FINAL JEE–MAIN EXAMINATION – APRIL, 2023
Held On Saturday 08th April, 2023
TIME : 09:00 AM to 12:00 PM

SECTION - A

31. A cylindrical wire of mass (0.4 ± 0.01)g has length (8 ± 0.04) cm and radius (6 ± 0.03) mm. The maximum error in its density will be:

(1) 4% (2) 1% (3) 3.5% (4) 5%

Sol. (1)

Cylindrical wire m = (0.4 ± 0.01) g

$\ell = (8 \pm 0.04)$ cm

r = (6 ± 0.03) mm

Density $\rho = \frac{m}{\pi r^2 \ell} \Rightarrow \rho r^2 \ell m^{-1} = \frac{1}{\pi}$ = const.

Differentiating after taking log on both side

$$\frac{d\rho}{\rho} + \frac{2dr}{r} + \frac{d\ell}{\ell} - \frac{dm}{m} = 0$$

$$\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} - \frac{\Delta\ell}{\ell} - \frac{2\Delta r}{r}$$

$$\left(\frac{\Delta\rho}{\rho}\right)_{max} = \left[\frac{0.01}{0.4} + \frac{0.04}{8} + 2\left(\frac{0.03}{6}\right)\right]$$

$$\left(\frac{\Delta\rho}{\rho}\right)_{max} = 0.04$$

Percentage error = 0.04 × 100 = 4%

32. The engine of a train moving with speed 10 ms^{-1} towards a platform sounds a whistle at frequency 400 Hz. The frequency heard by a passenger inside the train is : (neglect air speed. Speed of sound in air = 330 ms^{-1})

(1) 400 Hz (2) 388 Hz (3) 200 Hz (4) 412 Hz

Sol. (1)

The passenger inside the train is at rest wrt train so frequency heard by passenger inside the train is same as the source frequency i.e., 400 Hz.

33. The weight of a body on the earth is 400 N. Then weight of the body when taken to a depth half of the radius of the earth will be:

(1) 300 N (2) Zero (3) 100 N (4) 200 N

Sol. (4)

Weight on the earth surface = mg

mg = 400 N (given)

Weight at a depth d $w = m\left(\frac{GM(R-d)}{R^3}\right)$

$$W = mg\left(1 - \frac{d}{R}\right)$$

$$d = \frac{R}{2} \Rightarrow w = mg\left(1 - \frac{1}{2}\right) \Rightarrow w = \frac{mg}{2}$$

w = 200 N

34. A TV transmitting antenna is 98 m high and the receiving antenna is at the ground level. If the radius of the earth is 6400 km, the surface area covered by the transmitting antenna is approximately:

(1) 1240 km^2 (2) 1549 km^2 (3) 4868 km^2 (4) 3942 km^2

Sol. (4)

Max. distance covered $d = \sqrt{2Rh_T}$

(R = radius of earth, h_T = height of antenna)

Area $A = \pi d^2$

$A = \pi (2Rh_T)$

$A = 2 \times 3.14 \times 6400 \times 98 \times 10^{-3}$

$A \approx 3942\ km^2$

35. Certain galvanometers have a fixed core made of non magnetic metallic material. The function of this metallic material is
(1) To produce large deflecting torque on the coil
(2) To bring the coil to rest quickly
(3) To oscillate the coil in magnetic field for longer period of time
(4) To make the magnetic field radial

Sol. **(2)**
To bring the coil at rest quickly

36. Dimension of $\frac{1}{\mu_0 \in_0}$ should be equal to
(1) T/L (2) T^2 / L^2 (3) L/T (4) L^2/T^2

Sol. **(4)**
Dimension of $\frac{1}{\mu_0\varepsilon_0}$

$C = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \Rightarrow \frac{1}{\mu_0\varepsilon_0} = c^2$

