FINAL JEE–MAIN EXAMINATION – JANUARY, 2023
Held On Wednesday 01st February, 2023
TIME : 03:00 PM to 06:00 PM

SECTION-A

61. The sum $\sum_{n=1}^{\infty}\frac{2n^2+3n+4}{(2n)!}$ is equal to :

(1) $\frac{11e}{2}+\frac{7}{2e}$

(2) $\frac{13e}{4}+\frac{5}{4e}-4$

(3) $\frac{11e}{2}+\frac{7}{2e}-4$

(4) $\frac{13e}{4}+\frac{5}{4e}$

Official Ans. by NTA (2)

Ans. (2)

Sol. $\sum_{n=1}^{\infty}\frac{2n^2+3n+4}{(2n)!}$

$\frac{1}{2}\sum_{n=1}^{\infty}\frac{2n(2n-1)+8n+8}{(2n)!}$

$\frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{(2n-2)!}+2\sum_{n=1}^{\infty}\frac{1}{(2n-1)!}+4\sum_{n=1}^{\infty}\frac{1}{(2n)!}$

$e=1+1+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+.....$

$e^{-1}=1-1+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}+.....$

$\left(e+\frac{1}{e}\right)=2\left(1+\frac{1}{2!}+\frac{1}{4!}+.......\right)$

$e-\frac{1}{e}=\left(1+\frac{1}{3!}+\frac{1}{5!}+.....\right)$

Now

$\frac{1}{2}\left(\sum_{n=1}^{\infty}\frac{1}{(2n-2)!}\right)+2\sum_{n=1}^{\infty}\frac{1}{(2n-1)!}+4\sum_{n=1}^{\infty}\frac{1}{(2n)!}$

$=\frac{1}{2}\left[\frac{e+\frac{1}{e}}{2}\right]+2\left[\frac{e-\frac{1}{e}}{2}\right]+4\left(\frac{e+\frac{1}{e}-2}{2}\right)$

$=\frac{\left(e+\frac{1}{e}\right)}{4}+e-\frac{1}{e}+2e+\frac{2}{e}-4$

$=\frac{13}{4}e+\frac{5}{4e}-4$

62. Let $S=\left\{x\in R:0<x<1 \text{ and } 2\tan^{-1}\left(\frac{1-x}{1+x}\right)=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right\}$. If n(S) denotes the number of elements in S then :

(1) n(S) = 2 and only one element in S is less then $\frac{1}{2}$.

(2) n(S) = 1 and the element in S is more than $\frac{1}{2}$.

(3) n(S) = 1 and the element in S is less than $\frac{1}{2}$.

(4) n(S) = 0

Official Ans. by NTA (3)

Ans. (3)

Sol. $0<x<1$

$2\tan^{-1}\left(\frac{1-x}{1+x}\right)=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$\tan^{-1}x=\theta\in\left(0,\frac{\pi}{4}\right)\ \therefore x=\tan\theta$

$2\tan^{-1}\left(\tan\left(\frac{\pi}{4}-\theta\right)\right)=\cos^{-1}(\cos 2\theta)$

$2\left(\frac{\pi}{4}-\theta\right)=2\theta\ \therefore 4\theta=\frac{\pi}{2}\quad\therefore\theta=\frac{\pi}{8}$

$x=\tan\frac{\pi}{8}\ \therefore x=\sqrt{2}-1\simeq 0.414$

63. Let $\vec{a}=2\hat{i}-7\hat{j}+5\hat{k}$, $\vec{b}=\hat{i}+\hat{k}$ and $\vec{c}=\hat{i}+2\hat{j}-3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r}\times\vec{a}=\vec{c}\times\vec{a}$ and $\vec{r}\cdot\vec{b}=0$, then $|\vec{r}|$ is equal to :

(1) $\frac{11}{7}\sqrt{2}$

(2) $\frac{11}{7}$

(3) $\frac{11}{5}\sqrt{2}$

(4) $\frac{\sqrt{914}}{7}$

Official Ans. by NTA (1)

Ans. (1)

Sol. $\vec{a} = 2\hat{i} - 7\hat{j} + 5\hat{k}$

$\vec{b} = \hat{i} + \hat{k}$

$\vec{c} = \hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{r} \times \vec{a} = \vec{c} \times \vec{a} \Rightarrow (\vec{r} - \vec{c}) \times \vec{a} = 0$

$\therefore \vec{r} = \vec{c} + \lambda\vec{a}$

$\vec{r} \, . \, \vec{b} = 0 \Rightarrow \vec{c} \, .\vec{b} + \lambda \quad \vec{b} \, . \, \vec{a} = 0$

