FINAL JEE–MAIN EXAMINATION – APRIL, 2023
Held On Saturday 15th April, 2023
TIME : 09:00 AM to 12:00 PM

SECTION - A

1. Let S be the set of all values of λ, for which the shortest distance between the lines $\frac{x-\lambda}{0}=\frac{y-3}{4}=\frac{z+6}{1}$ and $\frac{x+\lambda}{3}=\frac{y}{-4}=\frac{z-6}{0}$ is 13. Then $8\left|\sum_{\lambda\in S}\lambda\right|$ is equal to

(1) 302 (2) 306 (3) 304 (4) 308

Sol. **(2)**

Short test distance $=\frac{\left|\begin{vmatrix}0 & 4 & 1\\ 3 & -4 & 0\\ 2\lambda & 3 & -12\end{vmatrix}\right|}{\left|\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 4 & 1\\ 3 & -4 & 0\end{vmatrix}\right|}$

$13=\frac{|153+8\lambda|}{|4\hat{i}+3\hat{j}-12\hat{k}|}$

$=\frac{|153+8\lambda|}{13}$

$|153 + 8\lambda| = 169$

$153 + 8\lambda = 169, -169$

$\lambda=\frac{16}{8},\frac{-322}{8}$

$8\left|\sum_{\lambda\in S}\lambda\right|=306$

2. Let S be the set of all (λ, μ) for which the vectors $\lambda\hat{i}-\hat{j}+\hat{k},\ \hat{i}+2\hat{j}+\mu\hat{k}$ and $3\hat{i}-4\hat{j}+5\hat{k}$, where $\lambda-\mu=5$, are coplanar, then $\sum_{(\lambda,\mu)\in S}80(\lambda^2+\mu^2)$ is equal to

(1) 2130 (2) 2210 (3) 2290 (4) 2370

Sol. **(3)**

$\begin{vmatrix}\lambda & -1 & 1\\ 1 & 2 & \mu\\ 3 & -4 & 5\end{vmatrix}=0$ & $\lambda - \mu = 5$

$\lambda(10 + 4\mu) + (5 - 3\mu) + (-10) = 0$

$(\mu + 5)(4\mu + 10) + 5 - 3\mu - 10 = 0$

$\mu = -15;\ \lambda = 5/4$

$\mu = -3;\ \lambda = 2$

Hence $\sum_{(\lambda,\mu)\in S}80\left(\lambda^2+\mu^2\right)$

$=80\left(\frac{250}{16}+13\right)$

$= 1250 + 1040$

$= 2290$

3. Let the foot of perpendicular of the point P(3, –2, –9) on the plane passing through the points (– 1, -2, -3), (9, 3, 4) , (9, -2, 1) be Q(α, β, γ). Then the distance of Q from the origin is

(1) $\sqrt{29}$ (2) $\sqrt{38}$ (3) $\sqrt{42}$ (4) $\sqrt{35}$

Sol. **(3)**

P(3, –2, –9)

Equation of plane through A,B,C.

$$\begin{vmatrix} x+1 & y+2 & z+3 \\ 10 & 5 & 7 \\ 10 & 0 & 4 \end{vmatrix} = 0$$

2x + 3y – 5z – 7 = 0

Foot of $\perp^r$ of P (3, –2, –9) is

$$\frac{x-3}{2} = \frac{y+2}{3} = \frac{z+9}{-5} = -\frac{(\cancel{6}-\cancel{6}+45-7)}{4+9+25}$$

$$\frac{x-3}{2} = \frac{y+2}{3} = \frac{z+9}{-5} = -1$$

Q(1, –5, –4) ≡ (α, β, γ)

$$OQ = \sqrt{\alpha^2+\beta^2+\gamma^2} = \sqrt{42}$$

4. If the set $\left\{\mathrm{Re}\left(\frac{z-\bar{z}+z\bar{z}}{2-3z+5\bar{z}}\right) : z \in C, \mathrm{Re}(z) = 3\right\}$ is equal to the interval (α,β] , then 24 (β–α) is equal to

