FINAL JEE–MAIN EXAMINATION – APRIL, 2023
Held On Thursday 06th April, 2023
TIME : 09:00 AM to 12:00 PM

SECTION - A

31. The kinetic energy of an electron, α -particle and a proton are given as 4 K, 2 K and K respectively. The de-Broglie wavelength associated with electron (λe), α -particle $(\lambda\alpha)$ and the proton (λp) are as follows :

(1) $\lambda\alpha > \lambda p > \lambda e$
(2) $\lambda\alpha = \lambda p > \lambda e$
(3) $\lambda\alpha = \lambda p < \lambda e$
(4) $\lambda\alpha < \lambda p < \lambda e$

Sol. **(4)**

According to De-Broglie, Momentum $P = \frac{h}{\lambda}$, where h is plank's constant and λ is wavelength.

Also, relation between Kinetic energy(KE) and momentum(P) is given by: $P = \sqrt{2mKE}$

Now, $\lambda = \frac{h}{P} = \frac{h}{\sqrt{2mKE}}$

$$\lambda_e = \frac{h}{\sqrt{2m_eKE_e}} = \frac{h}{\sqrt{2m_e \times 4k}} = \frac{h}{\sqrt{8m_ek}}$$

$$\lambda_p = \frac{h}{\sqrt{2m_pKE_p}} = \frac{h}{\sqrt{2m_pk}}$$

$$\lambda_\alpha = \frac{h}{\sqrt{2m_\alpha KE_\alpha}} = \frac{h}{\sqrt{2m_\alpha . 4k}} = \frac{h}{\sqrt{2 \times 2m_p . 2k}} = \frac{h}{\sqrt{8m_pk}}$$

From the above data, $\lambda_\alpha < \lambda_P < \lambda_e$

32. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A : Earth has atmosphere whereas moon doesn't have any atmosphere.

Reason R : The escape velocity on moon is very small as compared to that on earth.

In the light of the above statements. choose the correct answer from the options given below:

(1) Both A and R are correct and R is the correct explanation of A
(2) A is false but R is true
(3) Both A and R are correct but R is NOT the correct explanation of A
(4) A is true but R is false

Sol. **(1)**

$V_{esc} = \sqrt{\frac{2GM}{r}} = \sqrt{2gr}$

Radius of moon is less than that of earth and acceleration due to gravity is also less on moon as compared to that on earth.

Thus , V_{esc} of Moon < V_{esc} of Earth

This is also the reason behind escape of atmosphere from moon.

33. A source supplies heat to a system at the rate of 1000 W. If the system performs work at a rate of 200 W. The rate at which internal energy of the system increase is

(1) 500 W
(2) 600 W
(3) 800 W
(4) 1200 W

Sol. **(3)**

From Ist law of thermodynamics,

$dQ = dU + dW$

Also, we can write this as, $\frac{dQ}{dt} = \frac{dU}{dt} + \frac{dW}{dt}$

$\Rightarrow 1000W = \frac{dU}{dt} + 200W$

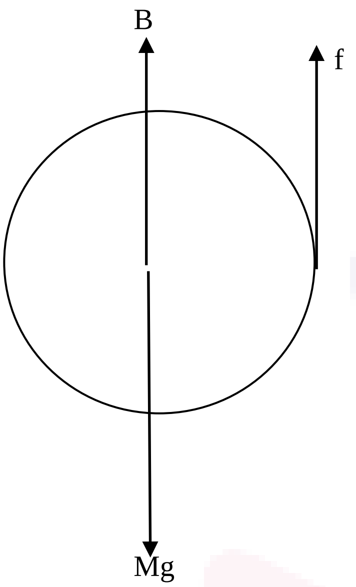
$\Rightarrow \frac{dU}{dt} = 800W$

34. A small ball of mass M and density ρ is dropped in a viscous liquid of density ρ_o. After some time, the ball falls with a constant velocity. What is the viscous force on the ball?

(1) $F = Mg\left(1 + \frac{\rho_0}{\rho}\right)$
(2) $F = Mg\left(1 + \frac{\rho}{\rho_o}\right)$
(3) $F = Mg\left(1 - \frac{\rho_o}{\rho}\right)$
(4) $F = Mg(1 \pm \rho\rho_o)$

Sol. **(3)**

At terminal velocity, net force on the ball is Zero.