$-2 + \lambda(7) = 0 \Rightarrow \lambda = \frac{2}{7}$

$\therefore \vec{r} = \vec{c} + \frac{2\vec{a}}{7} = \frac{1}{7}(11\hat{i} - 11\hat{k})$

$|\vec{r}| = \frac{11\sqrt{2}}{7}$

64. If $A = \frac{1}{2}\begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$, then :

(1) $A^{30} - A^{25} = 2I$

(2) $A^{30} + A^{25} + A = I$

(3) $A^{30} + A^{25} - A = I$

(4) $A^{30} = A^{25}$

Official Ans. by NTA (3)

Ans. (3)

Sol. $A = \frac{1}{2}\begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$

$A = \begin{bmatrix} \cos 60° & \sin 60° \\ -\sin 60° & \cos 60° \end{bmatrix}$

If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ Here $\alpha = \frac{\pi}{3}$

$A^2 = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$= \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}$

$A^{30} = \begin{bmatrix} \cos 30\alpha & \sin 30\alpha \\ -\sin 30\alpha & \cos 30\alpha \end{bmatrix}$

$A^{30} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$A^{25} = \begin{bmatrix} \cos 25\alpha & \sin 25\alpha \\ -\sin 25\alpha & \cos 25\alpha \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{-\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

$A^{25} = A$

$A^{25} - A = 0$

65. Two dice are thrown independently. Let A be the event that the number appeared on the 1st die is less than the number appeared on the 2nd die, B be the event that the number appeared on the 1st die is even and that on the second die is odd, and C be the event that the number appeared on the 1st die is odd and that on the 2nd is even. Then

(1) the number of favourable cases of the event $(A \cup B) \cap C$ is 6

(2) A and B are mutually exclusive

(3) The number of favourable cases of the events A, B and C are 15, 6 and 6 respectively

(4) B and C are independent

Official Ans. by NTA (1)

Ans. (1)

Sol. A : no. on 1st die < no. on 2nd die

A : no. on 1st die = even & no. of 2nd die = odd

C : no. on 1st die = odd & no. on 2nd die = even

$n(A) = 5 + 4 + 3 + 2 + 1 = 15$

$n(B) = 9$

$n(C) = 9$

$n((A \cup B) \cap C) = (A \cap C) \cup (B \cap C)$

$= (3 + 2 + 1) + 0 = 6.$

66. Which of the following statements is a tautology ?

(1) $p \to (p \land (p \to q))$

(2) $(p \land q) \to (\sim (p) \to q))$

(3) $(p \land (p \to q)) \to \sim q$

(4) $p \lor (p \land q)$

Official Ans. by NTA (2)

Ans. (2)

Sol. (i) $p \to (p \land (p \to q))$

$(\sim p) \lor (p \land (\sim p \lor q))$

$(\sim p) \lor (f \lor (p \land q))$

$\sim p \lor (p \land q) = (\sim p \lor p) \land (\sim p \lor q)$

$= \sim p \lor q$

(ii) $(p \land q) \to (\sim p \to q)$

$\sim (p \land q) \lor (p \lor q) = t$

$\{a, b, d\} \lor \{a, b, c\} = V$

Tautology

(iii) $(p \land (p \to q)) \to \sim q$

$\sim (p \land (\sim p \lor q)) \lor \sim q = \sim (p \land q) \lor \sim q = \sim p \lor \sim q$

Not tantology

(iv) $p \lor (p \land q) = p$

Not tautology.

67. The number of integral values of k, for which one root of the equation $2x^2 - 8x + k = 0$ lies in the interval (1, 2) and its other root lies in the interval (2, 3), is :

(1) 2

(2) 0

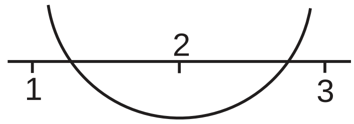
(3) 1

(4) 3

Official Ans. by NTA (3)

Ans. (3)

Sol. $2x^2 - 8x + k = 0$



$f(1) \cdot f(2) < 0$ & $f(2) \cdot f(3) < 0$

$(k - 6)(k - 8) < 0$ & $(k - 8)(k - 6) < 0$

$k \in (6, 8)$ $k \in (6, 8)$

integral value of k = 7

68. Let $f : R - \{0, 1\} \to R$ be a function such that $f(x) + f\left(\frac{1}{1-x}\right) = 1 + x$. Then f(2) is equal to :