(1) 36 (2) 27 (3) 30 (4) 42

Sol. **(3)**

Let $z_1 = \left(\frac{z-\bar{z}+z\bar{z}}{2-3z+5\bar{z}}\right)$

Let z = 3 + iy

$\bar{z} = 3 - iy$

$$z_1 = \frac{2iy+(9+y^2)}{2-3(3+iy)+5(3-iy)}$$

$$= \frac{9+y^2+i(2y)}{8-8iy}$$

$$= \frac{(9+y^2)+i(2y)}{8(1-iy)}$$

$$\mathrm{Re}(z_1) = \frac{(9+y^2)-2y^2}{8(1+y^2)}$$

$$= \frac{9-y^2}{8(1+y^2)}$$

$$=\frac{1}{8}\left[\frac{10-\left(1+y^2\right)}{\left(1+y^2\right)}\right]$$

$$=\frac{1}{8}\left[\frac{10}{\left(1+y^2\right)}-1\right]$$

$$1+y^2\in[1,\infty]$$

$$\frac{1}{1+y^2}\in(0,1]$$

$$\frac{10}{1+y^2}\in(0,10]$$

$$\frac{10}{1+y^2}-1\in(-1,9]$$

$$\text{Re}(z_1)\in\left(\frac{-1}{8},\frac{9}{8}\right]$$

$$\alpha=\frac{-1}{8},\beta=\frac{9}{8}$$

$$24(\beta-\alpha)=24\left(\frac{9}{8}+\frac{1}{8}\right)=30$$

5. Let x = x(y) be the solution of the differential equation 2(y+2) $\log_e$ (y+2) dx + (x+4–2$\log_e$ (y+2)) dy = 0, y > –1 with $x\left(e^4-2\right)=1$. Then $x\left(e^9-2\right)$ is equal to

(1) $\frac{4}{9}$ (2) $\frac{32}{9}$ (3) $\frac{10}{3}$ (4) 3

Sol. **(2)**

2(y+2) ln(y+2)dx + (x + 4 – 2 ln (y + 2)) dy = 0

$$2\ln(y+2)+(x+4-2\ln(y+2))\frac{1}{y+2}\cdot\frac{dy}{dx}=0$$

let, ln(y+2) = t

$$\frac{1}{y+2}\cdot\frac{dy}{dx}=\frac{dt}{dx}$$

$$2t+(x+4-2t)\cdot\frac{dt}{dx}=0$$

$$(x+4-2t)\frac{dt}{dx}=-2t$$

$$\frac{dx}{dt}=\frac{2t-4-x}{2t}$$

$$\frac{dx}{dt}+\frac{x}{2t}=\frac{2t-4}{2t}$$

$$x.t^{1/2}=\int\frac{2t-4}{2t}.t^{1/2}.dt$$

$$x.t^{1/2}=\int\left(t^{1/2}-\frac{2}{t^{1/2}}\right).dt$$

$$= \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - 2.\frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$x.t^{\frac{1}{2}} = \frac{2t^{\frac{3}{2}}}{3} - 4t^{\frac{1}{2}} + C$$

$$x = \frac{2}{3}.t - 4 + C.t^{\frac{-1}{2}}$$

$$x = \frac{2}{3}\ln(y+2) - 4 + C.(\ln(y+2))^{\frac{-1}{2}}$$

Put $y = e^4 - 2$, $x = 1$

$$1 = \frac{2}{3} \times 4 - 4 + C \times \frac{1}{2}$$

$$\frac{C}{2} = 5 - \frac{8}{3} = \frac{7}{3}$$

$$\Rightarrow C = \frac{14}{3}$$

$$x = \frac{2}{3} \times 9 - 4 + \frac{14}{3} \times \frac{1}{3}$$

$$= 2 + \frac{14}{9}$$

$$= \frac{32}{9}$$

6. If $\int_0^1 \frac{1}{(5+2x-2x^2)(1+e^{(2-4x)})}dx = \frac{1}{\alpha}\log_e\left(\frac{\alpha+1}{\beta}\right), \alpha, \beta > 0$, then $\alpha^4 - \beta^4$ is equal to

(1) 19 (2) –21 (3) 21 (4) 0

Sol. **(3)**

$$I = \int_0^1 \frac{dx}{(5+2x-2x^2)(1+e^{2-4x})} \quad \ldots(i)$$

$x \to 1 - x$

$$I = \int_0^1 \frac{e^{2-4x}dx}{(5+2x-2x^2)(1+e^{2-4x})} \ldots(ii)$$

Add (i) and (ii)

$$2I\int_0^1 \frac{dx}{5+2x-2x^2} = \int_0^1 \frac{dx}{2\left(\frac{11}{4} - \left(x - \frac{1}{2}\right)^2\right)}$$

$$I = \frac{1}{\sqrt{11}}\ln\left(\frac{\sqrt{11}+1}{\sqrt{10}}\right) \qquad \alpha = \sqrt{11}, \quad \beta = \sqrt{10}$$

