FINAL JEE(Advanced) EXAMINATION – 2023

(Held On Sunday 04th June, 2023)

PAPER-1 | **TEST PAPER WITH SOLUTION**

MATHEMATICS

SECTION-1 : (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;

Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;

Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;

Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : −2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

choosing ONLY (A), (B) and (D) will get +4 marks;

choosing ONLY (A) and (B) will get +2 marks;

choosing ONLY (A) and (D) will get +2 marks;

choosing ONLY (B) and (D) will get +2 marks;

choosing ONLY (A) will get +1 marks;

choosing ONLY (B) will get +1 marks;

choosing ONLY (D) will get +1 marks;

choosing no option (i.e. the question is unanswered) will get 0 marks; and

choosing any other combination of options will get –2 marks.

1. Let $S = (0, 1) \cup (1, 2) \cup (3, 4)$ and $T = \{0, 1, 2, 3\}$. Then which of the following statements is(are) true ?

(A) There are infinitely many functions from S to T

(B) There are infinitely many strictly increasing functions from S to T

(C) The number of continuous functions from S to T is at most 120

(D) Every continuous function from S to T is differentiable

Ans. (ACD)

Sol. $S = (0, 1) \cup (1, 2) \cup (3, 4)$

$T = \{0, 1, 2, 3\}$

Number of functions :

Each element of S have 4 choice

Let n be the number of element in set S.

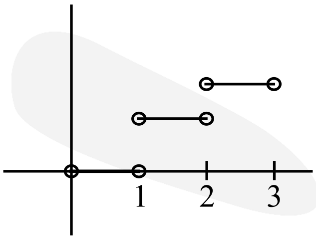
Number of function = 4^n

Here $n \to \infty$

$\Rightarrow$ Option (A) is correct.

Option (B) is incorrect (obvious)

(C)



For continuous function

Each interval will have 4 choices.

$\Rightarrow$ Number of continuous functions

$= 4 \times 4 \times 4 = 64$

$\Rightarrow$ Option (C) is correct.

(D) Every continuous function is piecewise constant functions

$\Rightarrow$ Differentiable.

Option (D) is correct.

2. Let T_1 and T_2 be two distinct common tangents to the ellipse $E : \frac{x^2}{6} + \frac{y^2}{3} = 1$ and the parabola $P : y^2 = 12x$. Suppose that the tangent T_1 touches P and E at the point A_1 and A_2, respectively and the tangent T_2 touches P and E at the points A_4 and A_3, respectively. Then which of the following statements is(are) true?

(A) The area of the quadrilateral $A_1A_2A_3A_4$ is 35 square units

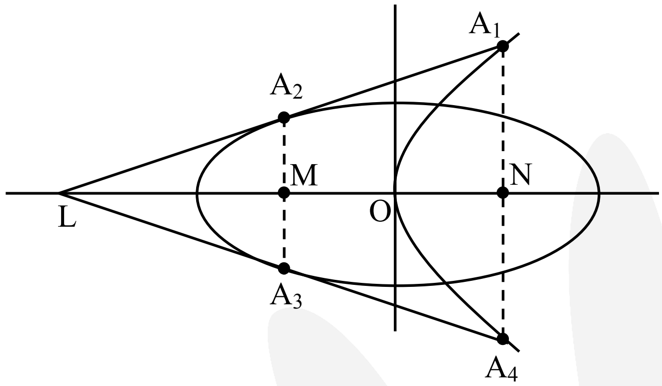
(B) The area of the quadrilateral $A_1A_2A_3A_4$ is 36 square units

(C) The tangents T_1 and T_2 meet the x-axis at the point (–3, 0)

(D) The tangents T_1 and T_2 meet the x-axis at the point (–6, 0)

Ans. (AC)

Sol.



$y = mx + \frac{3}{m}$

$C^2 = a^2m^2 + b^2$

$\frac{9}{m^2} = 6m^2 + 3 \qquad \Rightarrow m^2 = 1$

T_1 & T_2

$y = x + 3,\ y = -x - 3$

Cuts x-axis at (–3, 0)

$A_1(3, 6) \qquad A_4(3, -6)$

$A_2(-2, 1) \qquad A_3(-2, -1)$

$A_1A_4 = 12,\quad A_2A_3 = 2,\quad MN = 5$

Area $= \frac{1}{2}(12+2)\times 5 = 35$ sq.unit

Ans. (A, C)

3. Let $f : [0, 1] \to [0, 1]$ be the function defined by $f(x) = \frac{x^3}{3} - x^2 + \frac{5}{9}x + \frac{17}{36}$. Consider the square region $S = [0, 1]\times[0,1]$. Let $G = \{(x, y)\in S : y > f(x)\}$ be called the green region and $R = \{(x, y)\in S : y < f(x)\}$ be called the red region. Let $L_h = \{(x, h)\in S : x\in[0, 1]\}$ be the horizontal line drawn at a height $h \in[0, 1]$. Then which of the following statements is(are) ture?