(1) $\frac{9}{2}$

(2) $\frac{9}{4}$

(3) $\frac{7}{4}$

(4) $\frac{7}{3}$

Official Ans. by NTA (2)

Ans. (2)

Sol. $f(x) + f\left(\frac{1}{1-x}\right) = 1 + x$

$x = 2 \Rightarrow f(2) + f(-1) = 3$ (1)

$x = -1 \Rightarrow f(-1) + f\left(\frac{1}{2}\right) = 0$ (2)

$x = \frac{1}{2} \Rightarrow f\left(\frac{1}{2}\right) + f(2) = \frac{3}{2}$ (3)

$(1) + (3) - (2) \Rightarrow 2f(2) = \frac{9}{2}$

$\therefore f(2) = \frac{9}{4}$

69. Let the plane P pass through the intersection of the planes $2x + 3y - z = 2$ and $x + 2y + 3z = 6$, and be perpendicular to the plane $2x + y - z + 1 = 0$. If d is the distance of P from the point (–7, 1, 1), then d^2 is equal to :

(1) $\frac{250}{83}$

(2) $\frac{15}{53}$

(3) $\frac{25}{83}$

(4) $\frac{250}{82}$

Official Ans. by NTA (1)

Ans. (1)

Sol. $P \equiv P_1 + \lambda P_2 = 0$

$(2 + \lambda)x + (3 + 2\lambda)y + (3\lambda - 1)z - 2 - 6\lambda = 0$

Plane P is perpendicular to P_3 $\therefore$ $\vec{n} \cdot \vec{n}_3 = 0$

$2(\lambda + 2) + (2\lambda + 3) - (3\lambda - 1) = 0$

$\lambda = -8$

$P \equiv -6x - 13y - 25z + 46 = 0$

$6x + 13y + 25z - 46 = 0$

Dist from (–7, 1, 1)

$$d = \left|\frac{-42 + 13 + 25 - 46}{\sqrt{36 + 169 + 625}}\right| = \frac{50}{\sqrt{830}}$$

$$d^2 = \frac{50 \times 50}{830} = \frac{250}{83}$$

70. Let a, b be two real numbers such that ab < 0. If the complex number $\frac{1 + ai}{b + i}$ is of unit modulus and a + ib lies on the circle $|z - 1| = |2z|$, then a possible value of $\frac{1 + [a]}{4b}$, where [t] is greatest integer function, is :

(1) $-\frac{1}{2}$

(2) –1

(3) 1

(4) $\frac{1}{2}$

Official Ans. by NTA (DROP)

Sol. $ab < 0 \quad \left|\frac{1+ai}{b+i}\right| = 1$

$|1+ai| = |b+i|$

$a^2 + 1 = b^2 + 1 \Rightarrow a = \pm b \Rightarrow b = -a \quad$ as $ab < 0$

(a, b) lies on $|z-1| = |2z|$

$|a + ib - 1| = 2|a + ib|$

$(a-1)^2 + b^2 = 4(a^2 + b^2)$

$(a-1)^2 = a^2 = 4(2a^2)$

$1 - 2a = 6a^2 \Rightarrow 6a^2 + 2a - 1 = 0$

$$a = \frac{-2 \pm \sqrt{28}}{12} = \frac{-1 \pm \sqrt{7}}{6}$$

$$a = \frac{\sqrt{7}-1}{6} \;\&\; b = \frac{1-\sqrt{7}}{6}$$

$[a] = 0$

$$\therefore \frac{1+[a]}{4b} = \frac{6}{4\left(1-\sqrt{7}\right)} = -\left(\frac{1+\sqrt{7}}{4}\right)$$

or $[a] = 0$

Similarly it is not matching with $a = \frac{-1-\sqrt{7}}{6}$

No answer is matching.

71. The sum of the abosolute maximum and minimum values of the function $f(x) = |x^2 - 5x + 6| - 3x + 2$ in the interval $[-1, 3]$ is equal to :

(1) 10

(2) 12

(3) 13

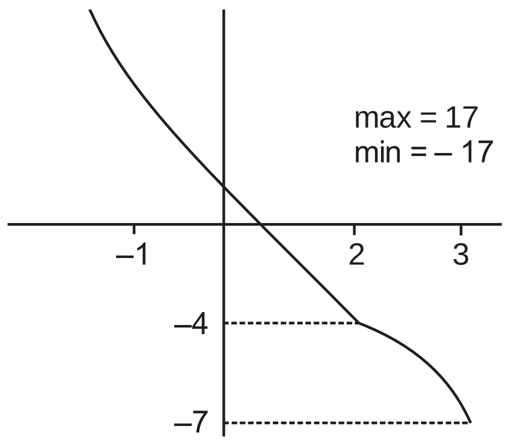
(4) 24

Official Ans. by NTA (1)