$Mg = f + B$

$\Rightarrow Mg = f + V_{ball}\rho_o g \quad ....(i)$

Volume of ball = $\frac{M}{\rho}$

From eq (i),

$Mg = f + \frac{M}{\rho}\rho_o g$

$\Rightarrow f = Mg - \frac{M}{\rho}\rho_o g$

$\Rightarrow f = Mg(1 - \frac{\rho_o}{\rho})$

35. A small block of mass 100 g is tied to a spring of spring constant 7.5 N/m and length 20 cm. The other end of spring is fixed at a particular point A. If the block moves in a circular path on a smooth horizontal surface with constant angular velocity 5 rad/s about point A, then tension in the spring is –

(1) 0.75 N (2) 1.5 N (3) 0.25 N (4) 0.50 N

Sol. **(1)**

$kx = m\omega^2 r$

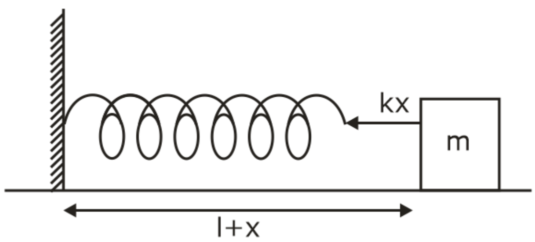
$\Rightarrow kx = 0.1 \times 25 \times (0.2 + x)$

$\Rightarrow 7.5x = 2.5(0.2 + x)$

$\Rightarrow 3x = 0.2 + x$

$\Rightarrow 2x = 0.2$

$\Rightarrow x = 0.1m$

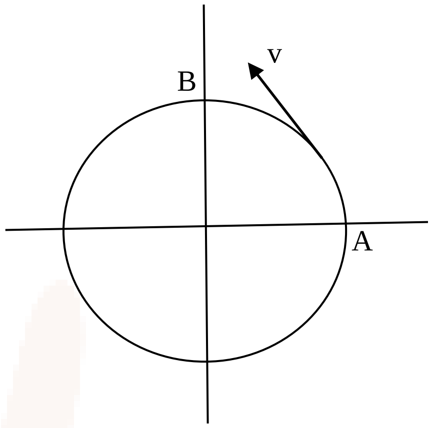


Now, tension in the spring = $kx = 7.5 \times 0.1\ N = 0.75\ N$

36. A particle is moving with constant speed in a circular path. When the particle turns by an angle 90°, the ratio of instantaneous velocity to its average velocity is $\pi : x\sqrt{2}$. The value of x will be -

(1) 7 (2) 2 (3) 1 (4) 5

Sol. **(2)**



$V_A = v\hat{j}$

And $V_B = -v\hat{i}$

Time to reach from A to B = $\frac{2\pi R}{4} \times \frac{1}{v} = \frac{\pi R}{2v}$

Displacement from A to B = $R\sqrt{2}$

Now, Average velocity from A to B = $\frac{Displacement}{Time} = \frac{R\sqrt{2}}{\frac{\pi R}{2v}} = \frac{2\sqrt{2}v}{\pi}$

Instantaneous velocity at B is $-v\hat{i}$

According to question, $\frac{instantaneous\ velocity}{average\ velocity} = \frac{\pi}{x\sqrt{2}}$

$\frac{v}{\frac{2\sqrt{2}v}{\pi}} = \frac{\pi}{x\sqrt{2}}$

$\Rightarrow \frac{\pi}{2\sqrt{2}} = \frac{\pi}{x\sqrt{2}}$

$\Rightarrow x = 2$

37. Two resistances are given as $R_1 = (10 \pm 0.5)\ \Omega$ and $R_2 = (15 \pm 0.5)\Omega$. The percentage error in the measurement of equivalent resistance when they are connected in parallel is -

(1) 2.33 (2) 4.33 (3) 5.33 (4) 6.33

Sol. **(2)**

In parallel combination, $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

$\Rightarrow \frac{1}{R_{eq}} = \frac{1}{10} + \frac{1}{15} = \frac{5}{30} = \frac{1}{6}$

Now, for error calculation,

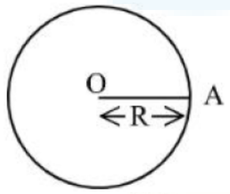
$\frac{dR_{eq}}{R_{eq}^2} = \frac{dR_1}{R_1^2} + \frac{dR_2}{R_2^2}$