$\alpha^4 - \beta^4$ =121-100 = 21

7. The number of common tangents, to the circles $x^2 + y^2 - 18x - 15y + 131 = 0$ and $x^2 + y^2 - 6x - 6y - 7 = 0$, is

(1) 4 (2) 1 (3) 3 (4) 2

Sol. **(3)**

$$C_1\left(9, \frac{15}{2}\right) r_1 = \sqrt{81 + \frac{225}{4} - 131} = \frac{5}{2}$$

C_2 (3,3) $r_2 = 5$

$$C_1C_2 = \sqrt{6^2 + \frac{81}{4}} = \frac{15}{2}$$

$$r_1 + r_2 = \frac{15}{2}$$

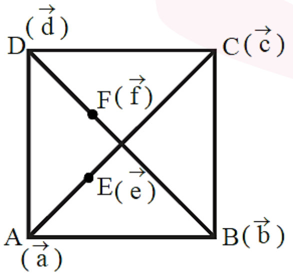
$C_1C_2 = r_1 + r_2$

Number of common tangents = 3

8. Let ABCD be a quadrilateral. If E and F are the mid points of the diagonals AC and BD respectively and $\left(\overrightarrow{AB} - \overrightarrow{BC}\right) + \left(\overrightarrow{AD} - \overrightarrow{DC}\right) = k\overrightarrow{FE}$, then k is equal to

(1) 4 (2) 2 (3) –2 (4) –4

Sol. **(4)**



$$\overrightarrow{AB} - \overrightarrow{BC} + \overrightarrow{AB} - \overrightarrow{DC} = k\overrightarrow{FE}$$

$$\left(\vec{b} - \vec{a}\right) - \left(\vec{c} - \vec{b}\right) + \left(\vec{d} - \vec{a}\right) - \left(\vec{c} - \vec{d}\right) = k\overrightarrow{FE}$$

$$2\left(\vec{b} + \vec{d}\right) - 2\left(\vec{a} - \vec{c}\right) = k\overrightarrow{FE}$$

$$2\left(2\vec{f}\right) - 2\left(2\vec{e}\right) = k\overrightarrow{FE}$$

$$4\left(\vec{f} - \vec{e}\right) = k\overrightarrow{FE}$$

$$-4\overrightarrow{FE} = k\overrightarrow{FE}$$

$k = -4$

9. Let $\left(a + bx + cx^2\right)^{10} = \sum_{i=0}^{20} p_i x^i$, a, b, c $\in$ N. If $p_1 = 20$ and $p_2 = 210$, then 2 (a+b+c) is equal to

(1) 8 (2) 12 (3) 6 (4) 15

Sol. **(2)**

$$\left(a + bx + cx^2\right)^{10} = \sum_{i=0}^{20} p_i x^i,$$

Coefficient of $x^1 = 20$

$$20 = \frac{10!}{9!1!} \times a^9 \times b^1$$

$a^9 . b = 2$

a = 2, b = 2

Coefficient of $x^2 = 210$

$$210 = \frac{10!}{9!1!} \times a^9 \times c^1 + \frac{10!}{8!2!} \times a^8 b^2$$

$210 = 10.c + 45 \times 4$

$10c = 30$

$c = 3$

$2(a + b = c) = 12$

10. Let [x] denote the greatest integer function and f(x) = max { 1 + x + [x], 2 + x, x+2[x]}, $0 \le x \le 2$. Let m be the number of points in [0,2], where f is not continuous and n be the number of points in (0, 2) , where f is not differentiable. Then $(m+n)^2 + 2$ is equal to

(1) 6 (2) 3 (3) 2 (4) 11

Sol. **2**

$$\text{Let } g(x) = 1 + x + [x] = \begin{cases} 1+x; & x \in [0,1) \\ 2+x; & x \in [1,2) \\ 5; & x = 2 \end{cases}$$

$$\lambda(x) = x + 2[x] = \begin{cases} x; & x \in [0,1) \\ x+2; & x \in [1,2) \\ 6; & x = 2 \end{cases}$$

