# FINAL JEE–MAIN EXAMINATION – APRIL, 2023
## Held On Thursday 06th April, 2023
## TIME : 03:00 PM to 06:00 PM

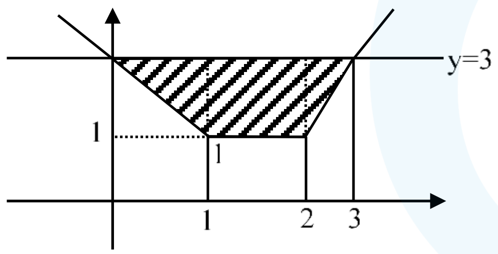
**SECTION-A**

**1.** If gcd (m, n) = 1 and
$1^2 - 2^2 + 3^2 - 4^2 + \ldots + (2021)^2 - (2022)^2 + (2023)^2 = 1012\, m^2n$
then $m^2 - n^2$ is equal to :
(1) 180 (2) 220 (3) 200 (4) 240

**Sol.** **(4)**
$(1-2)(1+2) + (3-4)(3+4) + \ldots + (2021-2022)(2021+2022) + (2023)^2 = (1012)\, m^2n$
$\Rightarrow (-1)\,[1 + 2 + 3 + 4 + \ldots + 2022] + (2023)^2 = (1012)\, m^2n$
$\Rightarrow (-1)\,\dfrac{(2022)(2023)}{2} + (2023)^2 = (1012)\, m^2n$
$\Rightarrow (2023)\,[\,2023 - 1011\,] = (1012)\, m^2n$
$\Rightarrow (2023)\,(1012) = (1012)\, m^2n$
$\Rightarrow m^2n = 2023$
$\Rightarrow m^2n = (17)^2 \times 7$
m = 17, n = 7
$m^2 - n^2 = (17)^2 - 7^2 = 289 - 49 = 240$
Ans. Option 4

**2.** The area bounded by the curves $y = |x-1| + |x-2|$ and $y = 3$ is equal to :
(1) 5 (2) 4 (3) 6 (4) 3

**Sol.** **(2)**
$y = |x-1| + |x-2|$




$A = \dfrac{1}{2}\,[1+3]\,[2]$
= 4
Ans. Option 2

**3.** For the system of equations
$x + y + z = 6$
$x + 2y + \alpha z = 10$
$x + 3y + 5z = \beta$, which one of the following is <u>NOT</u> true :
(1) System has a unique solution for $\alpha = 3, \beta \neq 14$.
(2) System has a unique solution for $\alpha = -3, \beta = 14$.
(3) System has no solution for $\alpha = 3, \beta = 24$.
(4) System has infinitely many solutions for $\alpha = 3, \beta = 14$.

**Sol.** **(1)**
$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & \alpha \\ 1 & 3 & 5 \end{vmatrix}$$
$= (10 - 3\alpha) - (5 - \alpha) + (3 - 2)$
$= 6 - 2\alpha$

$$\Delta x = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & \alpha \\ \beta & 3 & 5 \end{vmatrix}$$

$= 6(10 - 3\alpha) - (50 - \alpha 13) + (30 - 2\beta)$
$= 40 - 18\alpha + \alpha\beta - 2\beta$

$$\Delta y = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 10 & \alpha \\ 1 & \beta & 5 \end{vmatrix}$$

$= (50 - \alpha\beta) - 6(5 - \alpha) + (\beta - 10)$
$= 10 + 6\alpha + \beta - \alpha\beta$

$$\Delta z = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 1 & 3 & \beta \end{vmatrix}$$

$= (2\beta - 30) - (\beta - 10) + 6\,(1)$
$= \beta - 14$

for Infinite solution $\quad \Delta = 0, \quad \Delta_x = \Delta_y = \Delta_z = 0$
$\underline{\alpha = 3}, \quad \underline{\beta = 14}$
For unique solution $\alpha \neq 3$

Ans. Option 1

**4.** Among the statements :
(S1): $(p \Rightarrow q) \vee ((\sim p) \wedge q)$ is a tautology
(S2): $(q \Rightarrow p) \Rightarrow ((\sim p) \wedge q)$ is a contradiction
(1) only (S2) is True (2) only (S1) is True
(3) neigher (S1) and (S2) is True (4) both (S1) and (S2) are True

**Sol.** **(3)**
S1

| P | Q | ~p | ~p^q | p⇒q | (p⇒q) v (~p^q) |
|---|---|---|---|---|---|
| T | T | F | F | T | T |
| T | F | F | F | F | F |
| F | T | T | T | T | T |
| F | F | T | F | T | T |

S2

| P | Q | q⇒p | ~p | (~p)^q | (q⇒p) ⇒ (~p^q) |
|---|---|---|---|---|---|
| T | T | T | F | F | F |
| T | F | T | F | F | F |
| F | T | F | T | T | T |
| F | F | T | T | F | F |

Ans. Option 3

**5.** $\lim\limits_{n\to\infty}\left\{\left(2^{\frac{1}{2}} - 2^{\frac{1}{3}}\right)\left(2^{\frac{1}{2}} - 2^{\frac{1}{5}}\right)......\left(2^{\frac{1}{2}} - 2^{\frac{1}{2n+1}}\right)\right\}$ is equal to