$\Rightarrow \frac{dR_{eq}}{36} = \frac{0.5}{100} + \frac{0.5}{225}$

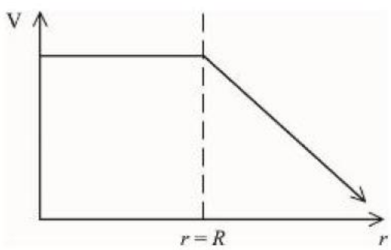
$dR_{eq} = 36 \times 0.5 \times \left(\frac{13}{900}\right) = 18 \times \frac{13}{900} = \frac{26}{100} = 0.26$

Now , $\frac{dR_{eq}}{R_{eq}} \times 100 = \frac{0.26}{6} \times 100 = \frac{26}{6} = 4.33$

38. For a uniformly charged thin spherical shell, the electric potential (V) radially away from the entre (O) of shell can be graphically represented as –



(1)



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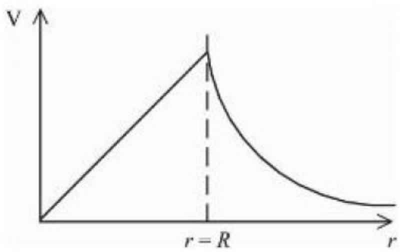

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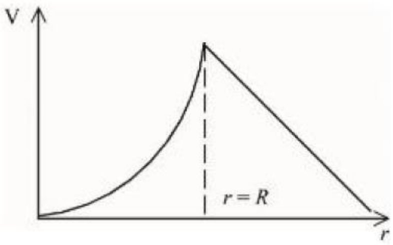

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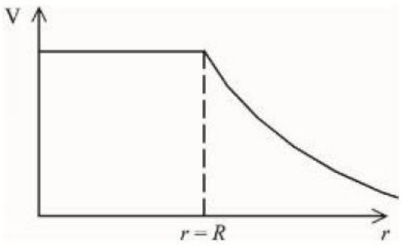


Sol. **(4)**

For $r \leq R, V = \frac{KQ}{R}$, i.e., Constant everywhere inside.

For $r > R, V = \frac{KQ}{r}$, i.e., Decreases with r.

39. A long straight wire of circular cross-section (radius a) is carrying steady current I. The current I is uniformly distributed across this cross-section. The magnetic field is

(1) zero in the region r < a and inversely proportional to r in the region r > a

(2) inversely proportional to r in the region r < a and uniform throughout in the region r > a

(3) directly proportional to r in the region r < a and inversely proportional to r in the region r > a

(4) uniform in the region r < a and inversely proportional to distance r from the axis, in the region r > a

Sol. **(3)**

It is a case of solid infinite current carrying wire.

Using ampere circuital law,

CASE I: if r ≤ R

$B = \frac{\mu_0 i}{2\pi R^2} r$

CASE II: r>R

$B = \frac{\mu_0 i}{2\pi r}$

40. By what percentage will the transmission range of a TV tower be affected when the height of the tower is increased by 21% ?

(1) 12% (2) 15% (3) 14% (4) 10%

Sol. **(4)**

New range is given by $\sqrt{2R(h + 0.21h)}$

$= \sqrt{2Rh \times 1.21}$

$= 1.1\sqrt{2Rh}$

It means new range increases by 10 %.

41. The number of air molecules per cm^3 increased from 3×10^{19} to 12×10^{19}. The ratio of collision frequency of air molecules before and after the increase in the number respectively is :

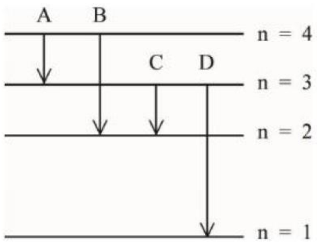
(1) 0.25 (2) 0.75 (3) 1.25 (4) 0.50

Sol. **(1)**

Collision frequency is given by $Z = n\pi d^2 V_{avg}$, where n is number of molecules per unit volume.

$\frac{Z_1}{Z_2} = \frac{n_1}{n_2} = \frac{3}{12} = \frac{1}{4} = 0.25$

42. The energy levels of an hydrogen atom are shown below. The transition corresponding to emission of shortest wavelength is



(1) A (2) D (3) C (4) B

Sol. **(2)**

$E = \frac{hc}{\lambda}$

$\Rightarrow \lambda = \frac{hc}{E}$

For λ_{min}, E must be maximum.