$r(x) = 2 + x$

$$f(x) = \begin{cases} 2+x; & x \in [0,2) \\ 6; & x = 2 \end{cases}$$

f(x) is discontinuous only at $x = 2 \Rightarrow m = 1$

f(x) is differentiable in $(0,2) \Rightarrow n = 0$

$(m + n)^2 + 2 = 3$

11. A bag contains 6 white and 4 black balls. A die is rolled once and the number of ball equal to the number obtained on the die are drawn from the bag at random. The probability that all the balls drawn are white is

(1) $\frac{1}{4}$ (2) $\frac{9}{50}$ (3) $\frac{11}{50}$ (4) $\frac{1}{5}$

Sol. **4**

6	W
4	R

$$\frac{1}{6} \times \left[\frac{^6C_1}{^{10}C_1} + \frac{^6C_2}{^{10}C_2} + \frac{^6C_3}{^{10}C_3} + \frac{^6C_4}{^{10}C_4} + \frac{^6C_5}{^{10}C_5} + \frac{^6C_6}{^{10}C_6} \right]$$

$$= \frac{1}{6}\left(\frac{126+70+35+15+5+1}{210}\right) = \frac{42}{210} = \frac{1}{5}$$

12. If the domain of the function $f(x) = \log_e\left(4x^2 + 11x + 6\right) + \sin^{-1}(4x+3) + \cos^{-1}\frac{10x+6}{3}$ is (α,β], then 36 |α+β| is equal to

(1) 72 (2) 63 (3) 45 (4) 54

Sol. **3**

$f(x) = \ln(4x^2 + 11x + 6) + \sin^{-1}(4x + 3)$

$+\cos^{-1}\left(\frac{10x+6}{3}\right)$

(i) $4x^2 + 11x + 6 > 0$

$4x^2 + 8x + 3x + 6 > 0$

$(4x + 3)(x + 2) > 0$

$x \in (-\infty, -2) \cup \left(-\frac{3}{4}, \infty\right)$

(ii) $4x + 3 \in [-1, 1]$

$x \in [-1, -1/2]$

(iii) $\frac{10x+6}{3} \in [-1, 1]$

$x \in \left[-\frac{9}{10}, -\frac{3}{10}\right]$

$x \in \left(-\frac{3}{4}, -\frac{1}{2}\right]$ $\qquad \alpha = -\frac{3}{4}, \beta = -\frac{1}{2}$

$\alpha + \beta = -\frac{5}{4}$

$36|\alpha + \beta| = 45$

13. Let the determinant of a square matrix A of order m be m – n, where m and n satisfy 4m + n = 22 and 17m + 4n = 93. If det (n adj (adj (mA))) = $3^a 5^b 6^c$, then a + b + c is equal to

(1) 101 (2) 84 (3) 109 (4) 96

Sol. **4**

$|A| = m - n$

$4m + n = 22$

$17m + 4n = 93$

$m = 5, n = 2$

$|A| = 3$

$|2\ \text{adj}(\text{adj } 5A))| = 2^5 |5A|^{16}$

$= 2^5 . 5^{80} |A|^{16}$

$= 2^5 . 5^{80} . 3^{16}$

$= 3^{11} . 5^{80} . 6^5$

$a + b + c = 96$

14. The mean and standard deviation of 10 observations are 20 and 8 respectively. Later on, it was observed that one observation was recorded as 50 instead of 40. Then the correct variance is

(1) 14 (2) 11 (3) 12 (4) 13

Sol. **4**

$\mu = 20, \sigma = 8$

$\mu_{\text{Corrected}} = \frac{200 - 50 + 40}{10} = 19$

$\sigma^2 = \frac{1}{10}\sum x_i^2 - 20^2$

$(64 + 400)10 = \sum x_i^2$

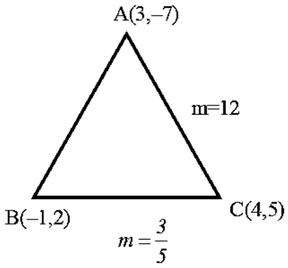
$\sigma^2_{\text{Corrected}} = \frac{1}{10}\left[(64 + 400)10 - 2500 + 1600\right] - 19^2$

$= 374 - 361 = 13$

15. If (α, β) is the orthocenter of the triangle ABC with vertices A (3, –7), B(–1,2) and C (4, 5), then $9\alpha - 6\beta + 60$ is equal to