(A) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the green region above the line L_h equals the area of the green region below the line L_h

(B) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the red region above the line L_h equals the area of the red region below the line L_h

(C) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the green region above the line L_h equals the area of the red region below the line L_h

(D) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the red region above the line L_h equals the area of the green region below the line L_h

Ans. (BCD)

Sol. $f(x)=\frac{x^3}{3}-x^2+\frac{5x}{9}+\frac{17}{36}$

$f'(x)=x^2-2x+\frac{5}{9}$

$f'(x) = 0$ at $x = \frac{1}{3}$ in [0, 1]

A_R = Area of Red region

A_G = Area of Green region

$A_R=\int_0^1 f(x)dx=\frac{1}{2}$

Total area = 1

$\Rightarrow A_G = \frac{1}{2}$

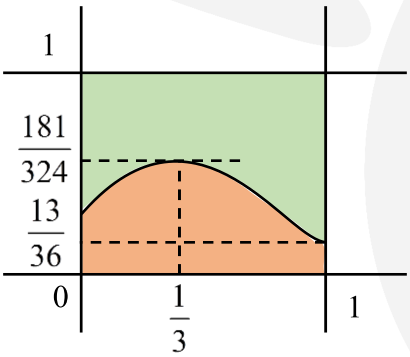
$\int_0^1 f(x)dx=\frac{1}{2}$

$A_G = A_R$

$f(0)=\frac{17}{36}$

$f(1)=\frac{13}{36}\approx 0.36$

$f\left(\frac{1}{3}\right)=\frac{181}{324}\approx 0.558$



(A) Correct when $h = \frac{3}{4}$ but $h\in\left[\frac{1}{4},\frac{2}{3}\right]$

$\Rightarrow$ (A) is incorrect

(B) Correct when $h=\frac{1}{4}$

$\Rightarrow$ (B) is correct

(C) When $h=\frac{181}{324}, A_R=\frac{1}{2}, A_G<\frac{1}{2}$

$h=\frac{13}{36}, A_R<\frac{1}{2}, A_G=\frac{1}{2}$

$\Rightarrow A_R = A_G$ for some $h\in\left(\frac{13}{36},\frac{181}{324}\right)$

$\Rightarrow$ (C) is correct

(D) Option (D) is remaining coloured part of option (C), hence option (D) is also correct.

SECTION-2 : (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 If **ONLY** the correct option is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : −1 In all other cases.

4. Let $f:(0,1)\to\mathbb{R}$ be the functions defined as $f(x)=\sqrt{n}$ if $x\in\left[\frac{1}{n+1},\frac{1}{n}\right)$ where $n\in\mathbb{N}$. Let $g:(0,1)\to\mathbb{R}$ be a function such that $\int_{x^2}^{x}\sqrt{\frac{1-t}{t}}\,dt<g(x)<2\sqrt{x}$ for all $x\in(0,1)$. Then $\lim_{x\to0}f(x)g(x)$

(A) does **NOT** exist
(B) is equal to 1
(C) is equal to 2
(D) is equal to 3

Ans. (C)

Sol. $\int_{x^2}^{x}\sqrt{\frac{1-t}{t}}\,dt.\sqrt{n}\le f(x)g(x)\le 2\sqrt{x}\sqrt{n}$

$\because \int_{x^2}^{x}\sqrt{\frac{1-t}{t}}\,dt=\sin^{-1}\sqrt{x}+\sqrt{x}\sqrt{1-x}-\sin^{-1}x-x\sqrt{1-x^2}$

$\Rightarrow \lim_{x\to0}\left(\frac{\sin^{-1}\sqrt{x}+\sqrt{x}\sqrt{1-x}-\sin^{-1}x-x\sqrt{1-x^2}}{\sqrt{x}}\le f(x)g(x)\le\frac{2\sqrt{x}}{\sqrt{x}}\right)$

