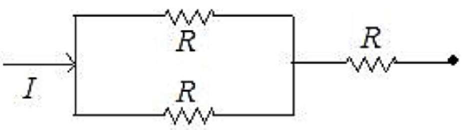
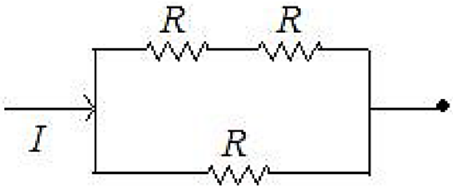
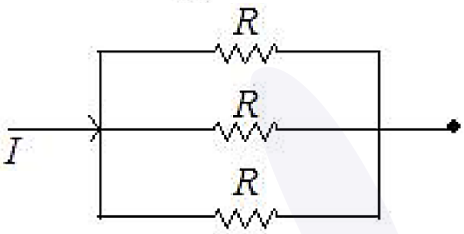
FINAL JEE–MAIN EXAMINATION – APRIL, 2023
Held On Thursday 13th April, 2023
TIME : 09:00 AM to 12:00 PM

SECTION - A

31. Different combination of 3 resistors of equal resistance R are shown in the figures. The increasing order for power dissipation is:

(A)


(B)


(C)


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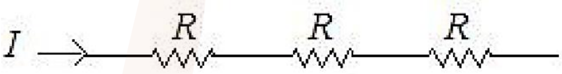


(1) $P_C < P_B < P_A < P_D$ (2) $P_C < P_D < P_A < P_B$ (3) $P_B < P_C < P_D < P_A$ (4) $P_A < P_B < P_C < P_D$

Sol. **(1)**

Power dissipation, $P = I^2R$

(A) $R_{eq} = \frac{R}{2} + R = \frac{3R}{2}$

(B) $R_{eq} = \frac{(2R)(R)}{2R + R} = \frac{2R}{3}$

(C) $R_{eq} = \frac{R}{3}$

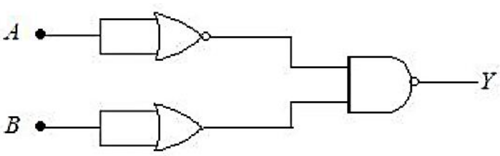
(D) $R_{eq} = 3R$

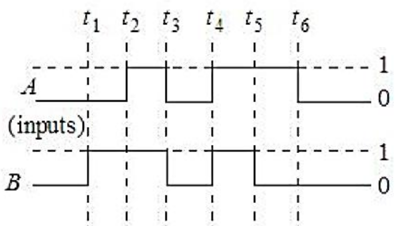
$R_D > R_A > R_B > R_C$

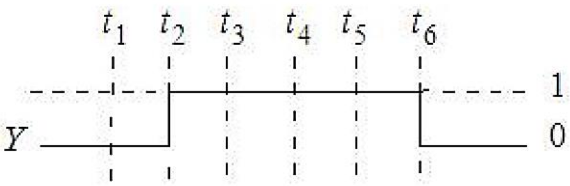
Since, $P \propto R_{eq}$

$P_D > P_A > P_B > P_C$

32. For the following circuit and given inputs A and B, chose the correct option for output 'Y'





(1)


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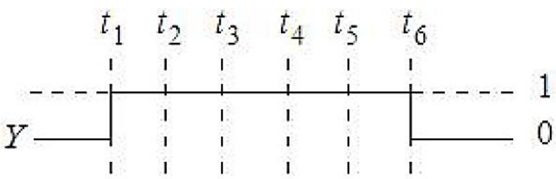

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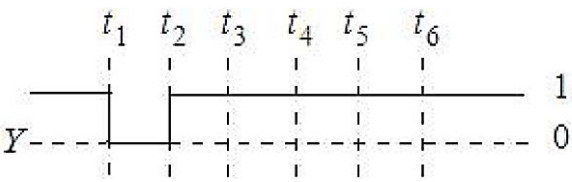