(1) 30 (2) 35 (3) 40 (4) 25

Sol. **4**



Altitude of BC : $y + 7 = \frac{-5}{3}(x - 3)$

$3y + 21 = -5x + 15$

$5x + 3y + 6 = 0$

Altitude of AC: $y - 2 = \frac{-1}{12}(x + 1)$

$12y - 24 = -x - 1$

$x + 12y = 23$

$\alpha = \frac{-47}{19}, \quad \beta = \frac{121}{57}$

$9\alpha - 6\beta + 60 = 25$

16. The number of real roots of the equation $x|x| - 5|x+2| + 6 = 0$, is

(1) 5 (2) 6 (3) 4 (4) 3

Sol. **4**

$x|x| - 5|x+2| + 6 = 0$

C–1 :- $x \in [0, \infty]$

$x^2 - 5x - 4 = 0$

$x = \frac{5 \pm \sqrt{25+16}}{2} = \frac{5 + \sqrt{41}}{2}$

$x = \frac{5 \pm \sqrt{41}}{2}$

C–2 :- :- $x \in [-2, 0)$

$-x^2 - 5x - 4 = 0$

$x^2 + 5x + 4 = 0$

$x = -1, -4$

$x = -1$

C–3 : $x \in [-\infty, -2)$

$-x^2 + 5x + 16 = 0$

$x^2 - 5x - 16 = 0$

$x = \frac{5 \pm \sqrt{25+64}}{2}$

$x = \frac{5 \pm \sqrt{89}}{2}$

$x = \frac{5 - \sqrt{89}}{2}$

17. Let the system of linear equations

$-x + 2y - 9z = 7$

$-x + 3y + 7z = 9$

$-2x + y + 5z = 8$

$-3x + y + 13z = \lambda$

has a unique solution $x = \alpha, y = \beta, z = \gamma$. Then the distance of the point (α, β, γ) from the plane $2x - 2y + z = \lambda$ is

(1) 7 (2) 9 (3) 13 (4) 11

Sol. **1**

$-x + 2y - 9z = 7$ –(1)

$-x + 3y - 7z = 9$ –(2)

$-2x + y + 5z = 8$ –(3)

(2) – (1)

$y + 16z = 2$ (4)

(3) – 2 × (1)

$-3y + 23z = -6$ – (5)

3 × (4) + (5)

$71z = 0 \Rightarrow z = 0$

$y = 2$

$x = -3$

$(-3, 2, 0) \to (\alpha, \beta, \gamma)$

Put in $-3x + y + 13z = l$

$\lambda = 9 + 2 = 11$

$$d = \left|\frac{-6-4-11}{3}\right| = 7$$

18. Let A_1 and A_2 be two arithmetic means and G_1, G_2, G_3 be three geometric means of two distinct positive numbers. Then $G_1^4 + G_2^4 + G_3^4 + G_1^2G_3^2$ is equal to

(1) $2(A_1 + A_2) G_1G_3$ (2) $(A_1 + A_2)^2 G_1G_3$ (3) $2(A_1 + A_2) G_1^2G_3^2$ (4) $(A_1 + A_2) G_1^2G_3^2$

Sol. **2**

a, A_1, A_2, b are in A.P.

$$d = \frac{b-a}{3}; A_1 = a + \frac{b-a}{3} = \frac{2a+b}{3}$$

$$A_2 = \frac{a+2b}{3}$$

$A_1 + A_2 = a + b$

a, G_1, G_2, G_3, b are in G.P.

$$r = \left(\frac{b}{a}\right)^{\frac{1}{4}}$$

$$G_1 = \left(a^3b\right)^{\frac{1}{4}}$$

$$G_2 = \left(a^2b^2\right)^{\frac{1}{4}}$$

$$G_3 = \left(ab^3\right)^{\frac{1}{4}}$$

$G_1^4 + _2^4 + G_3^4 + G_1^2 G_3^2 =$

$a^3b + a^2b^2 + ab^3 + \left(a^3b\right)^{\frac{1}{2}}.\left(ab^3\right)^{\frac{1}{2}}$

$= a^3b + a^2b^2 + ab^3 + a^2.b^2$

$= ab(a^2 + 2ab + b^2)$

$= ab(a + b)^2$

$= G_1.G_3.(A_1 + A_2)^2$

19. Negation of $p \wedge \left(q \wedge \sim\left(p \wedge q\right)\right)$ is

(1) $\left(\sim\left(p \wedge q\right)\right) \wedge q$ (2) $\sim\left(p \vee q\right)$ (3) $p \vee q$ (4) $\left(\sim\left(p \wedge q\right)\right) \vee p$