$\Rightarrow 2\le\lim_{x\to0}f(x)g(x)\le2$

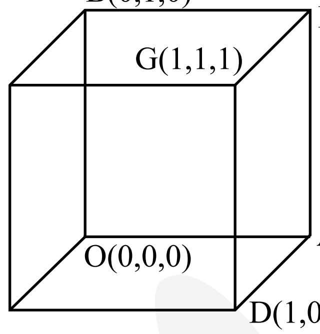
$\Rightarrow \lim_{x\to0}f(x)g(x)=2$

5. Let Q be the cube with the set of vertices $\{(x_1,x_2,x_3)\in\mathbb{R}^3:x_1,x_2,x_3\{0,1\}\}$. Let F be the set of all twelve lines containing the diagonals of the six faces of the cube Q. Let S be the set of all four lines containing the main diagonals of the cube Q; for instance, the line passing through the vertices (0, 0, 0) and (1, 1, 1) is in S. For lines ℓ_1 and ℓ_2, let $d(\ell_1,\ell_2)$ denote the shortest distance between them. Then the maximum value of $d(\ell_1,\ell_2)$, as ℓ_1 varies over F and ℓ_2 varies over S, is

(A) $\frac{1}{\sqrt{6}}$
(B) $\frac{1}{\sqrt{8}}$
(C) $\frac{1}{\sqrt{3}}$
(D) $\frac{1}{\sqrt{12}}$

Ans. (A)

Sol.



DR'S of OG = 1, 1, 1

DR'S of AF = –1, 1, 1

DR'S of CE = 1, 1, –1

DR'S of BD = 1, –1, 1

Equation of OG $\Rightarrow \frac{x}{1}=\frac{y}{1}=\frac{z}{1}$

Equation of AB $\Rightarrow \frac{x-1}{1}=\frac{y}{-1}=\frac{z}{0}$

Normal to both the line's

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 1 & -1 & 0\end{vmatrix}=\hat{i}+\hat{j}-2\hat{k}$$

$\overrightarrow{OA}=\hat{i}$

$$\text{S.D.} = \frac{\left|\hat{i}.(\hat{i}+\hat{j}-2\hat{k})\right|}{\left|\hat{i}+\hat{j}-2\hat{k}\right|}=\frac{1}{\sqrt{6}}$$

Ans. (A)

6. Let $X=\left\{(x,y)\in\mathbb{Z}\times\mathbb{Z}:\frac{x^2}{8}+\frac{y^2}{20}<1 \text{ and } y^2<5x\right\}$. Three distinct points P, Q and R are randomly chosen from X. Then the probability that P, Q and R form a triangle whose area is a positive integer, is

(A) $\frac{71}{220}$ (B) $\frac{73}{220}$ (C) $\frac{79}{220}$ (D) $\frac{83}{220}$

Ans. (B)

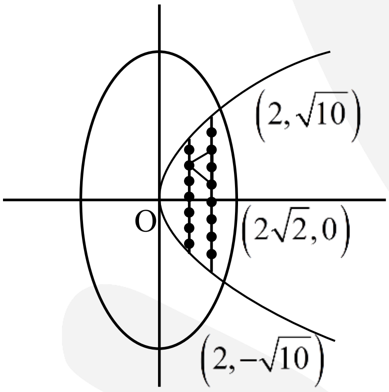
Sol. $\frac{x^2}{8}+\frac{y^2}{20}<1$ & $y^2<5x$

Solving corresponding equations

$\frac{x^2}{8}+\frac{y^2}{20}=1$ & $y^2=5x$

$\Rightarrow \left\{\begin{matrix} x=2 \\ y=\pm\sqrt{10} \end{matrix}\right\}$

X = {(1,1), (1,0), (1,–1), (1,2), (1,–2), (2, 3), (2,2), (2,1), (2,0), (2,–1), (2,–2), (2,–3)}



Let S be the sample space & E be the event n(S) = $^{12}C_3$

<u>For E</u>

Selecting 3 points in which 2 points are either or x = 1 & x = 2 but distance b/w then is even

Triangles with base 2 :

= 3×7 + 5×5 = 46

Triangles with base 4 :

= 1×7 + 3×5 = 22

Triangles with base 6 :

= 1×5 = 5

$P(E)=\frac{46+22+5}{^{12}C_3}=\frac{73}{220}$

Ans. (B)

7. Let P be a point on the parabola $y^2 = 4ax$, where a > 0. The normal to the parabola at P meets the x-axis at a point Q. The area of the triangle PFQ, where F is the focus of the parabola, is 120. If the slope m of the normal and a are both positive integers, then the pair (a,m) is

(A) (2, 3) (B) (1, 3) (C) (2, 4) (D) (3, 4)

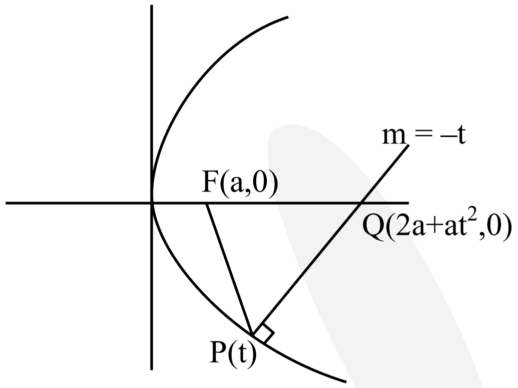
Ans. (A)