And E is maximum for D.

43. For the plane electromagnetic wave given by $E = E_0 \sin(\omega t - kx)$ and $B = B_0 \sin(\omega t - kx)$, the ratio of average electric energy density to average magnetic energy density is

(1) 2 (2) 1/2 (3) 1 (4) 4

Sol. **(3)**

In EM waves, average electric energy density is equal to average magnetic energy density.

$$\frac{1}{4}\epsilon_0 E_0^2 = \frac{1}{4\mu_0}B_0^2$$

44. A planet has double the mass of the earth. Its average density is equal to that of the earth. An object weighing W on earth will weigh on that planet:

(1) $2^{1/3}$ W (2) 2 W (3) W (4) $2^{2/3}$ W

Sol. **(1)**

Average Density of planet = average density of earth

$$\frac{M_e}{\frac{4}{3}\pi R_e^3} = \frac{M_p}{\frac{4}{3}\pi R_p^3}$$

$$\Rightarrow \frac{M_e}{R_e^3} = \frac{2M_e}{R_p^3}$$

$$\Rightarrow R_p = 2^{\frac{1}{3}} R_e \text{ ---------- } (i)$$

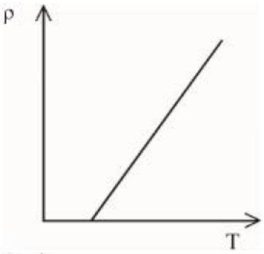
Now, $g = \frac{GM}{R^2}$

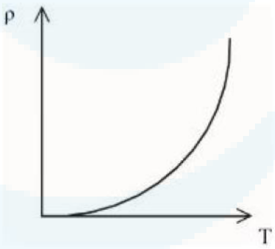
$$\frac{g_e}{g_p} = \frac{M_e}{R_e^2} \times \frac{R_p^2}{2M_e} = 2^{\frac{2}{3}-1} = 2^{-\frac{1}{3}}$$

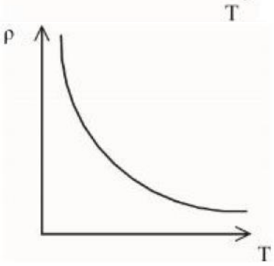
$$\Rightarrow g_p = 2^{\frac{1}{3}} g_e$$

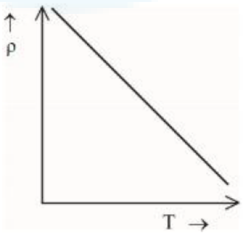
$$\Rightarrow W_p = 2^{\frac{1}{3}} W_e$$

45. The resistivity (ρ) of semiconductor varies with temperature. Which of the following curve represents the correct behavior

(1)


(2)


(3)


(4)


Sol. **(3)**

A semiconductor starts conduction more as the temperature increases. It means resistance decreases with increase in temperature. So, if temperature increases, its resistivity decreases.

Also, $\rho = \frac{m}{ne^2\tau}$

As Temperature increase, τ decreases but n increases and n is dominant over τ.

46. A monochromatic light wave with wavelength λ_1 and frequency ν_1 in air enters another medium. If the angle of incidence and angle of refraction at the interface are 45° and 30° respectively, then the wavelength λ_2 and frequency ν_2 of the refracted wave are :

(1) $\lambda_2 = \frac{1}{\sqrt{2}}\lambda_1, \nu_2 = \nu_1$ (2) $\lambda_2 = \lambda_1, \nu_2 = \frac{1}{\sqrt{2}}\nu_1$

(3) $\lambda_2 = \lambda_1, \nu_2 = \sqrt{2}\nu_1$ (4) $\lambda_2 = \sqrt{2}\lambda_1, \nu_2 = \nu_1$

Sol. **(1)**

$1 \times sin45 = \mu sin30$

$\Rightarrow \frac{1}{\sqrt{2}} = \mu \times \frac{1}{2}$

$\Rightarrow \mu = \sqrt{2} ----(i)$

Now, $\frac{\mu_1}{\mu_2} = \frac{V_2}{V_1} = \frac{\lambda_2}{\lambda_1} ------(ii)$