Sol. **4**

$\sim [p \wedge (q \wedge \sim (p \wedge q))]$

$\sim p \vee (\sim q \vee (p \wedge q))$

$\sim p \vee ((\sim q \vee p) \wedge (\sim q \vee q))$

$\sim p \vee (\sim q \vee p)$

$\sim (p \wedge q) \vee p$

20. The total number of three-digit numbers, divisible by 3, which can be formed using the digits 1, 3, 5, 8, if repetition of digits is allowed, is

(1) 21 (2) 18 (3) 20 (4) 22

Sol. **4**

(1,1,1) (3,3,3) (5,5,5) (8,8,,8)

(5,5,8) (8,8,5) (1,3,5) (1,3,8)

Total number $= 1 + 1 + 1 + 1 + \frac{3!}{2!} + \frac{3!}{2!} + 3! + 3! = 22$

SECTION - B

21. Let A = { 1 , 2 , 3, 4} and R be a relation on the set A×A defined by R={((a,b, (c,d):2a + 3b = 4c + 5d }. Then the number of elements in R is __________

Sol. **6**

A = {1,2,3,4}

R = {(a,b), (c,d)}

$2a + 3b = 4c + 5d = \alpha$ let

$2a = \{2,4,6,8\}$ $4c = \{4,8,12,16\}$

$3b = \{3,6,9,12\}$ $5d = \{5,10,15,20\}$

$$2a + 3b = \begin{Bmatrix} 5,8,11,14 \\ 7,10,13,16 \\ 9,12,15,18 \\ 11,14,17,20 \end{Bmatrix} 4c + 5d \begin{Bmatrix} 9,14,19,24 \\ 13,18.... \\ 17,22.... \\ 21,26.... \end{Bmatrix}$$

Possible value of α = 9, 13, 14, 14, 17, 18

Pairs of {(a,b), (c, d)} = 6

22. The number of elements in the set $\{n \in N : 10 \le n \le 100$ and $3^n - 3$ is a multiple of 7$\}$ is _________

Sol. **15**

$n \in [10, 100]$

$3^n - 3$ is multiple of 7

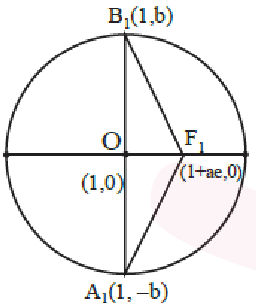
$3^n = 7\lambda + 3$

$n = 1, 7, 13, 20, \ldots\ldots 97$

Number of possible values of n = 15

23. Let an ellipse with centre (1,0) and latus rectum of length $\frac{1}{2}$ have its major axis along x-axis. If its minor axis subtends an angle 60° at the foci, then the square of the sum of the lengths of its minor and major axes is equal to ________

Sol. **9**



L.R. $= \frac{2b^2}{a} = \frac{1}{2}$

$4b^2 = a$ ...(i)

Ellipse $\frac{(x-1)^2}{a^2} + \frac{y^2}{b^2} = 1$

$m_{B_2F_1} = \frac{1}{\sqrt{3}}$

$\frac{b}{ae} = \frac{1}{\sqrt{3}}$

$3b^2 = a^2e^2 = a^2 - b^2$

$4b^2 = a^2$ ...(ii)

From (i) and (ii)

$a = a^2$

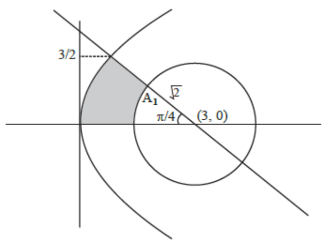
$\therefore\ a = 1$

$b^2 = \frac{1}{4}$

$((2a) + (2b))^2 = 9$

24. If the area bounded by the curve $2y^2 = 3x$, lines $x + y = 3$, $y = 0$ and outside the circle $(x-3)^2 + y^2 = 2$ is A , then $4(\pi + 4A)$ is equal to ________.