Sol. Let point P $(at^2, 2at)$

normal at P is $y = -tx + 2at + at^3$

$y = 0,\ x = 2a + at^2$

$Q(2a + at^2, 0)$



Area of $\Delta PFQ = \left|\frac{1}{2}(a+at^2)(2at)\right| = 120$

$\because\ m = -t$

$\because\ a^2[1 + m^2]\,m = 120$

$(a, m) = (2, 3)$ will satisfy

SECTION-3 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER.**
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 **ONLY** If the correct integer is entered;
  Zero Marks : 0 In all other cases.

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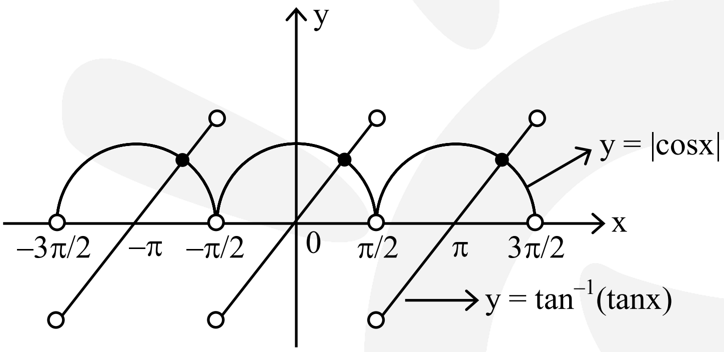
8. Let $\tan^{-1}(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, for $x \in \mathbb{R}$. Then the number of real solutions of the equation

$\sqrt{1+\cos(2x)} = \sqrt{2}\tan^{-1}(\tan x)$ in the set $\left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right) \cup \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ is equal to

Ans. (3)

Sol. $\sqrt{2}|\cos x| = \sqrt{2}.\tan^{-1}(\tan x)$

$|\cos x| = \tan^{-1}\tan x$



No. of solutions = 3

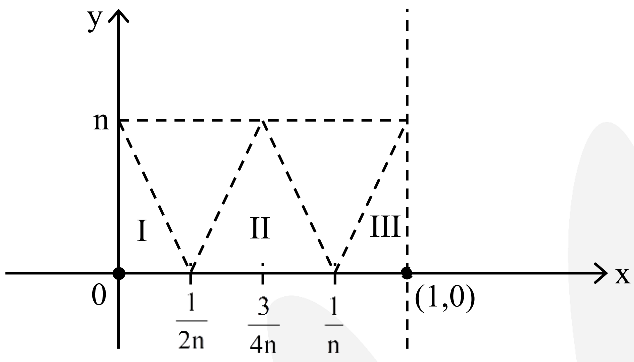
9. Let $n \geq 2$ be a natural number and $f : [0,1] \to \mathbb{R}$ be the function defined by

$$f(x) = \begin{cases} n(1-2nx) & \text{if } 0 \leq x \leq \frac{1}{2n} \\ 2n(2nx-1) & \text{if } \frac{1}{2n} \leq x \leq \frac{3}{4n} \\ 4n(1-nx) & \text{if } \frac{3}{4n} \leq x \leq \frac{1}{n} \\ \frac{n}{n-1}(nx-1) & \text{if } \frac{1}{n} \leq x \leq 1 \end{cases}$$

If n is such that the area of the region bounded by the curves $x = 0$, $x = 1$, $y = 0$ and $y = f(x)$ is 4, then the maximum value of the function f is

Ans. (8)

Sol.



Area = Area of (I + II + III) = 4

$$=\frac{1}{2}\times\frac{1}{2n}\times n+\frac{1}{2}\times\frac{1}{2n}\times n+\frac{1}{2}\left(1-\frac{1}{n}\right)\times n$$

$$=\frac{1}{4}+\frac{1}{4}+\frac{n-1}{2}=4$$

$\boxed{n=8}$

$\therefore$ maximum value of f(x) = 8

10. Let $\overset{r}{75\ldots57}$ denote the (r + 2) digit number where the first and the last digits are 7 and the remaining r digits are 5. Consider the sum S = 77 + 757 + 7557 + …+ $\overset{98}{75\ldots57}$. If $S=\frac{\overset{99}{75\ldots57}+m}{n}$, where m and n are natural numbers less than 3000, then the value of m + n is

Ans. (1219)