(1) $\frac{1}{\sqrt{2}}$ (2) $\sqrt{2}$ (3) 1 (4) 0

**Sol.** **(4)**

$$P = \lim_{n\to\infty}\left(2^{\frac{1}{2}} - 2^{\frac{1}{3}}\right)\left(2^{\frac{1}{2}} - 2^{\frac{1}{5}}\right)\ldots\ldots\left(2^{\frac{1}{2}} - 2^{\frac{1}{2n+1}}\right)$$

Let

$2^{\frac{1}{2}} - 2^{\frac{1}{3}}$ $\rightarrow$ Smallest

$2^{\frac{1}{2}} - 2^{\frac{1}{2n+1}}$ $\rightarrow$ Largest

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$$\left(2^{\frac{1}{2}} - 2^{\frac{1}{3}}\right)^n \le P \le \left(2^{\frac{1}{2}} - 2^{\frac{1}{2n+1}}\right)^n$$

$$\begin{pmatrix}\text{lie b / w}\\ \text{0 and 1}\end{pmatrix}^n$$

$$\lim_{n\to\infty}\left(2^{\frac{1}{2}} - 2^{\frac{1}{3}}\right)^n = 0$$

$$\lim_{n\to\infty}\left(2^{\frac{1}{2}} - 2^{\frac{1}{2n+1}}\right)^n = 0$$

$\therefore$ P = 0

**6.** Let P b a square matrix such that $P^2 = I - P$. For $\alpha, \beta, \gamma, \delta \in N$, if $P^\alpha + P^\beta = \gamma I - 29P$ and $P^\alpha - P^\beta = \delta I - 13P$, then $\alpha + \beta + \gamma - \delta$ is equal to :

(1) 40 (2) 22 (3) 24 (4) 18

**Sol.** **(3)**

$P^2 = I - P$

$P^\alpha + P^\beta = \gamma I - 29P$

$P^\alpha - P^\beta = \delta I - 13P$

$P^4 = (I - P)^2 = I + P^2 - 2P$

$P^4 = I + I - P - 2P = 2I - 3P$

$P^8 = (P^4)^2 = (2I - 3P)^2 = 4I + 9P^2 - 12P$

$= 4I + 9(I - P) - 12P$

$P^8 = 13I - 21P$ ....(1)

$P^6 = P^4.P^2 = (2I - 3P)(I - P)$

$= 2I - 5P + 3P^2$

$= 2I - 5P + 3(I - P)$

$= 5I - 8P$ ....(2)

(1) + (2) (1) – (2)

$P^8 + P^6 = 18I - 29P$ $\boxed{P^8 - P^6 = 8I - 13P}$

From (A) $\alpha = 8$, $\beta = 6$

$\gamma = 18$

$\delta = 8$

$\alpha + \beta + \gamma - \delta = 32 - 8 = 24$

**7.** A plane P contains the line of intersection of the plane $\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = 6$ and $\vec{r}.(2\hat{i} + 3\hat{j} + 4\hat{k}) = -5$. If P passes through the point (0, 2, –2), then the square of distance of the point (12, 12, 18) from the plane P is :

(1) 620 (2) 1240 (3) 310 (4) 155

**Sol.** **(1)**

eq$^n$ of plane $P_1 + \lambda P_2 = 0$

$(x + y + z - 6) + \lambda\ (2x + 3y + 4z + 5) = 0$

pass th. (0, 2, – 2)

$(-6) + \lambda\ (6 - 8 + 5) = 0$

$(-6) + \lambda\ [3] = 0 \qquad \Rightarrow \lambda = 2$

eq$^n$ of plane

$5x + 7y + 9z + 4 = 0$

distance from (12, 12, 18)

$$d = \left|\frac{60+84+162+4}{\sqrt{25+49+81}}\right|$$

$$d = \frac{310}{\sqrt{155}}$$

$$d^2 = \frac{310\times 310}{155}$$

$$\boxed{d^2 = 620}$$

Ans. Option 1

**8.** Let f(x) be a function satisfying $f(x) + f(\pi - x) = \pi^2,\ \forall x \in \mathbb{R}$. Then $\int_0^{\pi} f(x)\sin x\, dx$ is equal to :

(1) $\frac{\pi^2}{2}$ (2) $\pi^2$ (3) $2\pi^2$ (4) $\frac{\pi^2}{4}$

**Sol.** **(2)**

$I = \int_0^{\pi} f(x)\sin x\ dx \qquad .....(1)$

Apply king property

$I = \int_0^{\pi} f(\pi - x)\sin(\pi - x)\ dx \qquad .....(1)$

Add

$2I = \int_0^{\pi} f(x) + f(\pi - x)\ \sin x\, dx$

$2I = \int_0^{\pi} \pi^2\ \sin x\, dx$

$\cancel{2}I = \pi^2(\cancel{2})$

$\boxed{I = \pi^2}$

Ans. Option 2

**9.** If the coefficients of $x^7$ in $\left(ax^2 + \frac{1}{2bx}\right)^{11}$ and $x^{-7}$ in $\left(ax - \frac{1}{3bx^2}\right)^{11}$ are equal, then :