$\left[\frac{1}{\mu_0\varepsilon_0}\right] = [c^2]$

$= \left[\frac{L^2}{T^2}\right]$

37. Two projectiles A and B are thrown with initial velocities of 40 m/s and 60 m/s at angles 30° and 60° with the horizontal respectively. The ratio of their ranges respectively is ($g = 10\ m/s^2$)
(1) $2 : \sqrt{3}$ (2) $\sqrt{3} : 2$ (3) 4 : 9 (4) 1 : 1

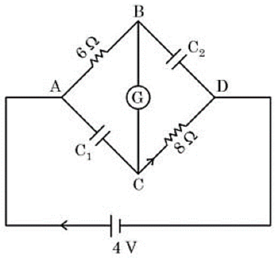
Sol. **(3)**
$R = \frac{u^2 \sin 2\theta}{g}$

$\{u_1 = 40\ m/s,\ \theta_1 = 30°,\ u_2 = 60\ m/s,\ \theta_2 = 60°\}$

$\frac{R_1}{R_2} = \left(\frac{u_1}{u_2}\right)^2 \frac{\sin 2\theta_1}{\sin 2\theta_2}$

$\frac{R_1}{R_2} = \left(\frac{40}{60}\right)^2 \times \frac{\sin 60°}{\sin 120°} \Rightarrow \frac{R_1}{R_2} = \frac{4}{9}$

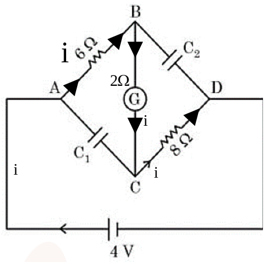
38. In this figure the resistance of the coil of galvanometer G is 2 Ω. The emf of the cell is 4 V. The ratio of potential difference across C_1 and C_2 is:



(1) $\frac{5}{4}$ (2) 1 (3) $\frac{4}{5}$ (4) $\frac{3}{4}$

Sol. **(3)**

At steady state current will not be in the capacitor branch.



$$i = \frac{4}{6+2+8}$$

$$i = \frac{1}{4}A$$

$$\Delta V_{C_1} = i(6+2)$$

$$\Delta V_{C_2} = i(2+8)$$

$$\frac{\Delta V_{C_1}}{\Delta V_{C_2}} = \frac{4}{5}$$

39. A charge particle moving in magnetic field B, has the components of velocity along B as well as perpendicular to B. The path of the charge particle will be

(1) Helical path with the axis along magnetic field B

(2) Straight along the direction of magnetic field B

(3) Helical path with the axis perpendicular to the direction of magnetic field B

(4) Circular path

Sol. **(1)**

Path will be helical with axis along uniform $\vec{B}$ -.

40. Proton (P) and electron (e) will have same de-Broglie wavelength when the ratio of their momentum is (assume, $m_p = 1849\ m_e$):

(1) 1 : 43 (2) 43 : 1 (3) 1 : 1849 (4) 1 : 1

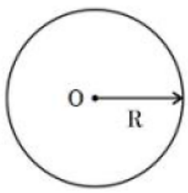
Sol. **(4)**

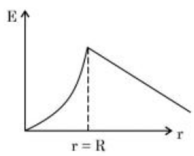
Debroglie wavelength $\lambda = \frac{h}{p}$

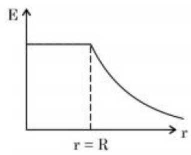
$\lambda_p = \lambda_e$

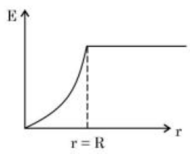
$$\frac{h}{p_p} = \frac{h}{p_e} \Rightarrow \frac{p_p}{p_e} = 1$$

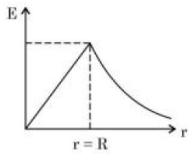
41. Graphical variation of electric field due to a uniformly charged insulating solid sphere of radius R, with distance r from the centre O is represented by:



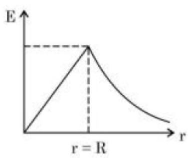
(1) 

(2) 

(3) 

(4) 

Sol. **(4)**



Electric field due to uniformly charged insulating solid sphere

$$E = \begin{cases} \frac{kQr}{R^3} & r \le R \\ \frac{kQ}{r^2} & r \ge R \end{cases}$$