Sol. **42**



$y^2 = \frac{3x}{2}, x + y = 3, y = 0$

$2y^2 = 3(3 - y)$

$2y^2 + 3y - 9 = 0$

$2y^2 - 3y + 6y - 9 = 0$

$(2y - 3)(y + 2) = 0;\ y = 3/2$

$\text{Area}\left(\int_0^{\frac{3}{2}}(x_R - x_2)dy\right) - A_1$

$= \int_0^{\frac{3}{2}}\left((3-y) - \frac{2y^2}{3}\right)dy - \frac{\pi}{8}(2)$

$A = \left(3y - \frac{y^2}{2} - \frac{2y^3}{9}\right)_0^{\frac{3}{2}} - \frac{\pi}{4}$

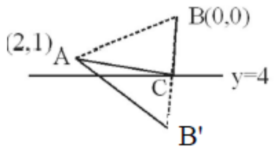
$4A + \pi = 4\left[\frac{9}{2} - \frac{9}{8} - \frac{3}{4}\right] = \frac{21}{2} = 10.50$

$\therefore 4(4A + \pi) = 42$

25. Consider the triangles with vertices A(2,1), B(0,0) and C(t, 4), $t \in [0,4]$. If the maximum and the minimum perimeters of such triangles are obtained at $t = \alpha$ and $t = \beta$ respectively, then $6\alpha + 21\beta$ is equal to ______

Sol. **48**

A (2,1), B (0,0), C (t, 4) : $t \in [0,4]$



$B1(0,8) \equiv$ image of B w.r.t. $y = 4$

for AC+BC+AB to be minimum

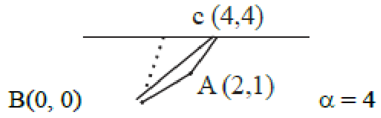
$m_{AB'} = \frac{-7}{2}$

line $AB_1 = 7x + 2y = 16$

$C\left(\frac{8}{7}, 4\right)$

$\beta = \frac{8}{7}$

For max. perimeter



$AB = \sqrt{5} : BC = 4\sqrt{2}, AC = \sqrt{13}$

$6\alpha + 21\beta = 24 + 24 = 48$

26. Let the plane P contain the line 2x + y–z–3 = 0 = 5x–3y + 4z + 9 and be parallel to the line $\frac{x+2}{2}=\frac{3-y}{-4}=\frac{z-7}{5}$. Then the distance of the point A(8,–1,–19) from the plane P measured parallel to the line $\frac{x}{-3}=\frac{y-5}{4}=\frac{2-z}{-12}$ is equal to _______

Sol. **26**

Plane ≡ $P_1 = \lambda P_2 = 0$

$(2x + y - z - 3) + \lambda(5x - 3y) + 4z + 9) = 0$

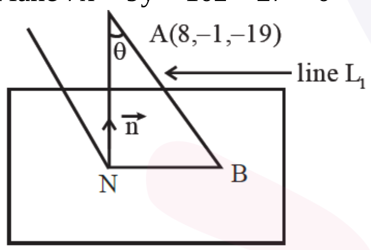
$(5\lambda + 2)x + (1 - 3\lambda)y + (4\lambda - 1)z + 9\lambda - 3 = 0$

$\vec{n}\cdot\vec{b}=0$ where $\vec{b}$ (2, 4, 5)

$2(5\lambda + 2) + 4(1 - 3\lambda) + 5(4\lambda - 1) = 0$

$\lambda=-\frac{1}{6}$

Plane 7x + 9y – 10z – 27 = 0



Equation of line AB is

$\frac{x-8}{-3}=\frac{y+1}{4}=\frac{z+19}{12}=\lambda$

Let B = (8–3λ,–1 + 4λ, –19 + 12λ) lies on plane P

$\therefore$ 7 (8 – 3λ) + 9 (4λ – 1) - 10 (12λ – 19) = 27

λ = 2

$\therefore$ Point B = (2, 7, 5)

$AB=\sqrt{6^2+8^2+24^2}=26$

27. If the sum of the series $\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{2^2}-\frac{1}{2.3}+\frac{1}{3^2}\right)+\left(\frac{1}{2^3}-\frac{1}{2^2.3}+\frac{1}{2.3^2}-\frac{1}{3^3}\right)+$ $\left(\frac{1}{2^4}-\frac{1}{2^2.3}+\frac{1}{2^2.3^2}-\frac{1}{2.3^2}+\frac{1}{3^4}\right)+......$ is $\frac{\alpha}{\beta}$, where α and β are co-prime, then α+3β is equal to __

Sol. **7**

$P\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{2^2}-\frac{1}{2.3}+\frac{1}{3^2}\right)+\left(\frac{1}{2^3}+\frac{1}{2^2.3}+\frac{1}{2.3^2}-\frac{1}{3^3}\right)+...$ $P\left(\frac{1}{2}+\frac{1}{3}\right)=\left(\frac{1}{2^2}-\frac{1}{3^2}\right)+\left(\frac{1}{2^3}+\frac{1}{3^3}\right)+\left(\frac{1}{2^4}-\frac{1}{3^4}\right)+...$