(1) 64 ab = 243 (2) 32 ab = 729 (3) 729 ab = 32 (4) 243 ab = 64

**Sol.** **(3)**

$\left(ax^2 + \frac{1}{2bx}\right)^{11}$

$r = \frac{11\times 2 - 7}{3} = 5$

Coefficient of $x^7$ is $= {}^{11}C_5 (a)^6 \left(\frac{1}{2b}\right)^5$

$$\left(ax - \frac{1}{3bx^2}\right)^{11}$$

$$r = \frac{11\times1-(-7)}{3} = 6$$

Coefficient of $x^{-7}$ is $= {}^{11}C_6 \cdot \frac{a5}{3^6 b^6}$

$$\because\ {}^{11}C_5(a^6)\left(\frac{1}{2^5b^5}\right) = {}^{11}C_6 \cdot \frac{a5}{3^6b^6}$$

$$\Rightarrow ab = \frac{2^5}{3^6}$$

$$\Rightarrow \boxed{729\,ab = 32}$$

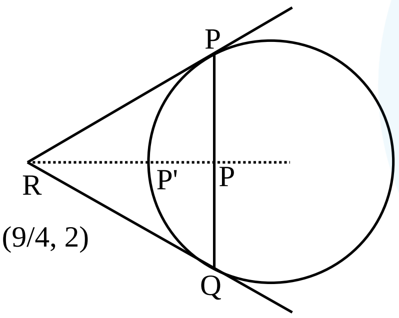
Ans. Opiton 3

**10.** If the tangents at the points P and Q are the circle $x^2 + y^2 -2x + y = 5$ meet at the point $R\left(\frac{9}{4},2\right)$, then the area of the triangle PQR is :

(1) $\frac{5}{4}$ (2) $\frac{13}{4}$ (3) $\frac{5}{8}$ (4) $\frac{13}{8}$

**Sol.** **(3)**

$x^2 + y^2 - 2x + y = 5$




with resperct to R PQ is C.O.C

eq$^n$ of C.O.C is T = 0

$$\frac{9}{4}x + 2y - \left(x + \frac{9}{4}\right) + \frac{1}{2}(y+2) - 5 = 0$$

$$\frac{5}{4}x + \frac{5}{2}y - \frac{25}{4} = 0$$

$$5x + 10y - 25 = 0$$

$$\boxed{x + 2y = 5}$$

Area $= \frac{1}{2}(P')(PQ)$

$(PQ) = 2\sqrt{r^2 - p^2} = \sqrt{5}$

$= \frac{1}{2}\left[\frac{\sqrt{5}}{4}\right](\sqrt{5})$

$P' = \frac{\frac{9}{4} + 4 - 5}{\sqrt{5}}$

$= \frac{5}{8}$

$= \left(\frac{5}{4\sqrt{5}}\right) = \frac{\sqrt{5}}{4}$

**Method II**

$$\text{area} = \frac{RL^3}{R^2 + L^2}$$

$$R = \frac{5}{2}$$

$$L = \sqrt{\frac{81}{16} + 4 - \frac{9}{2} + 2 - 5}$$

$$= \frac{5}{4}$$

$$\text{area} = = \frac{5}{8}$$

Ans. Option 3

**11.** Three dice are rolled. If the probability of getting different numbers on the three dice is $\frac{p}{q}$, where p and q are co-prime, then q – p is equal to :

(1) 1 (2) 2 (3) 4 (4) 3

**Sol.** **(3)**

$$\text{Fav.} = \frac{\left({}^6C_3\right)(3!)}{6 \times 6 \times 6}$$

$$= \frac{(20)(6)}{6 \cdot 6 \cdot 6} = \frac{20}{36} = \frac{5}{9} = \frac{p}{q}$$

$$\left.\begin{matrix} p = 5 \\ q = 9 \end{matrix}\right] \Rightarrow \boxed{q - p = 4}$$

Ans. Option 3

**12.** In a group of 100 persons 75 speak English and 40 speak Hindi. Each person speaks at least one of the two languages. If the number of persons, who speak only English is $\alpha$ and the number of persons who speak only Hindi is $\beta$, then the eccentricity of the ellipse $25\left(\beta^2 x^2 + \alpha^2 y^2\right) = \alpha^2 \beta^2$ is :

(1) $\frac{\sqrt{129}}{12}$ (2) $\frac{\sqrt{117}}{12}$ (3) $\frac{\sqrt{119}}{12}$ (4) $\frac{3\sqrt{15}}{12}$

**Sol.** **(3)**

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cap B) = 75 + 40 - 100$

$n(A \cap B) = 15$

Only E $\rightarrow$ 60 $\alpha = 60$

Only H $\rightarrow$ 25 $\beta = 25$

Both = 15

$$\frac{25x^2}{\alpha^2} + \frac{25y^2}{\beta^2} = 1$$

$$\frac{25x^2}{(60)^2} + \frac{(25y^2)}{(25)^2} = 1$$

$$e^2 = 1 - \left[\frac{25 \times 25}{(60)^2}\right]$$

$$e^2 = \frac{(60)^2 - (25)^2}{(60)^2}$$

$$e^2 = \frac{(60-25)(60+25)}{60 \times 60}$$

$$e^2 = \frac{(35)(85)}{60 \times 60} = \frac{119}{144}$$

$$e = \frac{\sqrt{119}}{12}$$

**13.** If the solution curve f(x, y) = 0 of the differential equation $(1 + \log_e x)\frac{dx}{dy} - x\log_e x = e^y, x > 0$, passes through the points (1, 0) and (α, 2), then $\alpha^\alpha$ is equal to :