Ans. (1)

Sol. $f(x) = |x^2 - 5x + 6| - 3x + 2$

$$f(x) = \begin{cases} x^2 - 8x + 8 & ; x \in [-1, 2] \\ -x^2 + 2x - 4 & ; x \in [2, 3] \end{cases}$$



72. Let P(S) denote the power set of $S=\{1,2,3, ..., 10\}$. Define the relations R_1 and R_2 on P(S) as AR_1B if $(A \cap B^c) \cup (B \cap A^c) = \varnothing$ and AR_2B if $A \cup B^c = B \cup A^c$, $\forall$ A, B $\in$ P(S). Then :

(1) both R_1 and R_2 are equivalence relations

(2) only R_1 is an equivalence relation

(3) only R_2 is an equivalence relation

(4) both R_1 and R_2 are not equivalence relations

Official Ans. by NTA (1)

Ans. (1)

Sol. $S = \{1, 2, 3, \ldots\ldots 10\}$

P(S) = power set of S

$AR, B \Rightarrow (A \cap \vec{B}) \cup (\vec{A} \cap B) = \phi$

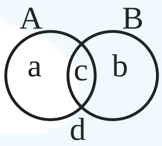
R1 is reflexive, symmetric

For transitive

$(A \cap \vec{B}) \cup (\vec{A} \cap B) = \phi$; $\{a\} = \phi = \{b\}$ $\;A = B$

$(B \cap \vec{C}) \cup (\vec{B} \cap C) = \phi \;\therefore\; B = C$

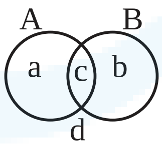
$\therefore$ A = C equivalence.



$R_2 \equiv A \cup \vec{B} = \vec{A} \cup B$

$R_2 \to$ Reflexive , symmetric

for transitive



$A \cup \vec{B} = \vec{A} \cup B \Rightarrow \{a, c, d\} = \{b, c, d\}$

$\{a\} = \{b\} \;\therefore\; A = B$

$B \cup \vec{C} = \vec{B} \cup C \Rightarrow B = C$

$\therefore A = C \qquad \therefore\; A \cup \vec{C} = \vec{A} \cup C \;\therefore$ Equivalence

73. The area of the region given by $\{(x, y) : xy \le 8, 1, \le y \le x^2\}$ is :

(1) $8\log_e 2 - \frac{13}{3}$ (2) $16\log_e 2 - \frac{14}{3}$

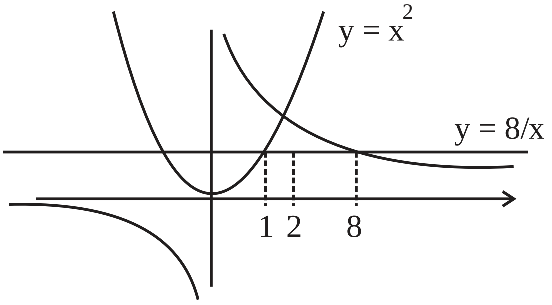
(3) $8\log_e 2 + \frac{7}{6}$ (4) $16\log_e 2 + \frac{7}{3}$

Official Ans. by NTA (2)

Ans. (2)

Sol.

$xy \le 8 \qquad 1 \le y \le x^2$



$$\text{Area} = \int_1^2 (x^2-1)dx + \int_2^8 \left(\frac{8}{x}-1\right)dx$$

$$= \left(\frac{x^3}{3}\right)_1^2 + 8(\ell n x)_2^8 - (x)_1^8$$

$$= \frac{7}{3} + 8(2\ell n 2) - 7$$

$$= 16\ \ell n2 - \frac{14}{3}$$

74. Let $\alpha x = \exp(x^{\beta}y^{\gamma})$ be the solution of the differential equation $2x^2y\ dy - (1 - xy^2)\ dx = 0$, $x > 0,\ y(2) = \sqrt{\log_e 2}$. Then $\alpha + \beta - \gamma$ equals :

(1) 1
(2) –1
(3) 0
(4) 3

Official Ans. by NTA (1)

Ans. (1)