Sol. $S = 77 + 757 + 7557 + \ldots + \underset{98}{75.....57}$

$$10S = \quad 770 + 7570 + \ldots + 75\ldots570 + 755\ldots..570$$

$$9S = -77 + \underbrace{13+13+.....+13}_{98\text{ times}} + \underset{98}{75.....570}$$

$$= -77 + 13\times 98 + \underset{99}{75.....57} + 13$$

$$S=\frac{\underset{99}{75.....57}+1210}{9}$$

m = 1210

n = 9

m + n = 1219

11. Let $A=\left\{\frac{1967+1686i\sin\theta}{7-3i\cos\theta}:\theta\in\mathbb{R}\right\}$. If A contains exactly one positive integer n, then the value of n is

Ans. (281)

Sol. $A=\frac{1967+1686i\sin\theta}{7-3i\cos\theta}$

$=\frac{281(7+6i\sin\theta)}{7-3i\cos\theta}\times\frac{7+3i\cos\theta}{7+3i\cos\theta}$

$=\frac{281(49-18\sin\theta\cos\theta+i(21\cos\theta+42\sin\theta))}{49+9\cos^2\theta}$

for positive integer

Im(A) = 0

$21\cos\theta+42\sin\theta=0$

$\tan\theta=\frac{-1}{2}$; $\sin2\theta=\frac{-4}{5}$, $\cos^2\theta=\frac{4}{5}$

$\text{Re}(A)=\frac{281(49-9\sin2\theta)}{49+9\cos^2\theta}$

$=\frac{281\left(49-9\times\frac{-4}{5}\right)}{49+9\times\frac{4}{5}}=281$ (+ve integer)

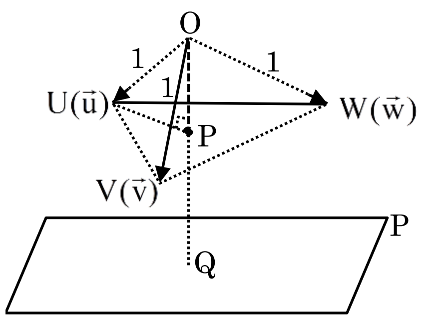
12. Let P be the plane $\sqrt{3}x+2y+3z=16$ and let

$S=\left\{\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}:\alpha^2+\beta^2+\gamma^2=1\text{ and the distance of }(\alpha,\beta,\gamma)\text{ from the plane P is }\frac{7}{2}\right\}$.

Let $\vec{u},\vec{v}$ and $\vec{w}$ be three distinct vectors in S such that $|\vec{u}-\vec{v}|=|\vec{v}-\vec{w}|=|\vec{w}-\vec{u}|$. Let V be the volume of the parallelepiped determined by vectors $\vec{u},\vec{v}$ and $\vec{w}$. Then the value of $\frac{80}{\sqrt{3}}V$ is

Ans. (45)

Sol.



Given $|\vec{u}-\vec{v}|=|\vec{v}-\vec{w}|=|\vec{w}-\vec{u}|$

$\Rightarrow\Delta UVW$ is an equilateral Δ

Now distances of U, V, W from $P = \frac{7}{2}$

$\Rightarrow PQ = \frac{7}{2}$

Also, Distance of plane P from origin

$\Rightarrow OQ = 4$

$\therefore OP = OQ - PQ \Rightarrow OP = \frac{1}{2}$

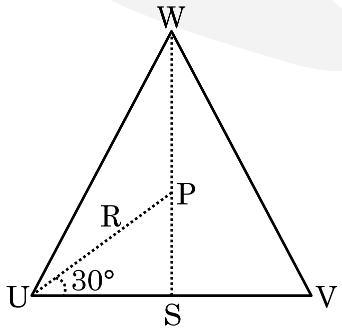
Hence, $PU = \sqrt{OU^2 - OP^2} \Rightarrow PU = \frac{\sqrt{3}}{2} = R$

Also, for ΔUVW, P is circumcenter

$\therefore$ for ΔUVW : $US = R\cos 30°$

$\Rightarrow UV = 2R\cos 30°$

$\Rightarrow UV = \frac{3}{2}$



$\therefore Ar(\Delta UVW) = \frac{\sqrt{3}}{4}\left(\frac{3}{2}\right)^2 = \frac{9\sqrt{3}}{16}$

$\therefore$ Volume of tetrahedron with coterminous edges $\vec{u}, \vec{v}, \vec{w}$

$= \frac{1}{3}(Ar.\Delta UVW) \times OP = \frac{1}{3} \times \frac{9\sqrt{3}}{16} \times \frac{1}{2} = \frac{3\sqrt{3}}{32}$

$\therefore$ parallelopiped with coterminous edges

$\vec{u}, \vec{v}, \vec{w} = 6 \times \frac{3\sqrt{3}}{32} = \frac{9\sqrt{3}}{16} = V$