(1) $e^{\sqrt{2}e^2}$ (2) $e^{e^2}$ (3) $e^{2e^{\sqrt{2}}}$ (4) $e^{2e^2}$

**Sol.** **(4)**

$$(1+\ell n\, x)\frac{dx}{dy} - x\,\ell n\, x = e^y$$

Let $x\,\ell n\, x = t$

$$(1+\ell n\, x)\frac{dx}{dy} = \frac{dt}{dy}$$

$$\frac{dt}{dy} - t = e^y$$ P = – 1, Q = $e^y$

$$I \cdot F = e^{\int -dy} = e^{-y}$$

Solution -

$$(t)(e^{-y}) = \int (e^{-y})(e^y)dy$$

$t(e^{-y}) = y + c$

$(x\,\ell n\, x)\, e^{-y} = y + c$ ⇒ pass (1, 0) ⇒ c = 0

pass (α, 2)

$$\boxed{\alpha^\alpha = e^{2e^2}}$$

Ans. Option 4

**14.** Let the sets A and B denote the domain and range respectively of the function $f(x) = \frac{1}{\sqrt{[x]-x}}$, where [x] denotes the smallest integer greater than or equal to x. Then among the statements :

(S1) : A ∩ B = (1, ∞) – N and

(S2) : A ∪ B = (1, ∞)

(1) only (S1) is true (2) neither (S1) nor (S2) is true

(3) only (S2) is true (4) both (S1) and (S2) are true

**Sol.** **(1)**

$$f(x) = \frac{1}{\sqrt{[x]-x}}$$

If $x \in I\,[x]$ = [x] (greatest integer function)

If $x \notin I\,[x] = [x]+1$

$$\Rightarrow f(x)=\begin{cases}\dfrac{1}{\sqrt{[x]-x}}, x\in I\\ \dfrac{1}{\sqrt{[x]+1-x}}, x\notin I\end{cases}$$

$$\Rightarrow f(x)=\begin{cases}\dfrac{1}{\sqrt{-\{x\}}}, x\in I, (\text{does not exist})\\ \dfrac{1}{\sqrt{1-\{x\}}}, x\notin I\end{cases}$$

$\Rightarrow$ domain of f(x) = R – I

Now, f(x) = $\dfrac{1}{\sqrt{1-\{x\}}}, x\notin I$

$\Rightarrow x < \{x\} < 1$

$\Rightarrow 0 < 1\sqrt{1-\{x\}} < 1$

$\Rightarrow \dfrac{1}{\sqrt{1-\{x\}}} > 1$

$\Rightarrow$ Range $(1, \infty)$

$\Rightarrow$ A = R – I

B = $(1\,\infty)$

So, $A\cap B=(1,\infty)-N$

$A\cup B\neq(1,\infty)$

$\Rightarrow$ S1 is only correct.

**15.** Let $a \neq b$ be two-zero real numbers. Then the number of elements in the set $X=\{z\in\mathbb{C}: \text{Re}(az^2+bz)=a \text{ and } \text{Re}(bz^2+az)=b\}$ is equal to :

(1) 0 (2) 2 (3) 1 (4) 3

**Sol.** **(1) Bonus**

$\because z+\bar{z}=2\text{Re}(z)$ If z = x + iy $\Rightarrow z+\bar{z}=2x$ $z^2+(\bar{z})^2=2(x^2-y^2)$

$(az^2+bz)+(a\bar{z}^2+b\bar{z})=2a$ ……….(1)

$(bz^2+az)+(b\bar{z}^2+a\bar{z})=2b$ ……….(2)

add (1) and (2)

$(a+b)z^2+(a+b)z+(a+b)\bar{z}^2+(a+b)\bar{z}=2(a+b)$

$(a+b)[z^2+z+(\bar{z})^2+\bar{z}]=2(a+b)$ ……….(3)

sub. (1) and (2)

$(a-b)[z^2-z+\bar{z}^2-\bar{z}]=2(a-b)$ ……….(4)

$z^2+\bar{z}^2-z-\bar{z}=2$

Case I: If $a+b\neq 0$

From (3) & (4)

$2x+2(x^2-y^2)=2 \Rightarrow x^2-y^2+x=1$ ……….(5)

$2(x^2-y^2)-2x=2 \Rightarrow x^2-y^2-x=1$ ……….(6)

(5) – (6)

$2x=0 \Rightarrow x=0$

from (5) $y^2=-1$ $\Rightarrow$ not possbible

$\therefore$ Ans = 0

Case II: If a + b = 0 then infinite number of solution.

So, the set X have infinite number of elements.