$\frac{5P}{6}=\frac{\frac{1}{4}}{1-\frac{1}{2}}-\frac{\frac{1}{9}}{1+\frac{1}{3}}$

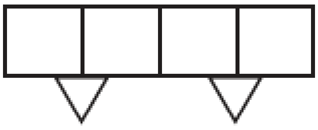
$\frac{5P}{6}=\frac{1}{2}-\frac{1}{12}=\frac{5}{12}$

$\therefore P=\frac{1}{2}=\frac{\alpha}{\beta}$ $\therefore \alpha = 1, \beta = 2$

α + 3β = 7

28. A person forgets his 4-digit ATM pin code. But he remembers that in the code all the digits are different, the greatest digit is 7 and the sum of the first two digits is equal to the sum of the last two digits. Then the maximum number of trials necessary to obtain the correct code is______

Sol. **72**



Sum of first two digits

Sum of last two digits = α

Case-I : $\alpha = 7$

$2 \times 12 = 24$ ways.

7 0	
0 7	1 6
	2 5
	3 4
	4 3
	5 2
	6 1

Case – II : $\alpha = 8$

17	26
71	62
	35
	53

2×8 ways

= 16 ways

Case-III : $\alpha = 9$

27	36
72	63
	45
	54

2×8 ways

= 16 ways

Case IV : $\alpha = 10$

37	46
73	64

2×4 ways

8 ways

Case V : $\alpha = 11$

47	56
74	65

2×4 ways

8 ways

Ans. $24 + 16 + 16 + 8 + 8 = 72$

29. If the line x = y = z intersects the line xsin A + ysin B + zsin C–18 = 0 = xsin 2A+ y sin 2B + z sin 2C – 9, where A, B, C are the angles of a triangle ABC, then 80 $\left(\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right)$ is equal to __

Sol. **5**

$\sin A + \sin B + \sin C = \frac{18}{x}$

$\sin 2A + \sin 2B + \sin 2C = \frac{9}{x}$

$\therefore$ sinA + sinB + sinC = 2(sin2A + sin2B + sin2C)

4cosA/2 cosB/2 cosC/2 = 2(4sinAsinBsinC)

16sinA/2sinB/2sinC/2 = 1

Hence Ans. = 5.

30. Let $f(x)=\int\frac{dx}{(3+4x^2)\sqrt{4-3x^2}}$, $|x| < \frac{2}{\sqrt{3}}$. If f (0) = 0 and f(1) = $\frac{1}{\alpha\beta}\tan^{-1}\left(\frac{\alpha}{\beta}\right)$ α, β > 0, then $\alpha^2 + \beta^2$ is equal to _____

Sol. **28**

$$f(x)=\int\frac{dx}{(3+4x^2)\sqrt{4-3x^2}}$$

$$x=\frac{1}{t}$$

$$=\int\frac{\frac{-1}{t^2}dt}{\frac{(3t^2+4)}{t^2}\frac{\sqrt{4t^2-3}}{t}}$$

$$=\int\frac{-dt.t}{(3t^2+4)\sqrt{4t^2-3}}$$: Put $4t^2 - 3 = z^2$

$$=-\frac{1}{4}\int\frac{zdx}{\left(3\left(\frac{z^2+3}{4}\right)+4\right)z}$$

$$=\int\frac{-dz}{3z^2+25}=-\frac{1}{3}\int\frac{dz}{z^2+\left(\frac{5}{\sqrt{3}}\right)^2}$$

$$=-\frac{1}{3}\frac{\sqrt{3}}{5}\tan^{-1}\left(\frac{\sqrt{3}z}{5}\right)+C$$

$$=-\frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}}{5}\sqrt{4t^2-3}\right)+C$$

$$f(x)=-\frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}}{5}\sqrt{\frac{4-3x^2}{x^2}}\right)+C$$

$$\because f(0)=0 \because c=\frac{\pi}{10\sqrt{3}}$$

$$f(1) = -\frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}}{5}\right) + \frac{\pi}{10\sqrt{3}}$$

$$f(1) = \frac{1}{5\sqrt{3}}\cot^{-1}\left(\frac{\sqrt{3}}{5}\right) = \frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{5}{\sqrt{3}}\right)$$

$$\alpha = 5 : \beta = \sqrt{3} \therefore \alpha^2 + \beta^2 = 28$$