$\therefore \frac{80}{\sqrt{3}} V = 45$

13. Let a and b be two nonzero real numbers. If the coefficient of x^5 in the expansion of $\left(ax^2+\frac{70}{27bx}\right)^4$ is equal to the coefficient of x^{-5} is equal to the coefficient of $\left(ax-\frac{1}{bx^2}\right)^7$, then the value of 2b is

Ans. (3)

Sol. $T_{r+1}={}^4C_r\left(a.x^2\right)^{4-r}.\left(\frac{70}{27bx}\right)^r$

$={}^4C_r.a^{4-r}.\frac{70^r}{(27b)^r}.x^{8-3r}$

here $8-3r=5$

$8-5=3r\Rightarrow r=1$

$\therefore$ coeff. $=4.a^3.\frac{70}{27b}$

$T_{r+1}={}^7C_r(ax)^{7-r}\left(\frac{-1}{bx^2}\right)^r$

$={}^7C_r.a^{7-r}\left(\frac{-1}{b}\right)^r.x^{7-3r}$

$7-3r=-5\Rightarrow 12=3r\Rightarrow r=4$

coeff. : ${}^7C_4.a^3.\left(\frac{-1}{b}\right)^4=\frac{35a^3}{b^4}$

now $\frac{35a^3}{b^4}=\frac{280a^3}{27b}$

$b^3=\frac{35\times 27}{280}=b=\frac{3}{2}\Rightarrow 2b=3$

SECTION-4 : (Maximum Marks : 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists : **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 **ONLY** if the option corresponding to the correct combination is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : –1 In all other cases.

14. Let α, β and γ be real numbers. consider the following system of linear equations

$x + 2y + z = 7$

$x + \alpha z = 11$

$2x - 3y + \beta z = \gamma$

Match each entry in **List - I** to the correct entries in **List-II**

List-I	**List-II**
(P) If $\beta = \frac{1}{2}(7\alpha - 3)$ and $\gamma = 28$, then the system has	(1) a unique solution
(Q) If $\beta = \frac{1}{2}(7\alpha - 3)$ and $\gamma \neq 28$, then the system has	(2) no solution
(R) If $\beta \neq \frac{1}{2}(7\alpha - 3)$ where $\alpha = 1$ and $\gamma \neq 28$, then the system has	(3) infinitely many solutions
(S) If $\beta \neq \frac{1}{2}(7\alpha - 3)$ where $\alpha = 1$ and $\gamma = 28$, then the system has	(4) $x = 11$, $y = -2$ and $z = 0$ as a solution
	(5) $x = -15$, $y = 4$ and $z = 0$ as a solution

The correct option is :

(A) (P) $\rightarrow$ (3) (Q) $\rightarrow$ (2) (R) $\rightarrow$ (1) (S) $\rightarrow$ (4)

(B) (P) $\rightarrow$ (3) (Q) $\rightarrow$ (2) (R) $\rightarrow$ (5) (S) $\rightarrow$ (4)

(C) (P) $\rightarrow$ (2) (Q) $\rightarrow$ (1) (R) $\rightarrow$ (4) (S) $\rightarrow$ (5)

(D) (P) $\rightarrow$ (2) (Q) $\rightarrow$ (1) (R) $\rightarrow$ (1) (S) $\rightarrow$ (3)

Ans. (A)

Sol. Given $x + 2y + z = 7$ .... (1)

$x + \alpha z = 11$ .... (2)

$2x - 3y + \beta z = \gamma$ .... (3)

Now, $\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 0 & \alpha \\ 2 & -3 & \beta \end{vmatrix} = 7\alpha - 2\beta - 3$

$\therefore$ if $\beta = \frac{1}{2}(7\alpha - 3)$

$\Rightarrow \boxed{\Delta = 0}$

Now, $\Delta_x = \begin{vmatrix} 7 & 2 & 1 \\ 11 & 0 & \alpha \\ \gamma & -3 & \beta \end{vmatrix}$

$= 21\alpha - 22\beta + 2\alpha\gamma - 33$

$\therefore$ if $\gamma = 28$

$\Rightarrow \Delta_x = 0$

$\Delta_y = \begin{vmatrix} 1 & 7 & 1 \\ 1 & 11 & \alpha \\ 2 & \gamma & \beta \end{vmatrix}$

$\Delta_y = 4\beta + 14\alpha - \alpha\gamma + \gamma - 22$

$\therefore$ if $\gamma = 28$

$\Rightarrow \Delta_y = 0$

Now, $\Delta_z = \begin{vmatrix} 1 & 2 & 7 \\ 1 & 0 & 11 \\ 2 & -3 & \gamma \end{vmatrix} = 56 - 2\gamma$