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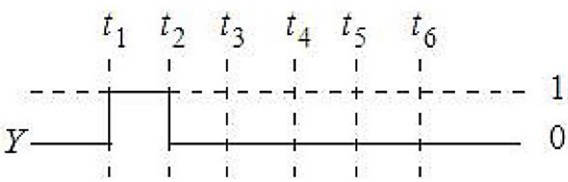
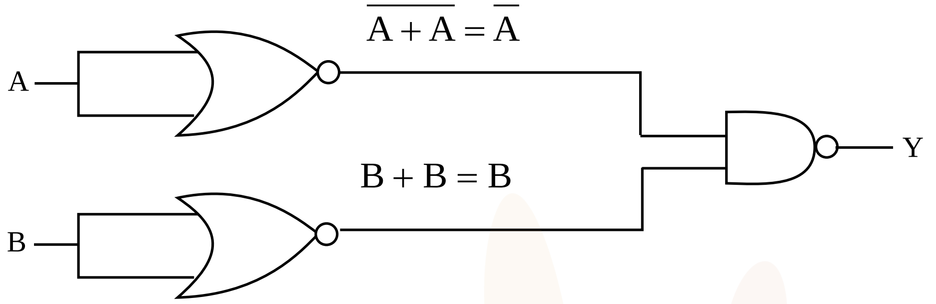


Sol. **(3)**



Output, $y=\overline{\overline{A}.B}=\overline{\overline{A}}+\overline{B}$

$y=A+\overline{B}$

t_1 to t_2 , A = 0, B = 1, Y = 0
t_2 to t_3 A=1, B = 1, Y = 1
t_3 to t_4 A = 0, B = 0, Y = 1
t_4 to t_5, A = 1, B = 1, Y = 1
t_5 to t_6, A = 1, B = 0, Y = 1
After t_6, A = 0, B = 0, Y = 1

33. A bullet of 10 g leaves the barrel of gun with a velocity of 600 m/s. If the barrel of gun is 50 cm long and mass of gun is 3 kg, then value of impulse supplied to the gun will be:
(1) 12 Ns (2) 6 Ns (3) 3 Ns (4) 36 Ns

Sol. **(2)**
Impulse, $|\vec{I}|=|\Delta\vec{p}|$
$= mV - 0$
$= (10 \times 10^{-3}\text{ kg})(600\text{ m/s})$
$I = 6$ N–S

34. Which of the following Maxwell's equation is valid for time varying conditions but not valid for static conditions:
(1) $\oint \vec{D}.\vec{dA}=Q$ (2) $\oint \vec{E}.\vec{dl}=-\frac{\partial\phi_B}{\partial t}$ (3) $\oint \vec{E}.\vec{dl}=0$ (4) $\oint \vec{B}.\vec{dl}=\mu_0 I$

Sol. **(2)**
For static conditions
$\oint \vec{E}.d\vec{l}=0$
For time varying condition,
$\oint \vec{E}.d\vec{l}=-\frac{\partial\phi_B}{\partial t}$

35. Match List – I with List – II

List – I (Layer of atmosphere)	**List – II (Approximate height over earth's surface)**
(A) F1 – Layer	(I) 10 km
(B) D – Layer	(II) 170 – 190 km
(C) Troposphere	(III) 100 km
(D) E – layer	(IV) 65 – 75 km

Choose the correct answer from the options given below:

(1) A – II, B – I, C – IV, D – III
(2) A – II, B – IV, C – III, D – I
(3) A – II, B – IV, C – I, D – III
(4) A – III, B – IV, C – I, D – II

Sol. **(3)**

$F_1 \rightarrow$ Lower part of F layer of ionosphere (170 – 190Km)

$D \rightarrow$ Lowest layer of ionosphere (65 – 75 Km)

Troposphere $\rightarrow$ Lowest layer of atmosphere (10 Km)

$E \rightarrow$ Middle part of ionosphere (100 Km)

36. The rms speed of oxygen molecule in a vessel at particular temperature is $\left(1+\frac{5}{x}\right)^{\frac{1}{2}} \nu$, where ν is the average speed of the molecule. The value of x will be: (Take $\pi = \frac{22}{7}$)

