 $42920 = 1$ 
 $42922 = 1$ 
 $42923 = 1$ 
 $= 2997$

82. Let  $a_1, a_2, \dots, a_n$  be in A.P. If  $a_5 = 2a_7$  and  $a_{11} = 18$ , then

$$12 \left( \frac{1}{\sqrt{a_{10}} + \sqrt{a_{11}}} + \frac{1}{\sqrt{a_{11}} + \sqrt{a_{12}}} + \dots + \frac{1}{\sqrt{a_{17}} + \sqrt{a_{18}}} \right)$$

is equal to \_\_\_\_\_.

**Official Ans. by NTA (8)**

**Ans. (8)**

**Sol.**  $2a_7 = a_5$  (given)

$$2(a_1 + 6d) = a_1 + 4d$$

$$a_1 + 8d = 0 \quad \dots\dots(1)$$

$$a_1 + 10d = 18 \quad \dots\dots(2)$$

By (1) and (2) we get  $a_1 = -72, d = 9$

$$a_{18} = a_1 + 17d = -72 + 153 = 81$$

$$a_{10} = a_1 + 9d = 9$$

$$12 \left( \frac{\sqrt{a_{11}} - \sqrt{a_{10}}}{d} + \frac{\sqrt{a_{12}} - \sqrt{a_{11}}}{d} + \dots + \frac{\sqrt{a_{18}} - \sqrt{a_{17}}}{d} \right)$$

$$12 \left( \frac{\sqrt{a_{18}} - \sqrt{a_{10}}}{d} \right) = \frac{12(9-3)}{9} = \frac{12 \times 6}{6} = 8$$

83. Let  $\theta$  be the angle between the planes

$$P_1 = \vec{r} \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 9 \text{ and } P_2 = \vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 15.$$

Let L be the line that meets  $P_2$  at the point

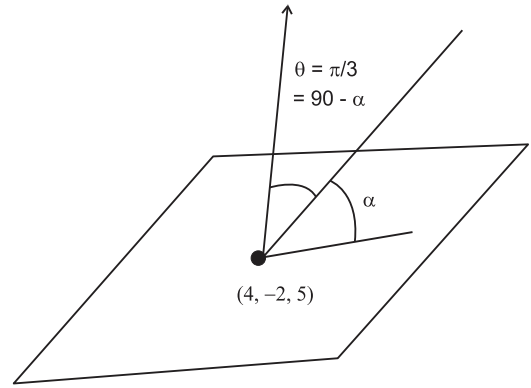
$(4, -2, 5)$  and makes an angle  $\theta$  with the normal of

$P_2$ . If  $\alpha$  is the angle between L and  $P_2$  then

$(\tan^2 \theta)(\cot^2 \alpha)$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (9)**

**Ans. (9)**



$$\cos \theta = \frac{(\hat{i} + \hat{j} + 2\hat{k}) \cdot (2\hat{i} - \hat{j} + \hat{k})}{6} = \frac{2 - 1 + 2}{6} = \frac{1}{2}$$

$$\theta = \pi/3 \quad \alpha = \pi/6$$

$$(\tan^2 \theta)(\cot^2 \alpha)$$

$$(3)(3) = 9$$

84. Let  $\alpha > 0$ , be the smallest number such that the

expansion of  $\left(x^{\frac{2}{3}} + \frac{2}{x^3}\right)^{30}$  has a term  $\beta x^{-\alpha}, \beta \in \mathbb{N}$ .

Then  $\alpha$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (2)**

**Ans. (2)**

**Sol.**  $T_{r+1} = {}^{30}C_r (x^{2/3})^{30-r} \left(\frac{2}{x^3}\right)^r$

$$= {}^{30}C_r \cdot 2^r \cdot x^{\frac{60-11r}{3}}$$

$$\frac{60-11r}{3} < 0 \Rightarrow 11r > 60 \Rightarrow r > \frac{60}{11} \Rightarrow r = 6$$

$$T_7 = {}^{30}C_6 \cdot 2^6 \cdot x^{-2}$$

We have also observed  $\beta = {}^{30}C_6 (2)^6$  is a natural number.