Sol. $\alpha x = e^{x^{\beta}.y^{\gamma}}$

$$2x^2y\frac{dy}{dx} = 1 - x.y^2 \qquad y^2 = t$$

$$x^2\frac{dt}{dx} = 1 - xt$$

$$\frac{dt}{dx} + \frac{t}{x} = \frac{1}{x^2} \qquad \text{I.F.} = e^{\ell nx} = x$$

$$t(x) = \int \frac{1}{x^2}.x\ dx$$

$$y^2 . x = \ell nx + C$$

$$\therefore 2.\ \ell n2 = \ell n2 + C$$

$$\therefore C = \ell n2$$

Hence, $xy^2 = \ell n2x$

$$\therefore 2x = e^{x.y^2}$$

Hence $\alpha = 2, \beta = 1, \gamma = 2$

75. The value of the integral $\int_{-\pi/4}^{\pi/4} \frac{x+\frac{\pi}{4}}{2-\cos 2x}dx$ is :

(1) $\frac{\pi^2}{6}$

(2) $\frac{\pi^2}{12\sqrt{3}}$

(3) $\frac{\pi^2}{3\sqrt{3}}$

(4) $\frac{\pi^2}{6\sqrt{3}}$

Official Ans. by NTA (4)

Ans. (4)

Sol. $I = \int_{-\pi/4}^{\pi/4} \frac{x+\frac{\pi}{4}}{2-\cos 2x}dx \qquad (1)$

$x \to -x$

$$I = \int_{-\pi/4}^{\pi/4} \frac{-x+\frac{\pi}{4}}{2-\cos 2x}dx \qquad (2)$$

(1) + (2)

$$2I = \int_{-\pi/4}^{\pi/4} \frac{\frac{\pi}{2}}{2-\cos 2x}dx$$

$$I = \frac{\pi}{4}.2\int_0^{\pi/4} \frac{dx}{2-\cos 2x}dx$$

$$I = \frac{\pi}{4}.2\int_0^{\pi/4} \frac{(1+\tan^2 x)dx}{2(1+\tan^2 x)-(1-\tan^2 x)}$$

$$I = \frac{\pi}{4}\int_0^1 \frac{dt}{3t^2+1}$$

$$\Rightarrow I = \frac{\pi}{2\sqrt{3}}\tan^{-1}\sqrt{3}$$

$$I = \frac{\pi^2}{6\sqrt{3}}$$

76. Let $9 = x_1 < x_2 < \ldots < x_7$ be in an A.P. with common difference d. If the standard deviation of $x_1, x_2 \ldots, x_7$ is 4 and the mean is $\bar{x}$, then $\bar{x} + x_6$ is equal to :

(1) $18\left(1+\frac{1}{\sqrt{3}}\right)$

(2) 34

(3) $2\left(9+\frac{8}{\sqrt{7}}\right)$

(4) 25

Official Ans. by NTA (2)

Ans. (2)

Sol. $9 = x_1 < x_2 < \ldots\ldots < x_7$

$9, 9 + d, 9 + 2d, \ldots\ldots\ldots 9 + 6d$

$0, d, 2d, \ldots\ldots\ldots 6d$

$$\bar{x}_{new} = \frac{21d}{7} = 3d$$

$$16 = \frac{1}{7}\left(0^2 + 1^2 + \ldots\ldots + 6^2\right)d^2 - 9d^2$$

$$= \frac{1}{\cancel{7}}\left(\frac{\cancel{6}\times\cancel{7}\times 13}{\cancel{6}}\right)d^2 - 9d^2$$

$16 = 4d^2$

$d^2 = 4$

$d = 2$

$\bar{x} + x_6 = 6 + 9 + 10 + 9$

77. For the system of linear equations $ax + y + z = 1$, $x + ay + z = 1$, $x + y + az = \beta$, which one of the following statements is NOT correct ?

(1) It has infinitely many solutions if $\alpha=2$ and $\beta = -1$

(2) It has no solution if $\alpha = -2$ and $\beta = 1$

(3) $x + y + z = \frac{3}{4}$ if $\alpha = 2$ and $\beta = 1$

(4) It has infinitely many solutions if $\alpha=1$ and $\beta=1$

Official Ans. by NTA (1)

Ans. (1)

Sol. $$\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0$$

$\alpha(\alpha^2 - 1) - 1(\alpha - 1) + 1(1 - \alpha) = 0$

$\alpha^3 - 3\alpha + 2 = 0$

$\alpha^2(\alpha - 1) + \alpha(\alpha - 1) - 2(\alpha - 1) = 0$

$(\alpha - 1)(\alpha^2 + \alpha - 2) = 0$

$\alpha = 1, \alpha = -2, 1$

For $\alpha = 1, \beta = 1$

$$\left.\begin{matrix} x + y + z = 1 \\ x + y + z = b \end{matrix}\right\}\text{infinite solution}$$