42. For a nucleus ${}^{A}_{Z}X$ having mass number A and atomic number Z

A. The surface energy per nucleon $(b_s) = -a_1 A^{2/3}$.

B. The Coulomb contribution to the binding energy $b_c = -a_2 \frac{Z(Z-1)}{A^{4/3}}$

C. The volume energy $b_v = a_3 A$

D. Decrease in the binding energy is proportional to surface area.

E. While estimating the surface energy, it is assumed that each nucleon interacts with 12 nucleons. (a_1, a_2 and a_3 are constants)

Choose the most appropriate answer from the options given below:

(1) B, C only (2) A, B, C, D only (3) B, C, E only (4) C, D only

Sol. **(4)**

$$E_B = a_v A - a_s A^{2/3} - a_A \frac{(A-2Z)^2}{A^{1/3}} - a_c \frac{Z(Z-1)}{A^{1/3}} + \delta(A,Z)$$

Volume term, **Surface term**, **Asymmetry term**, **Coulomb term**, **Pairing term**

Most appropriate is option (4)

43. At any instant the velocity of a particle of mass 500 g is $(2t\hat{i} + 3t^2\hat{j})\ ms^{-1}$. If the force acting on the particle at $t = 1s$ is $(\hat{i} + x\hat{j})N$. Then the value of x will be:

(1) 2 (2) 6 (3) 3 (4) 4

Sol. **(3)**

$\vec{V} = (2t\hat{i} + 3t^2\hat{j})m/s$, mass m = 500 gm

Force, $\vec{F} = m\vec{a}$ -

$$\vec{F} = \frac{1}{2}\left(\frac{d\vec{v}}{dt}\right) \Rightarrow \vec{F} = \frac{1}{2}(2\hat{i} + 6t\hat{j})$$

$$\vec{F} = (\hat{i} + 3t\hat{j})$$

At $t = 1\ s \Rightarrow \vec{F} = (\hat{i} + 3\hat{j})$

x = 3

44. Given below are two statements:

Statement I : If E be the total energy of a satellite moving around the earth, then its potential energy will be $\frac{E}{2}$

Statement II : The kinetic energy of a satellite revolving in an orbit is equal to the half the magnitude of total energy E.

In the light of the above statements, choose the most appropriate answer from the options given below

(1) Both Statement I and Statement II are incorrect

(2) Statement I is incorrect but Statement II is correct

(3) Statement I is correct but Statement II is incorrect

(4) Both Statement I and Statement II are correct

Sol. **(1)**

For satellite K.E. = $\frac{1}{2}mv^2 = \frac{1}{2}m\left(\sqrt{\frac{GM}{r}}\right)^2$

K.E. = $\frac{GMm}{2r}$

Potential energy U = $-\frac{GMm}{r}$

Total energy = K.E + U

$E = -\frac{GMm}{2r}$

U = 2E St I – incorrect

K.E. = |E| St II - incorrect

45. Two forces having magnitude A and $\frac{A}{2}$ are perpendicular to each other. The magnitude of their resultant is:

(1) $\frac{5A}{2}$ (2) $\frac{\sqrt{5}A^2}{2}$ (3) $\frac{\sqrt{5}A}{4}$ (4) $\frac{\sqrt{5}A}{2}$

Sol. **(4)**

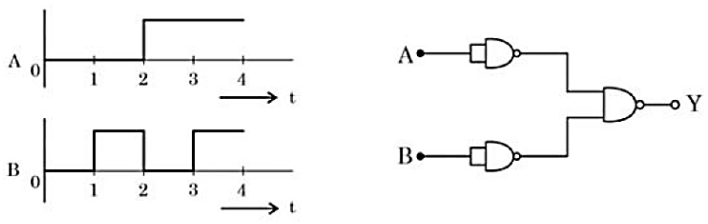
$|\vec{F}_1| = A,\ |\vec{F}_2| = \frac{A}{2}$ $\theta = \frac{\pi}{2}$