(1) 28
(2) 27
(3) 8
(4) 4

Sol. **(1)**

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$$V_{avg} = \nu = \sqrt{\frac{8RT}{\pi M}}$$

$$V_{rms} = \sqrt{\frac{3\pi}{8}}\nu$$

$$V_{rms} = \sqrt{\frac{3}{8}\times\frac{22}{7}}\nu = \left(\frac{33}{28}\right)^{1/2}\nu$$

$$V_{rms} = \left(1+\frac{5}{28}\right)^{1/2}\nu$$

x = 28

37. The ratio of powers of two motors is $\frac{3\sqrt{x}}{\sqrt{x}+1}$, that are capable of raising 300 kg water in 5 minutes and 50 kg water in 2 minutes respectively from a well of 100 m deep. The value of x will be

(1) 16
(2) 2
(3) 4
(4) 2.4

Sol. **(1)**

$$P = \frac{\text{Work}}{\text{Time}}$$

$$P_1 = \frac{mgh}{t_1} = \frac{(300)g(100)}{5}$$

$$P_2 = \frac{(50)g(100)}{2}$$

$$\frac{P_1}{P_2} = \frac{600}{250} = \frac{12}{5} = \frac{3\times 4}{4+1}$$

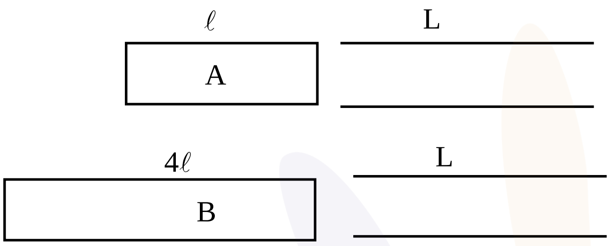
$$\frac{P_1}{P_2} = \frac{3\sqrt{16}}{\sqrt{16}+1}$$

x = 16

38. Two trains 'A' and 'B' of length '*l*' and '4*l*' are travelling into a tunnel of length 'L' in parallel tracks from opposite directions with velocities 108 km/h and 72 km/h, respectively. If train 'A' takes 35s less time than train 'B' to cross the tunnel then, length 'L' of tunnel is:
(Given L = 60 *l*)

(1) 2700 m (2) 1800 m (3) 1200 m (4) 900 m

Sol. **(2)**



$$V_A = 108 \times \frac{5}{18} = 30 \text{ m/s}$$

$$V_B = 72 \times \frac{5}{18} = 20 \text{ m/s}$$

$$T_A = \frac{\ell + L}{30}, T_B = \frac{4\ell + L}{20}$$

$$T_A = T_B - 35$$

$$\frac{\ell + L}{30} = \frac{4\ell + L}{20} - 35$$

Given, $L = 60\,\ell$

$$\frac{61\ell}{30} = \frac{64\ell}{20} - 35$$

$$\frac{192\ell - 122\ell}{60} = 35$$

$$70\ell = 60 \times 35$$

$$\ell = 30\text{m}$$

$$L = 60\ell = 1800\text{m}$$

39. Two bodies are having kinetic energies in the ratio 16 : 9. If they have same linear momentum, the ratio of their masses respectively is:

(1) 16 : 9 (2) 4 : 3 (3) 9 : 16 (4) 3 : 4

Sol. **(3)**

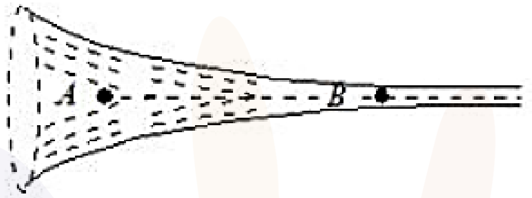
Kinetic energy, $KE = \frac{P^2}{2m}$

$$\frac{k_1}{k_2} = \frac{m_2}{m_1}$$

$$\frac{16}{9} = \frac{m_2}{m_1}$$

$$\frac{m_1}{m_2} = \frac{9}{16}$$

40. The figure shows a liquid of given density flowing steadily in horizontal tube of varying cross – section. Cross sectional areas at A is 1.5 cm^2, and B is 25 mm^2, if the speed of liquid at B is 60 cm/s then $(P_A - P_B)$ is:
(Given P_A and P_B are liquid pressures at A and B points)
Density $\rho = 1000\ kg\ m^{-3}$
A and B are on the axis of tube