If $\gamma = 28$

$\Rightarrow \boxed{\Delta_z = 0}$

$\therefore$ if $\gamma = 28$ and $\beta = \frac{1}{2}(7\alpha - 3)$

$\Rightarrow$ system has infinite solution

and if $\gamma \neq 28$

$\Rightarrow$ system has no solution

$\Rightarrow P \rightarrow (3)\ ;\ Q \rightarrow (2)$

Now if $\beta \neq \frac{1}{2}(7\alpha - 3)$

$\Rightarrow \Delta \neq 0$

and for $\alpha = 1$ clearly

$y = -2$ is always be the solution

$\therefore$ if $\gamma \neq 28$

System has a unique solution

if $\gamma = 28$

$\Rightarrow x = 11,\ y = -2$ and $z = 0$ will be one of the solution

$\therefore R \rightarrow 1\ ;\ S \rightarrow 4$

$\therefore$ option 'A' is correct

15. Consider the given data with frequency distribution

x_i	3	8	11	10	5	4
f_i	5	2	3	2	4	4

Match each entry in **List-I** to the correct entries in **List-II.**

List-I	**List-II**
(P) The mean of the above data is	(1) 2.5
(Q) The median of the above data is	(2) 5
(R) The mean deviation about the mean of the above data is	(3) 6
(S) The mean deviation about the median of the above data is	(4) 2.7
	(5) 2.4

The correct option is :

(A) (P) $\rightarrow$ (3) (Q) $\rightarrow$ (2) (R) $\rightarrow$ (4) (S) $\rightarrow$ (5)

(B) (P) $\rightarrow$ (3) (Q) $\rightarrow$ (2) (R) $\rightarrow$ (1) (S) $\rightarrow$ (5)

(C) (P) $\rightarrow$ (2) (Q) $\rightarrow$ (3) (R) $\rightarrow$ (4) (S) $\rightarrow$ (1)

(D) (P) $\rightarrow$ (3) (Q) $\rightarrow$ (3) (R) $\rightarrow$ (5) (S) $\rightarrow$ (5)

Ans. (A)

Sol.

x_i	3	4	5	8	10	11
f_i	5	4	4	2	2	3

(P) Mean

(Q) Median

(R) Mean deviation about mean

(S) Mean deviation about median

x_i	f_i	$x_i f_i$	C.F.	$\lvert x_i - \text{Mean}\rvert$	$f_i\lvert x_i - \text{Mean}\rvert$	$\lvert x_i - \text{Median}\rvert$	$f_i\lvert x_i - \text{Median}\rvert$
3	5	15	5	3	15	2	10
4	4	16	9	2	8	1	4
5	4	20	13	1	4	0	0
8	2	16	15	2	4	3	6
10	2	20	17	4	8	5	10
11	3	33	20	5	15	6	18
	$\Sigma f_i = 20$	$\Sigma x_i f_i = 120$			$\Sigma f_i\lvert x_i - \text{Mean}\rvert = 54$		$\Sigma f_i\lvert x_i - \text{Median}\rvert = 48$

(P) Mean $= \dfrac{\Sigma x_i f_i}{\Sigma f_i} = \dfrac{120}{20} = 6$

(Q) Median $= \left(\dfrac{20}{2}\right)^{th}$ observation $= 10^{th}$ observation $= 5$

(R) Mean deviation about mean $= \dfrac{\Sigma f_i \lvert x_i - \text{Mean}\rvert}{\Sigma f_i} = \dfrac{54}{20} = 2.70$

(S) mean deviation about median $= \dfrac{\Sigma f_i \lvert x_i - \text{Median}\rvert}{\Sigma f_i} = \dfrac{48}{20} = 2.40$

16. Let ℓ_1 and ℓ_2 be the lines $\vec{r}_1 = \lambda(\hat{i}+\hat{j}+\hat{k})$ and $\vec{r}_2 = (\hat{j}-\hat{k})+\mu(\hat{i}+\hat{k})$, respectively. Let X be the set of all the planes H that contain the line ℓ_1. For a plane H, let d(H) denote the smallest possible distance between the points of ℓ_2 and H. Let H_0 be plane in X for which $d(H_0)$ is the maximum value of d(H) as H varies over all planes in X.