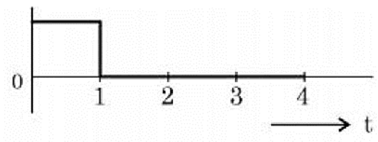
$|\vec{F}_{net}| = \sqrt{F_1^2 + F_2^2}$

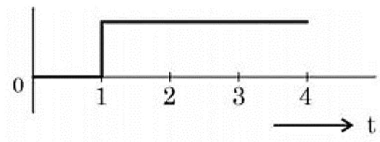
$= \sqrt{A^2 + \left(\frac{A}{2}\right)^2}$

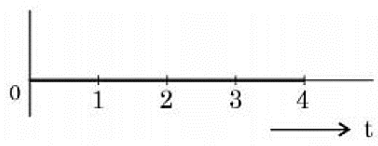
$|\vec{F}_{net}| = \frac{\sqrt{5}A}{2}$

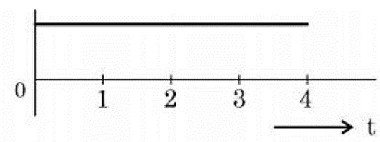
46. For the logic circuit shown, the output waveform at Y is:



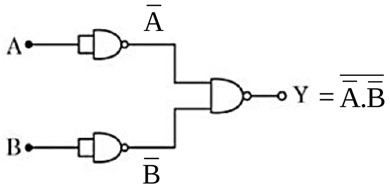
(1) 

(2) 

(3) 

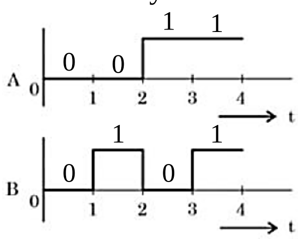
(4) 

Sol. **(2)**

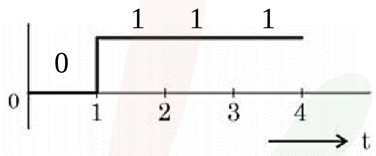


$y = \overline{\bar{A}.\bar{B}} \Rightarrow y = \bar{\bar{A}} + \bar{\bar{B}}$

$y = A + B$



A	B	y = A + B
0	0	0
0	1	1
1	0	1
1	1	1



47. An aluminium rod with Young's modulus $Y = 7.0 \times 10^{10}$ N/m^2 undergoes elastic strain of 0.04%. The energy per unit volume stored in the rod in SI unit is:

(1) 5600 (2) 2800 (3) 11200 (4) 8400

Sol. **(1)**

Aluminium rod Young's modulus

$y = 7.0 \times 10^{10} \frac{N}{m^2}$

strain 0.04%

strain $= \frac{0.04}{100}$

Energy per unit volume $= \frac{1}{2}$ stress × strain

$= \frac{1}{2}$ y strain × strain

$= \frac{1}{2}$ y (strain)^2

$= \frac{1}{2} \times 7 \times 10^{10} \times \left(\frac{0.04}{100}\right)^2$

Energy per unit volume $= 5600 \frac{J}{m^3}$

48. Given below are two statements:

Statement I : If heat is added to a system, its temperature must increase.

Statement II : If positive work is done by a system in a thermodynamic process, its volume must increase.

In the light of the above statements, choose the correct answer from the options given below

(1) Both Statement I and Statement II are true (2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false (4) Statement I is false but Statement II is true

Sol. **(4)**

St I False

Ex. in isothermal process temp. is constant but heat can be added.