**16.** The sum of all values of $\alpha$, for which the points whose position vectors are $\hat{i}-2\hat{j}+3k,\ 2\hat{i}-3\hat{j}+4k,\ (\alpha+1)\hat{i}+2k$ and $9\hat{i}+(\alpha-8)\hat{j}+6\hat{k}$ are coplanar, is equal to :

(1) –2 (2) 2 (3) 6 (4) 4

**Sol.** **(2)**

$$\left.\begin{array}{l} A=(1,-2,3) \\ B=(2,-3,4) \\ C=(\alpha+1,0,2) \\ D=(9,\alpha-8,6) \end{array}\right]$$

$$\left[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}\right]=0$$

$$\begin{vmatrix} 1 & -1 & 1 \\ \alpha & 2 & -1 \\ 8 & \alpha-6 & 3 \end{vmatrix}=0$$

$\Rightarrow (6 + \alpha - 6) + 1\ (3\alpha + 8) + (\alpha^2 - 6\alpha - 16) = 0$

$\Rightarrow \alpha^2 - 2\alpha - 8 = 0$

$\Rightarrow \alpha = 4, -2$

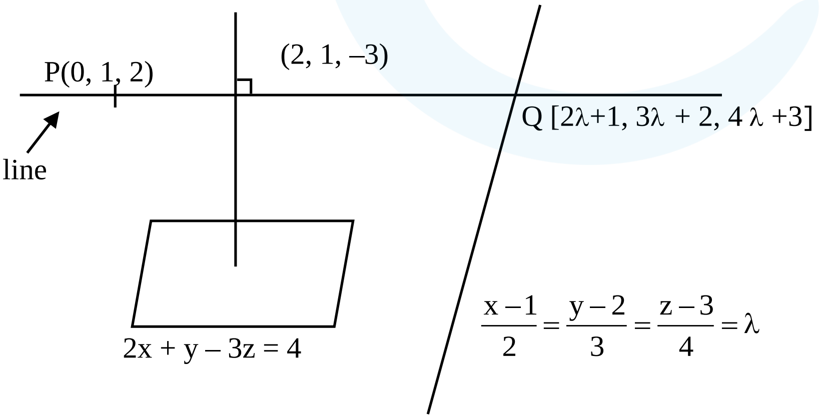
$\Rightarrow$ sum of all values of $\alpha = 2$

Ans. option 2

**17.** Let the line L pass through the point (0, 1, 2), intersect the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and be parallel to the plane $2x + y - 3z = 4$. Then the distance of the point P(1, –9, 2) from the line L is :

(1) 9 (2) $\sqrt{54}$ (3) $\sqrt{69}$ (4) $\sqrt{74}$

**Sol.** **(4)**




$\overrightarrow{PQ} = (2\lambda + 1, 3\lambda + 1, 4\lambda + 1)$

$\overrightarrow{PQ}\cdot\vec{n}=0 \quad \Rightarrow (2\lambda + 1).(2) + (3\lambda + 1)(1) + (4\lambda + 1)(-3) = 0$

$\Rightarrow -5\lambda = 0$

$\Rightarrow \lambda = 0$

Q = (1, 2, 3)

$eq^n$ of line

$$\frac{x-0}{1}=\frac{y-1}{1}=\frac{z-2}{1}=\mu$$

distance of line from (1, –9, 2)

(P'Q').(1, 1, 1) = 0

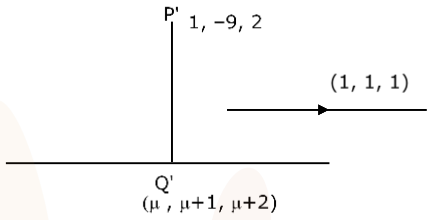
$\Rightarrow [\mu - 1, \mu + 10, \mu].[1, 1, 1] = 0$

$\Rightarrow \mu - 1 + \mu + 10 + \mu = 0$

$\mu = -3$

$Q' = (-3, -2, 1)$

$P'Q' = \sqrt{16+49+9} = \sqrt{74}$




**18.** All the letters of the word PUBLIC are written in all possible orders and these words are written as in a dictionary with serial numbers. Then the serial number of the word PUBLIC is :

(1) 580 (2) 578 (3) 576 (4) 582

**Sol.** **(4)**

B ———— = 5! = 120
C ———— = 5! = 120
I ———— = 5! = 120
L ———— = 5! = 120
PB ———— = 4! = 24
PC ———— = 4! = 24
PI ———— = 4! = 24
PL ———— = 4! = 24
PUBC ———— = 2! = 2
PUBI ———— = 2! = 2
PUBLC ———— = 1
PUBLIC ———— = 1
582

Rank = 582

Ans. Option 4

**19.** Let the vectors $\vec{a}, \vec{b}, \vec{c}$ represent three coterminous edges of a parallelepiped of volume V. Then the volume of the parallelepiped, whose coterminous edges are represented by $\vec{a}, \vec{b}+\vec{c}$ and $\vec{a}+2\vec{b}+3\vec{c}$ is equal to :