For $\alpha = 2, \beta = 1$

$\Delta = 4$

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = 3 - 1 - 1 = 1 \qquad \Rightarrow x = \frac{1}{4}$$

$$\Delta_2 = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 2 \end{vmatrix} = 2 - 1 = 1 \qquad \Rightarrow y = \frac{1}{4}$$

$$\Delta_3 = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 2 - 1 = 1 \qquad \Rightarrow z = \frac{1}{4}$$

For $\alpha = 2 \Rightarrow$ unique solution

78. Let $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} + 5\hat{k}$ be two vectors. Then which one of the following statements is TRUE?

(1) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{17}{\sqrt{35}}$ and the direction of the pr

(2) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-17}{\sqrt{35}}$ and the direction of the pr

(3) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{17}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\vec{b}$

(4) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-17}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\vec{b}$

Official Ans. by NTA (DROP)

Sol. $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$

$\vec{b} = \hat{i} - 3\hat{j} + 5\hat{k}$

$\vec{a}.\hat{b} = \frac{5-3-15}{\sqrt{35}} = -\frac{-13}{\sqrt{35}}$

79. Let $P(x_0, y_0)$ be the point on the hyperbola $3x^2 - 4y^2 = 36$, which is nearest to the line $3x + 2y = 1$. Then $\sqrt{2}\ (y_0 - x_0)$ is equal to :

(1) –3

(2) 9

(3) –9

(4) 3

Official Ans. by NTA (3)

Ans. (3)

Sol. $3x^2 - 4y^2 = 36$ $\quad\quad$ $3x + 2y = 1$

$m = -\frac{3}{2}$

$m = +\frac{\sec\theta\ 3}{\sqrt{12}.\tan\theta}$

$\Rightarrow \frac{3}{\sqrt{12}} \times \frac{1}{\sin\theta} = \frac{-3}{2}$

$\sin\theta = -\frac{1}{\sqrt{3}}$

$\left(\sqrt{12}.\sec\theta,\ 3\tan\theta\right)$

$\left(\sqrt{12}.\frac{\sqrt{3}}{\sqrt{2}}, -3\times\frac{1}{\sqrt{2}}\right) \Rightarrow \left(\frac{6}{\sqrt{2}}, \frac{-3}{\sqrt{2}}\right)$

80. If $y(x) = x^x$, $x > 0$, then $y''(2) - 2y'(2)$ is equal to :

(1) $8\log_e 2 - 2$

(2) $4\log_e 2 + 2$

(3) $4(\log_e 2)^2 - 2$

(4) $4(\log_e 2)^2 + 2$

Official Ans. by NTA (3)

Ans. (3)

Sol. $y' = x^x$

$y' = x^x(1 + \ell nx)$

$y'' = x^x(1 + \ell nx)^2 + x^x . \frac{1}{x}$

$y''(2) = 4(1 + \ell n2)^2 + 2$

$y'(2) = 4(1 + \ell n2)$

$y''(2) - 2y'(2) = 4(1 + \ell n2)^2 + 2 - 8(1 + \ell n2)$

$= 4(1 + \ell n2)[1 + \ell n2 - 2] + 2$

$= 4((\ell n2)^2 - 1) + 2$

$= 4(\ell n2)^2 - 2$

SECTION-B

81. The total number of six digit numbers, formed using the digits 4, 5, 9 only and divisible by 6, is ______.

Official Ans. by NTA (81)

Ans. (81)

Sol. Taking single digit → 444444 $\quad\quad \frac{6!}{6!} = 1$

Taking two digit →

(4, 5) $\quad$ 444555 $\quad\quad$ (4, 9) $\quad$ 444999

$\frac{5!}{3!2!} = 10 \quad\quad \frac{5!}{3!2!} = 10$

Taking three digit

$4, 5, 9, 4, 4, 4 \Rightarrow \frac{5!}{3!} = 20$

$4, 5, 9, 5, 5, 5 \Rightarrow \frac{5!}{4!} = 5$

$4, 5, 9, 9, 9, 9 \Rightarrow \frac{5!}{4!} = 5$

$4, 5, 9, 4, 5, 9 \Rightarrow \frac{5!}{2!2!} = 30$

Total = 81

82. Number of integral solutions to the equation $x + y + z = 21$, where $x \geq 1$, $y \geq 3$, $z \geq 4$, is equal to ______.