ST II True

$w = \int PdV$

If volume increases the w = +ve

49. An air bubble of volume 1 cm^3 rises from the bottom of a lake 40 m deep to the surface at a temperature of 12°C. The atmospheric pressure is 1×10^5 Pa, the density of water is 1000 kg/m^3 and g = 10 m/s^2. There is no difference of the temperature of water at the depth of 40 m and on the surface. The volume of air bubble when it reaches the surface will be:

(1) 3 cm^3 (2) 4 cm^3 (3) 2 cm^3 (4) 5 cm^3

Sol. **(4)**

Pressure at surface = $P_{atm} = 1 \times 10^5$ Pa

$v_{surface} = ?$

Pressure at h = 40 m depth

$P = P_{atm} + \rho gh$

$P = 10^5 + 10^3 \times 10 \times 40$

$P = 5 \times 10^5$ Pa

$v = 1\ cm^3$

Temp. is constant

$P_1V_1 = P_2V_2$

$10^5 \times v = 5 \times 10^5 \times 1$

$v = 5\ cm^3$

50. In a reflecting telescope, a secondary mirror is used to:

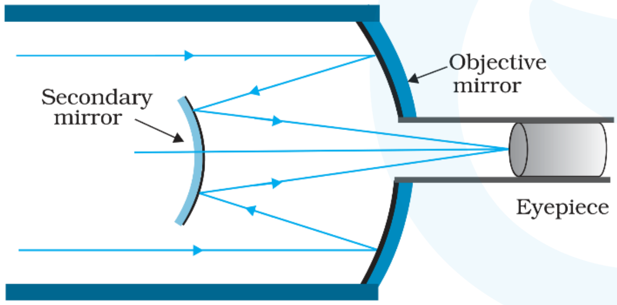
(1) Make chromatic aberration zero

(2) Reduce the problem of mechanical support

(3) Move the eyepiece outside the telescopic tube

(4) Remove spherical aberration

Sol. **(3)**



To move the eye piece outside the telescopic tube

SECTION – B

51. The momentum of a body is increased by 50%. The percentage increase in the kinetic energy of the body is ________ %.

Sol. **(125)**

$$K_i = \frac{P_i^2}{2m}$$

$$K_f = \frac{\left(P_i + \frac{P_i}{2}\right)^2}{2m} \Rightarrow K_f = \frac{9}{4}\frac{P_i^2}{2m}$$

$$\text{Percentage increase in K.E.} = \frac{K_f - K_i}{K_i} \times 100$$

$$= \frac{\frac{9}{4} - 1}{1} \times 100$$

$$= \frac{5}{4} \times 100 = 125\%$$

52. A nucleus with mass number 242 and binding energy per nucleon as 7.6 MeV breaks into two fragment each with mass number 121. If each fragment nucleus has binding energy per nucleon as 8.1 MeV, the total gain in binding energy is __________ MeV.

Sol. **(121)**

Gain in binding energy = $B.E_f - BE_i$

$= 2(121 \times 8.1) - 242 \times 7.6$

$= 121$ MeV

53. An electric dipole of dipole moment is 6.0×10^{-6} C m placed in a uniform electric field of 1.5×10^{3} NC^{-1} in such a way that dipole moment is along electric field. The work done in rotating dipole by 180° in this field will be __________ mJ.

Sol. **(18)**

$W_{ext} = U_f - U_i \quad \{U = -\vec{P}.\vec{E}\}$

$= -PE\cos\pi - (-PE\cos 0)$

$= 2PE$

$= 2 \times 6 \times 10^{-6} \times 1.5 \times 10^{3}$

$= 18$ mJ

54. An organ pipe 40 cm long is open at both ends. The speed of sound in air is 360 ms^{-1}. The frequency of the second harmonic is __________ Hz.

Sol. **(900)**

Open organ pipe $\ell = 40$ cm

Speed of sound v = 360 m/s

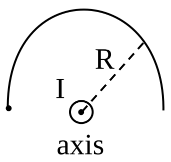
Frequency of second harmonics $f_2 = \frac{2v}{2\ell}$

$f_2 = \frac{v}{\ell} \Rightarrow f_2 = \frac{360}{0.4}$

$f_2 = 900\,Hz$

55. The moment of inertia of a semicircular ring about an axis, passing through the center and perpendicular to the plane of ring, is $\frac{1}{x}$ MR^2, where R is the radius and M is the mass of the semicircular ring. The value of x will be __________.