(1) 2 V (2) 6 V (3) 3 V (4) V

**Sol.** **(4)**

$v = [\vec{a}\ \vec{b}\ \vec{c}]$

$v_1 = [\vec{a}\ \ \vec{b}+\vec{c}\ \ \vec{a}+2\vec{b}+3\vec{c}]$

$$v_1 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} [\vec{a}\ \vec{b}\ \vec{c}]$$

$v_1 = (3-2)v$

$= v$

Ans. Option 4

**20.** Among the statements :

(S1) : $2023^{2022} - 1999^{2022}$ is divisible by 8

(S2) : $13(13)^n - 11n - 13$ is divisible by 144 for infinitely many $n \in \mathbb{N}$

(1) only (S2) is correct (2) only (S1) is correct

(3) both (S1) and (S2) are incorrect (4) both (S1) and (S2) are correct

**Sol.** **(4)**

$\because$ $x^n - y^n = (x - y)[x^{n-1} + x^{n-2}y + x^{n-3}y^2 + \ldots\ldots + y^{n-1}]$

$x^n - y^n$ is divisible by $x - y$

Stat 1 $\rightarrow$ $(2023)^{2022} - (1999)^{2022}$

$(2023) - (1999) = 24$

$\therefore$ $(2023)^{2022} - (1999)^{2022}$

is divisible by 8

Stat 2 $\rightarrow$ $13(1 + 12)^n - 11n - 13$

$$13\left[1 + {}^nC_1(12) + {}^nC_2(12)^2 + \ldots\right] - 11n - 13$$

$$\Rightarrow (156n - 11n) + 13 \cdot {}^nC_2(12)^2 + 13 \cdot {}^nC_3(12)^3 + \ldots.$$

$$\Rightarrow 145n + 13 \cdot {}^nC_2(12)^2 + 13 \cdot {}^nC_3(12)^3 + \ldots.$$

If ($n = 144m$, $m \in N$) then it is divisible by 144 for infinite values of n.

Ans. Option 4

## SECTION-B

**21.** The value of tan 9° – tan 27° – tan 63° + tan 81° is ____ :

**Sol.** **4**

$(\tan 9^o + \cot 9^o) - (\tan 27^o + \cot 27^o)$

$$\frac{1}{\sin 9^o \cos 9^o} - \frac{1}{\sin 27^o \cos 27^o}$$

$$\frac{2}{\sin 18^o} - \frac{2}{\sin 54^o}$$

$$\frac{2(4)}{\sqrt{5} - 1} - \frac{2(4)}{(\sqrt{5} + 1)}$$

$$\frac{8(\sqrt{5} + 1)}{4} - \frac{8(\sqrt{5} - 1)}{4}$$

$$2\left[(\sqrt{5} + 1) - (\sqrt{5} - 1)\right]$$

$= 4$

**22.** If $(20)^{19} + 2(21)(20)^{18} + 3(21)^2(20)^{17} + \ldots + 20(21)^{19} = k(20)^{19}$, then k is equal to _____ :

**Sol.** **400**

$S = (20)^{19} + 2(21)(20)^{18} + \ldots\ldots. + 20(21)^{19}$

$$\frac{21}{20}S = 21(20)^{18} + 2(21)^9(20)^{17} + \ldots\ldots. + (21)^{20}$$

Subtract

$$\left(1 - \frac{21}{20}\right)S = (20)^{19} + (21)(20)^{18} + (21)^2(20)^{17} + \ldots\ldots. + (21)^{19} - (21)^{20}$$

$$\left(\frac{-1}{20}\right)S = (20)^{19}\left[\frac{1 - \left(\frac{21}{20}\right)^{20}}{1 - \frac{21}{20}}\right] - (21)^{20}$$

$$\left(\frac{-1}{20}\right)S = (21)^{20} - (20)^{20} - (21)^{20}$$

$S = (20)^{21} = K\,(20)^{19}$ (given)
$K = (20)^2 = 400$

**23.** Let the eccentricity of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is reciprocal to that of the hyperbola $2x^2 - 2y^2 = 1$. If the ellipse intersects the hyperbola at right angles, then square of length of the latus-rectum of the ellipse is ______:

**Sol.** **2**

$E : \frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \rightarrow e$

$H : x^2 - y^2 = \frac{1}{2} \Rightarrow e' = \sqrt{2}$

$\boxed{e = \frac{1}{\sqrt{2}}}$

$\because e^2 = \frac{1}{2}$

$1-\frac{b^2}{a^2}=\frac{1}{2} \Rightarrow \frac{b^2}{a^2}=\frac{1}{2}$

$\boxed{a^2 = 2b^2}$

E & H are at right angle
they are confocal
Focus of Hyperbola = focus of ellipse

$\left(\pm\frac{1}{\sqrt{2}}\cdot\sqrt{2},0\right)=\left(\pm\frac{a}{\sqrt{2}},0\right)$

$\boxed{a=\sqrt{2}}$

$\because a^2 = 2b^2 \Rightarrow b^2 = 1$

Length of LR $=\frac{2b^2}{a}=\frac{2(1)}{\sqrt{2}}$

$= \sqrt{2}$

Square of LR = 2

**24.** For $\alpha, \beta, z \in \mathbb{C}$ and $\lambda > 1$, if $\sqrt{\lambda - 1}$ is the radius of the circle $|z - \alpha|^2 + |z - \beta|^2 = 2\lambda$, then $|\alpha - \beta|$ is equal to ____:

**Sol.** **2**

$|z-z_1|^2+|z-z_2|^2=|z_1-z_2|^2$

$z_1=\alpha,\ z_2=\beta$

$|\alpha-\beta|^2=2\lambda$

$|\alpha-\beta|=\sqrt{2\lambda}$

$2r=\sqrt{2\lambda}$

$2\sqrt{\lambda-1}=\sqrt{2\lambda}$

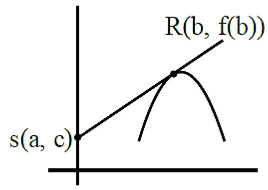
$\Rightarrow 4(\lambda-1)=2\lambda$

$\boxed{\lambda=2}$

$\boxed{|\alpha-\beta|=2}$

**25.** Let a curve y = f(x), $x \in (0, \infty)$ pass through the points $P\left(1,\frac{3}{2}\right)$ and $Q\left(a,\frac{1}{2}\right)$. If the tangent at any point R (b,f(b)) to the given curve cuts the y-axis at the points S(0, c) such that bc = 3, then $(PQ)^2$ is equal to _____:

**Sol.** **5**




Equation of tangent at R(b, f(2)) is

$y - f(b) = f'(b).(x - b)$

which passes through (0, c)

$\Rightarrow c - f(b) = f'(b).(-b)$

$\Rightarrow \frac{3}{b} - f(b) = f(b).(-b)$

$\Rightarrow \frac{bf'(b) - f(b)}{b^2} = -\frac{3}{b^3}$

$\Rightarrow d\left(\frac{f(b)}{b}\right) = -\frac{3}{b^3} \Rightarrow \frac{f(b)}{b} = \frac{3}{2b^2} + \lambda$

Which passes through (1, 3/2)

$\Rightarrow \frac{3}{2} = \frac{3}{2} + \lambda \Rightarrow \lambda = 0$

$\Rightarrow f(b) = \frac{3}{2b}$

$f(a) = \frac{1}{2} \Rightarrow \frac{1}{2} = \frac{3}{2b} \Rightarrow b = 3$

$\Rightarrow c = 1 \Rightarrow Q(3, 1/2)$

$\Rightarrow PQ^2 = 2^2 + (1)^2 = 5$

**26.** If the lines $\frac{x-1}{2} = \frac{2-y}{-3} = \frac{z-3}{\alpha}$ and $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{\beta}$ intersect, then the magnitude of the minimum value of $8\alpha\beta$ is _____:

**Sol.** **18**

If the lines $\frac{x-1}{2} = \frac{2-y}{-3} = \frac{z-3}{\alpha}$ and $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{\beta}$ intersect Point of first line (1, 2, 3) and point on second line (4, 1, 0).

Vector joining both points is $-3\hat{i} + \hat{j} + 3k$

Now vector along second line is $2\hat{i} + 3\hat{j} + \alpha k$

Also vector along second line is $5\hat{i} + 2\hat{j} + \beta k$

Now these three vectors must be coplanar

$$\Rightarrow \begin{vmatrix} 2 & 3 & \alpha \\ 5 & 2 & \beta \\ -3 & 1 & 3 \end{vmatrix}$$

$\Rightarrow 2(6-\beta) - 3(15+3\beta) + \alpha(11) = 0$

$\Rightarrow \alpha - \beta = 3$

Now $\alpha = 3 + \beta$

Given expression $8(3+\beta).\ \beta = 8(\beta^2 + 3\beta)$

$$= 8\left(\beta^2 + 3\beta + \frac{9}{4} - \frac{9}{4}\right) = 8\left(\beta + \frac{3}{2}\right)^2 - 18$$

So magnitude of minimum value = 18

**27.** Let $f(x) = \frac{x}{\left(1+x^n\right)^{\frac{1}{n}}}, x \in \mathbb{R} - \{-1\}, n \in \mathbb{N}, n > 2.$ If $f^n(x)$ =n (fofof….. upto n times) (x), then

$\lim\limits_{n\to\infty} \int_0^1 x^{n-2}\left(f^n(x)\right)dx$ is equal to _____:

**Sol.** **0**

Let $f(x) = \frac{x}{\left(1+x^n\right)^{\frac{1}{n}}}, x \in \mathbb{R} - \{-1\}, n \in \mathbb{N}, n > 2.$

If $f^n(x)$ =n (fofof….. upto n times) (x)

then $\lim\limits_{n\to\infty} \int_0^1 x^{n-2}\left(f^n(x)\right)dx$

$$f\left(f\left(x\right)\right) = \frac{x}{\left(1+2x^n\right)^{1/n}}$$

$$f\left(f\left(f\left(x\right)\right)\right) = \frac{x}{\left(1+3x^n\right)^{1/n}}$$

Similarly $f^n\left(x\right) = \frac{x}{\left(1+n\cdot x^n\right)^{1/n}}$

Now $\lim\limits_{n\to\infty} \int \frac{x^{n-2}\cdot xdx}{\left(1+n\cdot x^n\right)^{1/n}} = \lim\limits_{n\to\infty} \int \frac{x^{n-1}\cdot dx}{\left(1+n\cdot x^n\right)^{1/n}}$