Official Ans. by NTA (105)

Ans. (105)

Sol. $^{15}C_2 = \frac{15 \times 14}{2} = 105$

83. The line $x = 8$ is the directrix of the ellipse $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with the corresponding focus $(2, 0)$. If the tangent to E at the point P in the first quadrant passes through the point $\left(0, 4\sqrt{3}\right)$ and intersects the x-axis at Q, then $(3PQ)^2$ is equal to ______.

Official Ans. by NTA (39)

Ans. (39)

Sol. $\frac{a}{e} = 8$ ....(1) $\qquad ae = 2$ ....(2)

$8e = \frac{2}{e}$

$e^2 = \frac{1}{4} \Rightarrow e = \frac{1}{2}$

$a = 4$

$b^2 = a^2(1 - e^2)$

$= 16\left(\frac{3}{4}\right) \quad = 12$

$\frac{x\cos\theta}{4} + \frac{y\sin\theta}{2\sqrt{3}} = 1$

$\sin\theta = \frac{1}{2}$

$\theta = 30°$

$P\left(2\sqrt{3}, \sqrt{3}\right)$

$Q\left(\frac{8}{\sqrt{3}}, 0\right)$

$(3PQ)^2 = 39$

84. If the x-intercept of a focal chord of the parabola $y^2 = 8x + 4y + 4$ is 3, then the length of this chord is equal to ______.

Official Ans. by NTA (16)

Ans. (16)

Sol. $y^2 = 8x + 4y + 4$

$(y - 2)^2 = 8(x + 1)$

$y^2 = 4ax$

$a = 2,\ X = x + 1,\ Y = y - 2$

focus $(1, 2)$

$y - 2 = m(x - 1)$

Put $(3, 0)$ in the above line

$m = -1$

Length of focal chord $= 16$

85. If $\int_0^{\pi} \frac{5^{\cos x}\left(1 + \cos x\cos 3x + \cos^2 x + \cos^3 x\cos 3x\right)dx}{1 + 5^{\cos x}} = \frac{k\pi}{16}$, then k is equal to ______.

Official Ans. by NTA (13)

Ans. (13)

Sol

$$I = \int_0^{\pi} \frac{5^{\cos x}(1 + \cos x\cos 3x + \cos^2 x + \cos^3 x\cos 3x)}{1 + 5^{\cos x}}dx$$

$$I = \int_0^{\pi} \frac{5^{-\cos x}(1 + \cos x\cos 3x + \cos^2 x + \cos^3 x\cos 3x)}{1 + 5^{-\cos x}}dx$$

$$2I = \int_0^{\pi} (1 + \cos x\cos 3x + \cos^2 x + \cos^3 x\cos 3x)dx$$

$$\cancel{2}I = \cancel{2}\int_0^{\frac{\pi}{2}} (1 + \cos x\cos 3x + \cos^2 x + \cos^3 x\cos 3x)dx$$

$$I = \int_0^{\frac{\pi}{2}} (1 + \sin x(-\sin 3x) + \sin^2 x - \sin^3 x\sin 3x)dx$$

$$2I = \int_0^{\frac{\pi}{2}} (3 + \cos 4x + \cos^3 x\cos 3x - \sin^3 x\sin 3x)dx$$

$$2I = \int_0^{\frac{\pi}{2}} 3 + \cos 4x + \left(\frac{\cos 3x + 3\cos x}{4}\right)\cos 3x - \sin 3x\left(\frac{3\sin x - \sin 3x}{4}\right)dx$$

$$2I = \int_0^{\frac{\pi}{2}} \left(3 + \cos 4x + \frac{1}{4} + \frac{3}{4}\cos 4x\right)dx$$

$$2I = \frac{13}{4} \times \frac{\pi}{2} + \frac{7}{4}\left(\frac{\sin 4x}{4}\right)_0^{\frac{\pi}{2}} \Rightarrow I = \frac{13\pi}{16}$$

86. Let the sixth term in the binomial expansion of $\left(\sqrt{2^{\log_2}\left(10-3^x\right)}+\sqrt[5]{2^{(x-2)\log_2 3}}\right)^m$, in the increasing powers of $2^{(x-2)\log_2 3}$, be 21. If the binomial coefficients of the second, third and fourth terms in the expansion are respectively the first, third and fifth terms of an A.P., then the sum of the squares of all possible values of x is ______.

Official Ans. by NTA (4)

Ans. (4)

Sol. $T_6 = {}^mC_5\left(10-3^x\right)^{\frac{m-5}{2}}.\left(3^{x-2}\right)=21 \quad ....(1)$

${}^mC_1, {}^mC_2, {}^mC_3$ are in A.P.