Sol. **(1)**



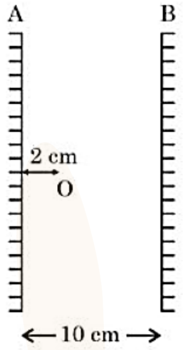
$I = \int dmR^2 \Rightarrow R^2\int dm = MR^2$

$I = MR^2$

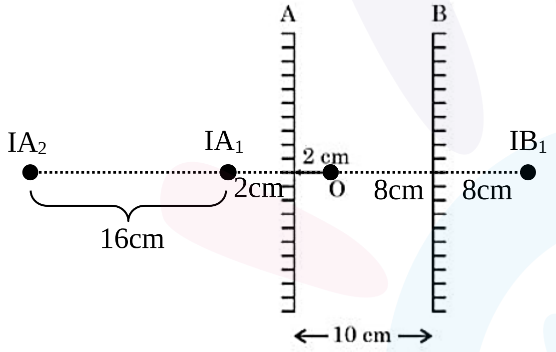
Given $I = \frac{1}{x}MR^2$

$x = 1$

56. Two vertical parallel mirrors A and B are separated by 10 cm. A point object O is placed at a distance of 2 cm from mirror A. The distance of the second nearest image behind mirror A from the mirror A is _________ cm.



Sol. **(18)**



d = 2 + 16

d = 18 cm

57. The magnetic intensity at the center of a long current carrying solenoid is found to be 1.6×10^3 Am^{-1}. If the number of turns is 8 per cm, then the current flowing through the solenoid is __________ A.

Sol. **(2)**

$H = 1.6 \times 10^3$ A/m, n = 8 per cm = 800 per m

$$H = nI \Rightarrow I = \frac{H}{n}$$

$$I = \frac{1.6 \times 10^3}{8 \times 10^2} \Rightarrow I = 2\text{ A}$$

58. A current of 2 A through a wire of cross-sectional area 25.0 mm^2. The number of free electrons in a cubic meter are 2.0×10^{28}. The drift velocity of the electrons is ________ $\times 10^{-6}$ ms^{-1} (given, charge on electron = 1.6×10^{-19} C).

Sol. **(25)**

$I = neAV_d$

$$V_d = \frac{I}{neA} \Rightarrow V_d = \frac{2}{2 \times 10^{28} \times 1.6 \times 10^{-19} \times 25 \times 10^{-6}}$$

$V_d = 25$ m/s

59. An oscillating LC circuit consists of a 75 mH inductor and a 1.2 μF capacitor. If the maximum charge to the capacitor is 2.7 μC. The maximum current in the circuit will be __________ mA.

Sol. **(9)**

LC oscillation L = 75 mH

C = 1.2 μF

$U_{max\,L} = U_{max\,C}$

$$\frac{1}{2}LI^2_{max} = \frac{1}{2}\frac{q^2_{max}}{C}$$

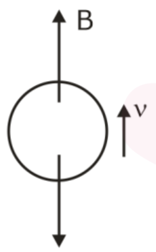
$$I_{max} = \frac{q_{max}}{\sqrt{LC}} \Rightarrow I_{max} = \frac{2.7\times10^{-6}}{\sqrt{75\times10^{-3}\times1.2\times10^{-6}}}$$

$I_{max} = 9 \times 10^{-3}$ A

$I_{max} = 9$ mA

60. An air bubble of diameter 6 mm rises steadily through a solution of density 1750 kg/m^3 at the rate of 0.35 cm/s. TGe co-efficient of viscosity of the solution (neglect density of air) is __________ Pas (given, g = 10 ms^{-2}).

Sol. **(10)**



F_v=6pηrv

For uniform velocity net force = 0

B = 6πηrv

$$\rho\frac{4}{3}\pi r^3 g = 6\pi\eta rv$$

$$\eta = \frac{2\rho r^2 g}{9v}$$

$$\eta = \frac{2\times1750\times(3\times10^{-3})^2\times10}{9\times0.35\times10^{-2}}$$

η = 10 Pa-s