(1) 175 Pa (2) 36 Pa (3) 27 Pa (4) 135 Pa

Sol. **(1)**
By equation of continuity,

$A_1V_1 = A_2V_2$

$(1.5 \times 10^{-4})\ V_A = (25 \times 10^{-6})\ 60$ cm/s

$V_A = 10$ cm/s

By Bernoulli's theorem,

$$P_A + \frac{1}{2}\rho V_A^2 = P_B + \frac{1}{2}\rho V_B^2$$

$$P_A - P_B = \frac{\rho}{2}(V_B^2 - V_A^2)$$

$$P_A - P_B = \frac{1000}{2}(60^2 - 10^2)\times 10^{-4}$$

$$P_A - P_B = 175\text{Pa}$$

41. $${}^{238}_{92}A \rightarrow {}^{234}_{90}B + {}^{4}_{2}D + Q$$
In the given nuclear reaction, the approximate amount of energy released will be:
[Given, mass of ${}^{238}_{92}A$ = 238.05079 × 931.5 MeV/c^2,
mass of ${}^{234}_{90}B$ = 234.04363 × 931.5 MeV/c^2,
mass of ${}^{4}_{2}D$ = 4.00260 × 931.5 MeV/c^2]

(1) 4.25 MeV (2) 5.9 MeV (3) 3.82 MeV (4) 2.12 MeV

Sol. **(1)**

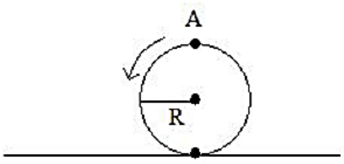
$Q = \Delta m C^2$

$Q = (238.05079 - 234.04363 - 4.00260) \times 931.5\text{MeV}$

$Q = 0.00456 \times 931.5\text{MeV}$

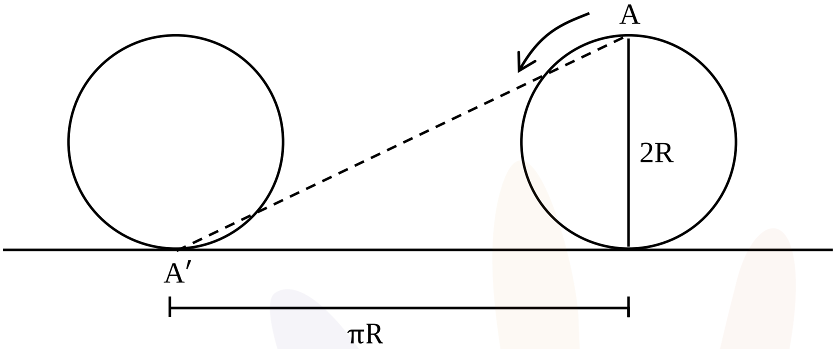
Q = 4.25 MeV

42. A disc is rolling without slipping on a surface. The radius of the disc is R. At t = 0, the top most point on the disc is A as shown in figure. When the disc completes half of its rotation, the displacement of point A from its initial position is



(1) $2R\sqrt{(1+4\pi^2)}$ (2) $R\sqrt{(\pi^2+4)}$ (3) 2R (4) $R\sqrt{(\pi^2+1)}$

Sol. **(2)**

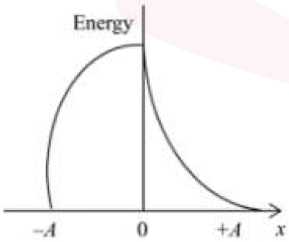


Displacement = $A'A$

$$A'A = \sqrt{(\pi R)^2 + (2R)^2}$$

$$A'A = R\sqrt{\pi^2 + 4}$$

43. Which graph represents the difference between total energy and potential energy of a particle executing SHM vs it's distance from mean position?

(1)


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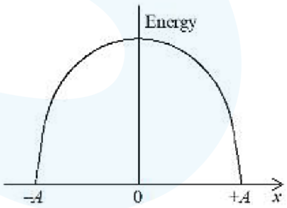

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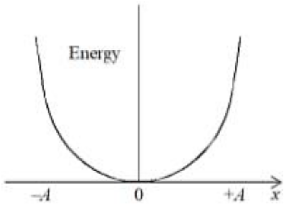

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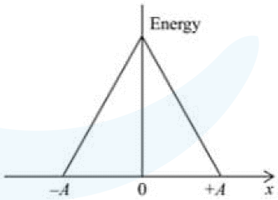