Using eq (i) and (ii),

$\lambda_2 = \frac{1}{\sqrt{2}}\lambda_1$

And $V_2 = \frac{1}{\sqrt{2}}V_1$

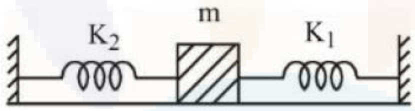
Now, for relation between frequencies,

Frequency, $v = \frac{V}{\lambda}$

Or $\frac{v_1}{v_2} = \frac{V_1}{V_2} \times \frac{\lambda_2}{\lambda_1} = 1$

$v_1 = v_2$

47. A mass m is attached to two strings as shown in figure. The spring constants of two springs are K_1 and K_2. For the frictionless surface, the time period of oscillation of mass m is



(1) $2\pi\sqrt{\frac{m}{K_1 - K_2}}$ (2) $\frac{1}{2\pi}\sqrt{\frac{K_1 - K_2}{m}}$ (3) $\frac{1}{2\pi}\sqrt{\frac{K_1 + K_2}{m}}$ (4) $2\pi\sqrt{\frac{m}{K_1 + K_2}}$

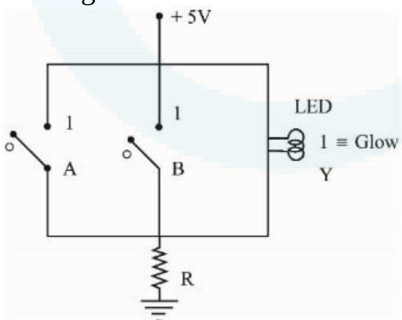
Sol. **(4)**

Both the springs are in parallel.

$K_{eq} = K_1 + K_2$

$T = 2\pi\sqrt{\frac{m}{K_{eq}}} = 2\pi\sqrt{\frac{m}{K_1+K_2}}$

48. Name the logic gate equivalent to the diagram attached



(1) NOR (2) OR (3) NAND (4) AND

Sol. **(1)**

A	B	Y
0	0	1
1	0	0
0	1	0
1	1	0

NOR gate

49. The induced emf can be produced in a coil by

A. moving the coil with uniform speed inside uniform magnetic field

B. moving the coil with non uniform speed inside uniform magnetic field

C. rotating the coil inside the uniform magnetic field

D. changing the area of the coil inside the uniform magnetic field

Choose the correct answer from the options given below :

(1) B and D only (2) C and D only (3) B and C only (4) A and C only

Sol. **(2)**

Induced emf can be induced in a coil by changing magnetic flux.

And $\phi = \vec{B}.\overrightarrow{dA}$

By rotating coil, angle between coil and magnetic field changes and hence flux changes.

By changing area, magnetic flux changes.

50. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : When a body is projected at an angle 45°, it's range is maximum.

Reason R : For maximum range, the value of sin 2θ should be equal to one.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both A and R are correct but R is NOT the correct explanation of A

(2) A is false but R is true

(3) Both A and R are correct and R is the correct explanation of A

(4) A is true but R is false

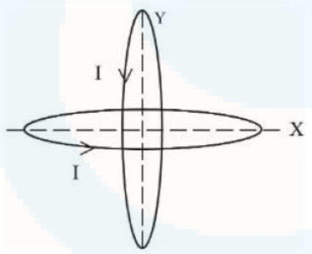
Sol. **(3)**

For a ground to ground projectile, Horizontal range is given by $R = \frac{u^2 sin2\theta}{g}$

And for R_{max}, $sin2\theta$ must be maximum.

SECTION - B

51. Two identical circular wires of radius 20 cm and carrying current $\sqrt{2}$ A are placed in perpendicular planes as shown in figure. The net magnetic field at the centre of the circular wires is _________ $\times 10^{-8}$ T.