Match each entry in **List-I** to the correct entries in **List-II.**

List-I	**List-II**
(P) The value of $d(H_0)$ is	(1) $\sqrt{3}$
(Q) The distance of the point (0,1,2) from H_0 is	(2) $\frac{1}{\sqrt{3}}$
(R) The distance of origin from H_0 is	(3) 0
(S) The distance of origin from the point of intersection of planes y = z, x = 1 and H_0 is	(4) $\sqrt{2}$
	(5) $\frac{1}{\sqrt{2}}$

The correct option is :

(A) (P) $\to$ (2) (Q) $\to$ (4) (R) $\to$ (5) (S) $\to$ (1)

(B) (P) $\to$ (5) (Q) $\to$ (4) (R) $\to$ (3) (S) $\to$ (1)

(C) (P) $\to$ (2) (Q) $\to$ (1) (R) $\to$ (3) (S) $\to$ (2)

(D) (P) $\to$ (5) (Q) $\to$ (1) (R) $\to$ (4) (S) $\to$ (2)

Ans. (B)

Ans. ( )

Sol. $L_1 : \vec{r}_1 = \lambda(\hat{i}+\hat{j}+\hat{k})$

$L_2 : \vec{r}_2 = \hat{j}-\hat{k}+\mu(\hat{i}+\hat{k})$

Let system of planes are

$ax + by + cz = 0$ .... (1)

$\because$ It contain L_1

$\therefore a + b + c = 0$ .... (2)

For largest possible distance between plane (1) and L_2 the line L_2 must be parallel to plane (1)

$\therefore a + c = 0$ .... (3)

$\Rightarrow \boxed{b = 0}$

$\therefore$ Plane H_0 : $\boxed{x - z = 0}$

Now $d(H_0)$ = ⊥ distance from point (0, 1, –1) on L_2 to plane.

$$\Rightarrow d(H_0) = \left|\frac{0+1}{\sqrt{2}}\right| = \frac{1}{\sqrt{2}}$$

$\therefore P \rightarrow 5$

for 'Q' distance $= \left|\frac{2}{\sqrt{2}}\right| = \sqrt{2}$

$\therefore Q \rightarrow 4$

$\therefore$ (0, 0, 0) lies on plane

$\therefore R \rightarrow 3$

for 'S' x = z ; y = z ; x = 1

$\therefore$ point of intersection p(1, 1, 1).

$\therefore \ OP = \sqrt{1+1+1} = \sqrt{3}$

$\therefore S \rightarrow 2$

$\therefore$ option [B] is correct

17. Let z be complex number satisfying $|z|^3 + 2z^2 + 4\bar{z} - 8 = 0$, where $\bar{z}$ denotes the complex conjugate of z. Let the imaginary part of z be nonzero.

Match each entry in **List-I** to the correct entries in **List-II.**

List-I	List-II
(P) $|z|^2$ is equal to	(1) 12
(Q) $|z-\bar{z}|^2$ is equal to	(2) 4
(R) $|z|^2 + |z+\bar{z}|^2$ is equal to	(3) 8
(S) $|z+1|^2$ is equal to	(4) 10
	(5) 7

The correct option is :

(A) (P) → (1) (Q) → (3) (R) → (5) (S) → (4)

(B) (P) → (2) (Q) → (1) (R) → (3) (S) → (5)

(C) (P) → (2) (Q) → (4) (R) → (5) (S) → (1)

(D) (P) → (2) (Q) → (3) (R) → (5) (S) → (4)

Ans. (B)

Sol. $\because |z|^3 + 2z^2 + 4\bar{z} - 8 = 0$ .... (1)

Take conjugate both sides

$\Rightarrow |z|^3 + 2\bar{z}^2 + 4z - 8 = 0$ .... (2)

By (1) – (2)

$\Rightarrow 2(z^2 - \bar{z}^2) + 4(\bar{z} - z) = 0$

$\Rightarrow \boxed{z + \bar{z} = 2}$ .... (3)

$\Rightarrow |z + \bar{z}| = 2$ .... (4)

Let z = x + iy

$\therefore \boxed{x = 1}$ $\therefore z = 1 + iy$

Put in (1)

$\Rightarrow (1 + y^2)^{3/2} + 2(1 - y^2 + 2iy) + 4(1 - iy) - 8 = 0$

$\Rightarrow (1 + y^2)^{3/2} = 2(1 + y^2)$

$\Rightarrow \sqrt{1 + y^2} = 2 = |z|$

Also $\boxed{y = \pm\sqrt{3}}$

$\therefore z = 1 \pm i\sqrt{3}$

$\Rightarrow z - \bar{z} = \pm 2i\sqrt{3}$

$\Rightarrow |z - \bar{z}| = 2\sqrt{3}$

$\Rightarrow |z - \bar{z}|^2 = 12$

Now $z + 1 = 2 + i\sqrt{3}$

$|z + 1|^2 = 4 + 3 = 7$

$\therefore P \rightarrow 2 ; Q \rightarrow 1 ; R \rightarrow 3 ; S \rightarrow 5$

$\therefore$ Option [B] is correct.