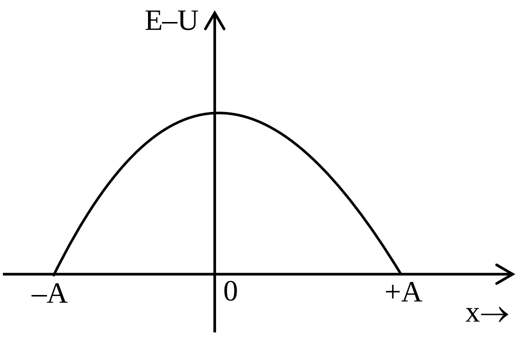
Sol. **(2)**

Total energy in SHM = E

$E = K + U$

$E - U = K$

$$E - U = \frac{1}{2}m\omega^2(A^2 - x^2)$$



44. Two charges each of magnitude 0.01 C and separated by a distance of 0.4 mm constitute an electric dipole. If the dipole is placed in an uniform electric field '$\vec{E}$' of 10 dyne/C making 30° angle with $\vec{E}$, the magnitude of torque acting on dipole is

(1) 1.5×10^{-9} Nm (2) 2.0×10^{-10} Nm (3) 1.0×10^{-8} Nm (4) 4.0×10^{-10} Nm

Sol. **(2)**

Dipole moment, P = qd

$P = 0.01 \times 0.4 \times 10^{-3}$

$P = 4 \times 10^{-6}$ C-m

Torque, $\tau = pE\sin\theta$

$\tau = 4\times10^{-6} \times (10\times10^{-5})\times \sin 30°$

$\tau = 4\times10^{-10}\,N-m$

45. Under isothermal condition, the pressure of a gas is given by $P = aV^{-3}$, where a is a constant and V is the volume of the gas. The bulk modulus at constant temperature is equal to

(1) $\frac{P}{2}$ (2) 2P (3) P (4) 3P

Sol. **(4)**

$P = aV^{-3}$

$\frac{dP}{dV} = -3aV^{-4}$

Bulk modulus, $B = -V\frac{dP}{dV}$

$B = -V\left(\frac{-3a}{V^4}\right)$

$B = 3\frac{a}{V^3} = 3P$

46. A planet having mass 9 Me and radius 4Re, where Me and Re are mass and radius of earth respectively, has escape velocity in km/s given by:

(Given escape velocity on earth $V_e = 11.2 \times 10^3$ m/s)

(1) 11.2 (2) 67.2 (3) 33.6 (4) 16.8

Sol. **(4)**

Escape velocity, $v_e = \sqrt{\frac{2GM}{R}}$

$V_p = \sqrt{\frac{2G(9m_e)}{4R_e}} = \frac{3}{2}(V_e)_{earth}$

$v_p = \frac{3}{2}\times 11.2\text{ km/s}$

$v_p = 16.8\text{ km/s}$

47. A body of mass (5 ± 0.5) kg is moving with a velocity of (20 ± 0.4) m/s. Its kinetic energy will be

(1) (1000 ± 140) J (2) (500 ± 140) J (3) (500 ± 0.14) J (4) (1000 ± 0.14) J

Sol. **(1)**

Kinetic energy, $KE = \frac{1}{2}mv^2$

$KE = \frac{1}{2}\times 5\times 20^2$

$$KE = 1000\,J$$

$$\frac{\Delta K}{K} = \frac{\Delta m}{m} + \frac{2\Delta v}{v}$$

$$\frac{\Delta K}{1000} = \frac{0.5}{5} + 2 \times \frac{0.4}{20}$$

$$\Delta K = 1000(0.1 + 0.04)$$

$$\Delta K = 1000 \times 0.14$$

$$\Delta K = 140\,J$$

$$KE = (1000 \pm 140)\,J$$

48. The difference between threshold wavelengths for two metal surfaces A and B having work function $\phi_A = 9$ eV and $\phi_B = 4.5$ eV in nm is:
{Given, hc = 1242 eV nm}

(1) 276 (2) 264 (3) 540 (4) 138

Sol. (4)

$$\phi = \frac{hc}{\lambda}$$

$$\lambda_A = \frac{1242}{9} = 138\,nm$$

$$\lambda_B = \frac{1242}{4.5} = 276\,nm$$

$$\lambda_B - \lambda_A = 276 - 138 = 138\,nm$$

49. The source of time varying magnetic field may be
(A) A permanent magnet
(B) An electric field changing linearly with time
(C) Direct current
(D) A decelerating charge particle
(E) An antenna fed with a digital signal
Choose the correct answer from the options given below:

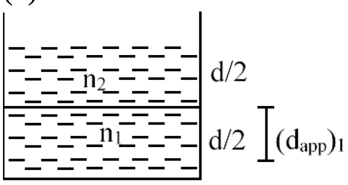
(1) (B) and (D) only (2) (C) and (E) only (3) (D) only (4) (A) only

Sol. **(3)**
Accelerated charge particle produces EMW which has time varying E and B.
If E is linear function of time then B will be constant.

50. A vessel of depth 'd' is half filled with oil of refractive index n_1 and the other half is filled with water of refractive index n_2. The apparent depth of this vessel when viewed from above will be -

(1) $\frac{d(n_1+n_2)}{2n_1n_2}$ (2) $\frac{d\,n_1n_2}{(n_1+n_2)}$ (3) $\frac{d\,n_1n_2}{2(n_1+n_2)}$ (4) $\frac{2d(n_1+n_2)}{n_1n_2}$

Sol. (1)



$$\left(d_{app}\right)_1 = \frac{d}{2\left(\frac{n_1}{n_2}\right)} = \frac{n_2 d}{2n_1}$$

$$\left(d_{app}\right)_2 = \frac{\left(d_{app}\right)_1 + \frac{d}{2}}{n_2}$$

$$= \frac{\left(\frac{n_2}{n_1}+1\right)\frac{d}{2}}{n_2}$$

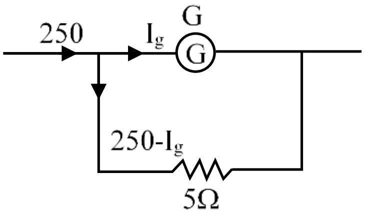
$$\left(d_{app}\right)_2 = \frac{(n_1+n_2)d}{2n_1 n_2}$$

SECTION – B

51. When a resistance of 5 Ω is shunted with a moving coil galvanometer, it shows a full scale deflection for a current of 250 mA, however when 1050 Ω resistance is connected with it in series, it gives full scale deflection for 25 volt. The resistance of galvanometer is ___________ Ω.

Sol. (50)

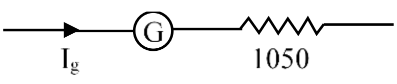
For ammeter,



$$I_g(G) = (250 - I_g)5$$

$$I_g = \frac{1250}{5+G}\,\text{mA}$$

For voltmeter,



$$V = I_g R$$

$$25 = I_g(G+1050)$$

From equation (1),

$$25 = \frac{1250\times10^{-3}}{G+5}(G+1050)$$

$$20(G+5) = G+1050$$

19 G = 1050 – 100

$$G = \frac{950}{19} = 50\Omega$$

52. The radius of 2nd orbit of He^+ of Bohr's model is r_1 and that of fourth orbit of Be^{3+} is represented as r_2. Now the ratio $\frac{r_2}{r_1}$ is x : 1. The value of x is __________

Sol. **(2)**

$$r \propto \frac{n^2}{Z}$$

$$\frac{r_2}{r_1} = \left(\frac{n_2}{n_1}\right)^2 \times \frac{z_1}{z_2}$$

$$\frac{r_2}{r_1} = \left(\frac{4}{2}\right)^2 \times \frac{2}{4}$$

$$\frac{r_2}{r_1} = 2$$

$$x = 2$$

53. A solid sphere is rolling on a horizontal plane without slipping. If the ratio of angular momentum about axis of rotation of the sphere to the total energy of moving sphere is π : 22 the, the value of its angular speed will be rad/s.