(Take $\pi = 3.14$)

Sol. **(628)**

$\overrightarrow{B_{net}} = \frac{\mu_0 i}{2r}\hat{\imath} + \frac{\mu_0 i}{2r}\hat{\jmath}$

$\Rightarrow B_{net} = \frac{\mu_0 i}{2r}\sqrt{2} = 4\pi \times 10^{-7} \times \sqrt{2} \times \sqrt{2} \times \frac{1}{2\times 0.2} = 2 \times 3.14 \times 10^{-6} = 628 \times 10^{-8}\, T$

52. A steel rod bas a radius of 20 mm and a length of 2.0 m. A force of 62.8 kN stretches it along its length. Young's modulus of steel is 2.0×10^{11} N/m². The longitudinal strain produced in the wire is ________ $\times 10^{-5}$

Sol. **(25)**

$Y = \frac{stress}{strain}$

$\Rightarrow strain = \frac{stress}{Y} = \frac{F}{AY} = \frac{62.8\times 1000}{\pi r^2 \times 2\times 10^{11}} = \frac{62.8\times 1000}{3.14\times 400\times 10^{-6}\times 2\times 10^{11}} = \frac{200}{8} \times 10^{-5} = 25 \times 10^{-5}$

53. The length of a metallic wire is increased by 20% and its area of cross section is reduced by 4%. The percentage change in resistance of the metallic wire is _______

Sol. **(25)**

$R = \frac{\rho l}{A}$

$R' = \frac{\rho \times 1.2 l}{0.96 A} = \frac{10}{8} \times R = 1.25\, R$

It means 25 % increase in Resistance.

54. The radius of fifth orbit of the Li^{++} is __________ $\times 10^{-12}$ m.
Take : radius of hydrogen atom = 0.51 Å

Sol. **(425)**

$r_n = \frac{0.51n^2}{Z} A^0$

For Li^{++}, z=3.

So $r_5 = 0.51 \times \frac{25}{3} \times 10^{-10} m = 17 \times 25 \times 10^{-12} m = 425 \times 10^{-12} m$

55. A particle of mass 10 g moves in a straight line with retardation 2x, where x is the displacement in SI units. Its loss of kinetic energy for above displacement is $\left(\frac{10}{x}\right)^{-n}$ J. The value of n will be ____________

Sol. **(2)**

Given, $a = -2x$

$\Rightarrow \frac{vdv}{dx} = -2x$

$\Rightarrow vdv = -2xdx$

$\Rightarrow \int_{v_1}^{v_2} vdv = -2\int_0^x xdx$

$\Rightarrow \frac{v_2^2}{2} - \frac{v_1^2}{2} = -\frac{2x^2}{2}$

$\Rightarrow \frac{mv_1^2}{2} - \frac{mv_2^2}{2} = mx^2 = \frac{10}{1000}x^2 = 10^{-2}x^2 = \left(\frac{10}{x}\right)^{-2}$

n=2.

56. An ideal transformer with purely resistive load operates at 12 kV on the primary side. It supplies electrical energy to a number of nearby houses at 120 V. The average rate of energy consumption in the houses served by the transformer is 60 kW. The value of resistive load (Rs) required in the secondary circuit will be ________ mΩ .

Sol. **(240)**

$\frac{V_s}{V_p} = \frac{N_s}{N_p}$

$\Rightarrow \frac{120}{12000} = \frac{N_s}{N_p}$

$\Rightarrow \frac{N_s}{N_p} = \frac{1}{100} - - - (i)$

For an ideal transformer, input power = Output power

And power is given by $P = iV$

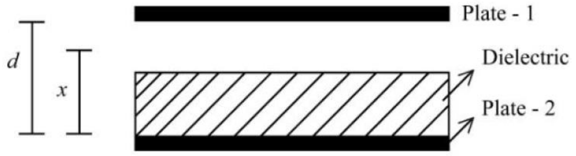
$i_p V_p = i_s V_s = 60000W$

$i_p = \frac{60000}{12000} = 5$

Now, $R_p = \frac{V_p}{i_p} = \frac{12000}{5} = 2400\ \Omega$

$R_s = \frac{V_s}{i_s} = \frac{120}{60000/120} = 120 \times \frac{120}{60000} = \frac{120}{500} = 0.240\Omega = 240\ m\Omega$

57. A parallel plate capacitor with plate area A and plate separation d is filed with a dielectric material of dielectric constant K = 4. The thickness of the dielectric material is x, where x < d.