$\therefore \alpha = 2$

85. Let  $\vec{a}$  and  $\vec{b}$  be two vector such that  $|\vec{a}| = \sqrt{14},$

$|\vec{b}| = \sqrt{6}$  and  $|\vec{a} \times \vec{b}| = \sqrt{48}$ . Then  $(\vec{a} \cdot \vec{b})^2$  is equal to

\_\_\_\_\_.

**Official Ans. by NTA (36)**

**Ans. (36)**



**Sol.**  $|\vec{a}| = \sqrt{14}$ ,  $|\vec{b}| = \sqrt{6}$   $|\vec{a} \times \vec{b}| = \sqrt{48}$

$$|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = |\vec{a}|^2 \times |\vec{b}|^2$$

$$\Rightarrow (\vec{a} \cdot \vec{b})^2 = 84 - 48 = 36$$

**86.** Let the line  $L: \frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{1}$  intersect the plane  $2x + y + 3z = 16$  at the point P. Let the point Q be the foot of perpendicular from the point  $R(1, -1, -3)$  on the line L. If  $\alpha$  is the area of triangle PQR. then  $\alpha^2$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (180)**

**Ans. (180)**

**Sol.** Any point on L  $((2\lambda + 1), (-\lambda - 1), (\lambda + 3))$

$$2(2\lambda + 1) + (-\lambda - 1) + 3(\lambda + 3) = 16$$

$$6\lambda + 10 = 16 \Rightarrow \lambda = 1$$

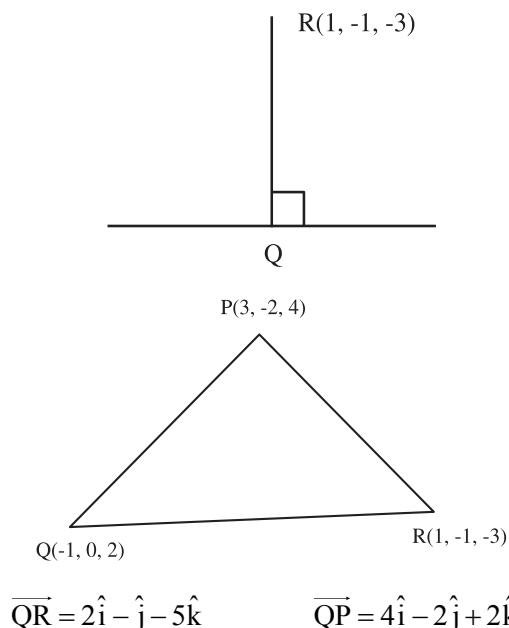
$$\therefore P = (3, -2, 4)$$

$$\text{DR of QR} = \langle 2\lambda, -\lambda, \lambda + 6 \rangle$$

$$\text{DR of L} = \langle 2, -1, 1 \rangle$$

$$4\lambda + \lambda + \lambda + 6 = 0 \quad 6\lambda + 6 = 0 \Rightarrow \lambda = -1$$

$$Q = (-1, 0, 2)$$



$$\vec{QR} \times \vec{QP} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & -5 \\ 4 & -2 & 2 \end{vmatrix} = -12\hat{i} - 24\hat{j}$$

$$\alpha = \frac{1}{2} \times \sqrt{144 + 576} \Rightarrow \alpha^2 = \frac{720}{4} = 180$$

**87.** The remainder on dividing  $5^{99}$  by 11 is \_\_\_\_\_.

**Official Ans. by NTA (9)**

**Ans. (9)**

**Sol.**  $5^{99} = 5^4 \cdot 5^{95}$   
 $= 625[5^5]^{19}$   
 $= 625[3125]^{19}$   
 $= 625[3124+1]^{19}$   
 $= 625[11k \times 19 + 1]$