Now $1 + nx^n = t$

$n^2 \cdot x^{n-1}dx = dt$

$x^{n-1}dx = \frac{dt}{n^2}$

$$\Rightarrow \lim_{n\to\infty} \frac{1}{n^2}\int_1^{1+n} \frac{dt}{t^{1/n}}$$

$$\Rightarrow \lim_{n\to\infty} \frac{1}{n^2}\left[\frac{t^{1-\frac{1}{n}}}{1-\frac{1}{n}}\right]_1^{1+n}$$

$\Rightarrow \lim\limits_{n\to\infty} \frac{1}{n(n-1)}\left(\left(1+n\right)^{\frac{n-1}{n}} - 1\right)$ Now let $n = \frac{1}{h}$

$$\Rightarrow \lim_{h\to 0}\frac{\left(1+\frac{1}{h}\right)^{1-h}-1}{\frac{1}{h}\frac{(1-h)}{h}}$$

Using series expansion

$\Rightarrow 0$

**28.** If the mean and variance of the frequency distribution.

| $x_i$ | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 |
|---|---|---|---|---|---|---|---|---|
| $f_i$ | 4 | 4 | $\alpha$ | 15 | 8 | $\beta$ | 4 | 5 |

are 9 and 15.08 respectively, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is ______:

**Sol.** **25**

| $x_i$ | $f_i$ | $f_ix_i$ | $f_ix_i^2$ |
|---|---|---|---|
| 2 | 4 | 8 | 16 |
| 4 | 4 | 16 | 64 |
| 6 | $\alpha$ | $6\alpha$ | $36\alpha$ |
| 8 | 15 | 120 | 960 |
| 10 | 8 | 80 | 800 |
| 12 | $\beta$ | $12\beta$ | $144\beta$ |
| 14 | 4 | 56 | 784 |
| 16 | 5 | 80 | 1280 |

$$N=\sum f_i = 40+\alpha+\beta$$

$$\sum f_i x_i = 360 + 6\alpha + 12\beta$$

$$\sum f_i x_i^2 = 3904 + 36\alpha + 144\beta$$

$$\text{Mean } (\bar{x}) = \frac{\sum f_i x_i}{\sum f_i} = 9$$

$\Rightarrow 360 + 6\alpha + 12\beta = 9\,(40 + \alpha + \beta)$

$3\alpha = 3\beta \Rightarrow \alpha = \beta$

$$\sigma^2 = \frac{\sum f_i x_i^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{\sum f_i}\right)^2$$

$$\Rightarrow \frac{3904+36\alpha+144\beta}{40+\alpha+\beta} - (\bar{x})^2 = 15.08$$

$$\Rightarrow \frac{3904+180\alpha}{40+2\alpha} - (9)^2 = 15.08$$

$\Rightarrow \alpha = 5$

Now, $\alpha^2 + \beta^2 - \alpha\beta = \alpha^2 = 25$

**29.** The number of points, where the curve $y = x^5 - 20x^3 + 50x + 2$ crosses the x-axis is _____:

**Sol.** **5**

$y = x^5 - 20x^3 + 50x + 2$

$$\frac{dy}{dx} = 5x^4 - 60x^2 + 50 = 5(x^4 - 12x^2 + 10)$$

$$\frac{dy}{dx} = 0 \Rightarrow x^4 - 12x^2 + 10 = 0$$

$\Rightarrow x^2 = \dfrac{12 \pm \sqrt{144-40}}{2}$

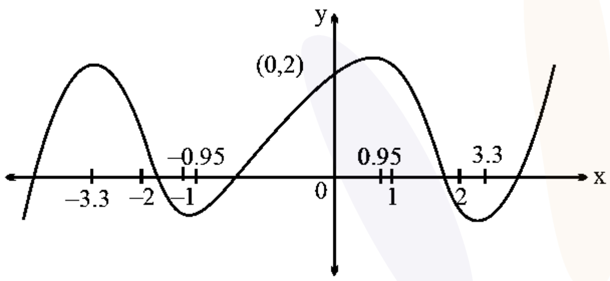
$\Rightarrow x^2 = 6 \pm \sqrt{26} \Rightarrow x^2 \approx 6 \pm 5.1$

$\Rightarrow x^2 \approx 11.1, 0.9$

$\Rightarrow x \approx \pm 3.3, \pm 0.95$

f(0) = 2, f (1) = + ve, f (2) = – ve

f(–1) = – ve, f (–2) = +ve




The number of points where the curve cuts the x-axis = 5.

**30.** The number of 4-letter words, with or without meaning, each consisting of 2 vowels and 2 consonants, which can be formed from the letters of the word UNIVERSE without repetition is _____:

**Sol.** **432**

UNIVERSE

| Vowels | Consonant |
|---|---|
| E, E | N, V, |
| I, U | R, S |

Case I  2 vowels different, 2 consonant different

$\left({}^{3}C_{2}\right)\left({}^{4}C_{2}\right)$ (4!)

= (3) (6) (24)

= 432