Let C_1 and C_2 be the capacitance of the system for $x = \frac{1}{3}d$ and $x = \frac{2d}{3}$, respectively. If $C_1 = 2\mu F$ the value of C_2 is __________ μF

Sol. **(3)**

$$C_1 = \frac{\frac{\frac{\epsilon_0 A}{2d}}{3} \times \frac{\frac{4\epsilon_0 A}{d}}{3}}{\frac{\epsilon_0 A}{2d/3} + \frac{4\epsilon_0 A}{d/3}} = \frac{18}{\frac{3}{2}+12}\frac{\epsilon_0 A}{d} = 18 \times \frac{2}{27}\frac{\epsilon_0 A}{d} = \frac{4}{3}\frac{\epsilon_0 A}{d}$$

According to qn, $\frac{4}{3}\frac{\epsilon_0 A}{d} = 2 \Rightarrow \frac{\epsilon_0 A}{d} = \frac{3}{2} - - - - - - (i)$

Now, $C_2 = \frac{\frac{\frac{\epsilon_0 A}{d}}{3} \times \frac{\frac{4\epsilon_0 A}{2d}}{3}}{\frac{\epsilon_0 A}{d/3} + \frac{4\epsilon_0 A}{2d/3}} = \frac{18}{3+6}\frac{\epsilon_0 A}{d} = 2 \times \frac{\epsilon_0 A}{d} = 2 \times \frac{3}{2} = 3$

58. Two identical solid spheres each of mass 2 kg and radii 10 cm are fixed at the ends of a light rod. The separation between the centres of the spheres is 40 cm. The moment of inertia of the system about an axis perpendicular to the rod passing through its middle point is __________ $\times 10^{-3}$ kg-m^2

Sol. **(176)**

Using parallel axis theorem,

$I_{sys} = \left(\frac{2}{5}mr^2 + md^2\right) \times 2$

$\Rightarrow I_{sys} = \left(\frac{2}{5} \times 2 \times 0.01 + 2 \times 0.04\right) \times 2 = (0.008 + 0.08) \times 2 = 0.088 \times 2 = 176 \times 10^{-3}$

59. A person driving car at a constant speed of 15 m/s is approaching a vertical wall. The person notices a change of 40 Hz in the frequency of his car's horn upon reflection from the wall. The frequency of horn is ______ Hz.

Sol. **(420)**

$f' = f_0 + 40$

$\Rightarrow f_0\left(\frac{330+15}{330-15}\right) = f_0 + 40$

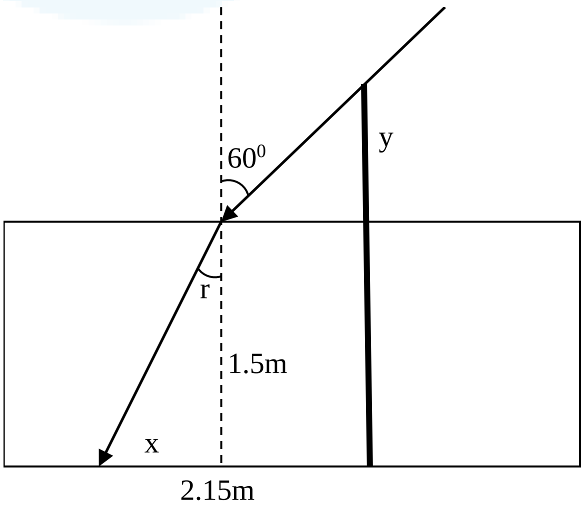
$\Rightarrow f_0 \times \frac{345}{315} = f_0 + 40$

$\Rightarrow f_0 \times \frac{30}{315} = 40$

$\Rightarrow f_0 = 40 \times \frac{315}{30} = 420\, Hz$

60. A pole is vertically submerged in swimming pool, such that it gives a length of shadow 2.15 m within water when sunlight is incident at an angle of 30° with the surface of water. If swimming pool is filled to a height of 1.5 m, then the height of the pole above the water surface in centimeters is ($n_w = 4/3$) ____________

Sol. **(50)**



$sin60 = \frac{4}{3} sinr$

$\Rightarrow sinr = \frac{3}{4} \times \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{8} - - - (i)$

$cosr = \sqrt{1 - \frac{27}{64}} = \frac{\sqrt{37}}{8} = 0.75$

$\Rightarrow tanr = \sqrt{\frac{27}{37}}$

$\Rightarrow \frac{x}{1.5} = 0.85$

$\Rightarrow x = 0.85 \times 1.5 = 1.275m$

$tan30 = \frac{y}{2.15-1.275} = \frac{y}{0.875}$

$y = \frac{0.875}{1.732} = 0.50$

So length of pole above water surface = 0.50m=50cm