$$= 625 \times 11k \times 19 + 625$$

$$= 11k_1 + 616 + 9$$

$$= 11(k_2) + 9$$

$$\text{Remainder} = 9$$

**88.** If the variance of the frequency distribution

$x_i$	2	3	4	5	6	7	8
Frequency $f_i$	3	6	16	$\alpha$	9	5	6

**Official Ans. by NTA (5)**

**Ans. (5)**

**Sol.**

$x_i$	$f_i$	$d_i = x_i - 5$	$f_i d_i^2$	$f_i d_i$
2	3	-3	27	-9
3	6	-2	24	-12
4	16	-1	16	-16
5	$\alpha$	0	0	0
6	9	1	9	9
7	5	2	20	10
8	6	3	54	18

$$\sigma_x^2 = \sigma_d^2 = \frac{\sum f_i d_i^2}{\sum f_i} - \left( \frac{\sum f_i d_i}{\sum f_i} \right)^2$$

$$= \frac{150}{45 + \alpha} - 0 = 3$$

$$\Rightarrow 150 = 135 + 3\alpha$$

$$\Rightarrow 3\alpha = 15 \Rightarrow \alpha = 5$$

89. Let for  $x \in \mathbb{R}$

$$f(x) = \frac{x + |x|}{2} \text{ and } g(x) = \begin{cases} x, & x < 0 \\ x^2 & x \geq 0 \end{cases}$$

Then area bounded by the curve  $y = (f \circ g)(x)$  and the lines  $y = 0, 2y - x = 15$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (72)**

**Ans. (72)**

**Sol.**  $f(x) = \frac{x + |x|}{2} = \begin{cases} x & x \geq 0 \\ 0 & x < 0 \end{cases}$

$$g(x) = \begin{cases} x^2 & x \geq 0 \\ x & x < 0 \end{cases}$$

$$f \circ g(x) = f[g(x)] = \begin{cases} g(x) & g(x) \geq 0 \\ 0 & g(x) < 0 \end{cases}$$

$$f \circ g(x) = \begin{cases} x^2 & x \geq 0 \\ 0 & x < 0 \end{cases}$$

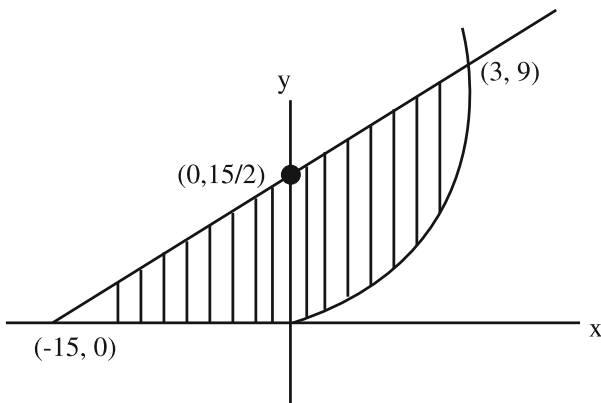
$$2y - x = 15$$

$$A = \int_0^3 \left( \frac{x+15}{2} - x^2 \right) dx + \frac{1}{2} \times \frac{15}{2} \times 15$$

$$\left. \frac{x^2}{4} + \frac{15x}{2} - \frac{x^3}{3} \right|_0^3 + \frac{225}{4}$$

$$= \frac{9}{4} + \frac{45}{2} - 9 + \frac{225}{4} = \frac{99 - 36 + 225}{4}$$

$$= \frac{288}{4} = 72$$



90. Number of 4-digit numbers that are less than or equal to 2800 and either divisible by 3 or by 11, is equal to \_\_\_\_\_.

**Official Ans. by NTA (710)**

**Ans. (710)**

**Sol.** 1000 – 2799

Divisible by 3

$$1002 + (n - 1) 3 = 2799$$

$$n = 600$$

Divisible by 11

$$1 - 2799 \rightarrow \left[ \frac{2799}{11} \right] = [254] = 254$$

$$1 - 999 = \left[ \frac{999}{11} \right] = 90$$

$$1000 - 2799 = 254 - 90 = 164$$

Divisible by 33

$$1 - 2799 \rightarrow \left[ \frac{2799}{33} \right] = 84$$

$$1 - 999 \rightarrow \left[ \frac{999}{33} \right] = 30$$

$$1000 - 2799 \rightarrow 54$$

$$\therefore n(3) + n(11) - n(33)$$

$$600 + 164 - 54 = 710$$