$2. {}^mC_2 = {}^mC_1 + {}^mC_3$

Solving for m, we get

m = 2(rejected), 7

Put in equation (1)

$21 . (10-3^x)\ \dfrac{3^x}{9} = 21$

$3^x = 3^0, 3^2$

x = 0, 2

Sum of the squares of all possible values of x = 4

87. If the term without x in the expansion of $\left(x^{\frac{2}{3}}+\dfrac{\alpha}{x^3}\right)^{22}$ is 7315, then |α| is equal to _______.

Official Ans. by NTA (1)

Ans. (1)

Sol. $T_{r+1} = {}^{22}C_r.\left(x^{\frac{2}{3}}\right)^{22-r}.(\alpha)^r, x^{-3r}$

$= {}^{22}C_r.x^{\frac{44}{3}-\frac{2r}{3}-3r}(\alpha)^r$

$\dfrac{44}{3}=\dfrac{11r}{3}$

r = 4

${}^{22}C_4.\alpha^4 = 7315$

$\dfrac{22\times21\times20\times19}{24}.\alpha^4 = 7315$

$\alpha = 1$

88. The sum of the common terms of the following three arithmetic progressions.

3, 7, 11, 15, …………, 399,

2, 5, 8, 11,………….., 359 and

2, 7, 12, 17,……., 197, is equal to ______.

Official Ans. by NTA (321)

Ans. (321)

Sol. 3, 7, 11, 15, …………, 399 $\quad d_1 = 4$

2, 5, 8, 11,………….., 359 $\quad d_2 = 3$

2, 7, 12, 17,……., 197 $\quad d_3 = 5$

LCM $(d_1, d_2, d_3) = 60$

Common terms are 47, 107, 167

Sum = 321

89. Let αx + βy + yz = 1 be the equation of a plane passing through the point (3, –2, 5) and perpendicular to the line joining the points (1, 2, 3) and (–2, 3, 5). Then the value of αβy is equal to ______.

Official Ans. by NTA (6)

Ans. (Bonus)

Sol. Given Equation is not equation of plane as yz is present. If we consider y is γ then answer would be 6.

Normal vector of plane = $3\hat{i}-\hat{j}-2\hat{k}$

Plane : $3x - y - 2z + \lambda = 0$

Point (3, –2, 5) satisfies the plane

$\lambda = -1$

$3x - y - 2z = 1$

$\alpha\beta y = 6$

90. The point of intersection C of the plane $8x + y + 2z = 0$ and the line joining the points $A(-3, -6, 1)$ and $B(2, 4, -3)$ divides the line segment AB internally in the ratio $k : 1$. If a, b, c ($|a|, |b|, |c|$ are coprime) are the direction ratios of the perpendicular from the point C on the line $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$, then $|a + b + c|$ is equal to ______.

Official Ans. by NTA (10)

Ans. (10)

Sol. Plane : $8x + y + 2z = 0$

Given line AB : $\frac{x-2}{5} = \frac{y-4}{10} = \frac{z+3}{-4} = \lambda$

Any point on line $(5\lambda + 2, 10\lambda + 4, -4\lambda - 3)$

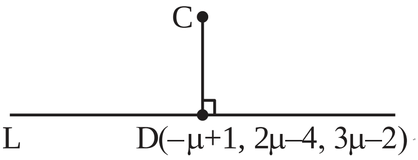
Point of intersection of line and plane

$8(5\lambda + 2) + 10\lambda + 4 - 8\lambda - 6 = 0$

$\lambda = -\frac{1}{3}$

$C\left(\frac{1}{3}, \frac{2}{3}, -\frac{5}{3}\right)$

$L : \frac{x-1}{-1} = \frac{y+4}{2} = \frac{z+2}{3} = \mu$



$\overrightarrow{CD} = \left(-\mu + \frac{2}{3}\right)\hat{i} + \left(2\mu - \frac{14}{3}\right)\hat{j} + \left(3\mu - \frac{1}{3}\right)\hat{k}$

$\left(-\mu + \frac{2}{3}\right)(-1) + \left(2\mu - \frac{14}{3}\right)2 + \left(3\mu - \frac{1}{3}\right)3 = 0$

$\mu = \frac{11}{14}$

$\overrightarrow{CD} = \frac{-5}{42}, \frac{-130}{42}, \frac{85}{42}$

Direction ratios $\rightarrow (-1, -26, 17)$

$|a + b + c| = 10$