Sol. **(4)**

Angular momentum,

$$L = I\omega$$

$$L = \frac{2}{5}MR^2\omega$$

$$\text{Energy } = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2$$

$$E = \frac{1}{2}M(\omega R)^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2$$

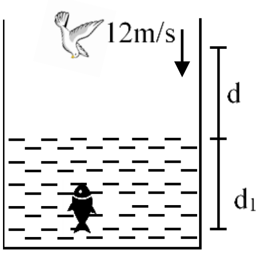
$$= \frac{7}{10}M\omega^2R^2$$

$$\frac{L}{E} = \frac{4}{7\omega} = \frac{\pi}{22}$$

$$\omega = \frac{88}{7\pi} = \frac{88}{7 \times \frac{22}{7}} = 4 \text{ rad/s}$$

54. A fish rising vertically upward with a uniform velocity of 8 ms^{-1}, observes that a bird is diving vertically downward towards the fish with the velocity of 12 ms^{-1}. If the refractive index of water is $\frac{4}{3}$, then the actual velocity of the diving bird to pick the fish, will be _____________ ms^{-1}.

Sol. **(3)**



$d_{app} = d_1 + \mu d$

$v_{app} = v_1 + \mu v$

$12 = 8 + \frac{4}{3}v$

$4 = \frac{4}{3}v$

$v = 3 m/s$

55. The elastic potential energy stored in a steel wire of length 20 m stretched through 2 cm is 80 J. The cross sectional area of the wire is ___________ mm^2.
(Given, $y = 2.0 \times 10^{11}$ Nm^{-2})

Sol. **(40)**

Energy, $U = \frac{1}{2}kx^2$

$80 = \frac{1}{2}k(2\times10^{-2})^2$

$k = \frac{160}{4\times10^{-4}}$

$k = 4\times10^5 N/m$

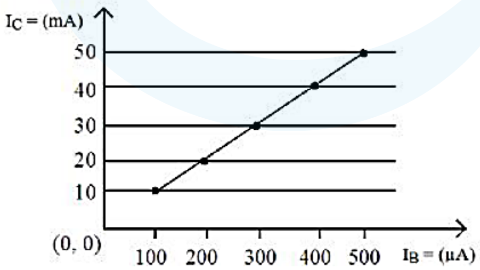
$\frac{yA}{\ell} = 4\times10^5$

$A = \frac{4\times10^5\times20}{2\times10^{11}}$

$A = 40\times10^{-6} m^2$

$A = 40 mm^2$

56. From the given transfer characteristic of a transistor in CE configuration, the value of power gain of this configuration is 10^x, for $R_B = 10$ k Ω, $R_C = 1$ k Ω. The value of x is ______________



Sol. **(3)**

Current gain, $\beta = \frac{\Delta I_C}{\Delta I_B}$

$\beta = \frac{10mA}{100\mu A}$

$\beta = 100$

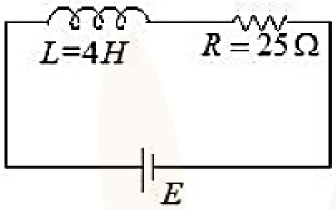
Power gain $\beta^2 \frac{R_C}{R_B}$

$= 10^4 \times \frac{1}{10}$

$= 10^3$

So, $x = 3$

57. In the given figure, an inductor and a resistor are connected in series with a batter of emf E volt. $\frac{E^a}{2b}$ J/s represents the maximum rate at which the energy is stored in the magnetic field (inductor). The numerical value of $\frac{b}{a}$ will be ______________



Sol. **(25)**

$U = \frac{1}{2}LI^2$

$I = I_0\left(1 - e^{-t/\tau}\right)$

Rate of energy, $P = \frac{dU}{dt}$

$P = LI\frac{dI}{dt}$

$\frac{dP}{dt} = L\left(I\frac{d^2I}{dt^2} + \left(\frac{dI}{dt}\right)^2\right)$

For maximum rate, $\frac{dP}{dt} = 0$

$I\frac{d^2I}{dt^2} = -\left(\frac{dI}{dt}\right)^2$ ….. (1)

$I = I_0\left(1 - e^{t/\tau}\right)$

$\frac{dI}{dt} = \frac{I_0}{\tau}e^{-t/\tau}$

$\frac{d^2I}{dt^2} = -\frac{I_0}{\tau^2}e^{-t/\tau}$

By equation (1),

$I_0\left(1 - e^{-t/\tau}\right) \times \frac{I_0}{\tau^2}e^{-t/\tau} = \frac{-I_0^2}{\tau^2}e^{-2t/\tau}$

Let $e^{-t/\tau} = x$

$x - x^2 = x^2$

$x = \frac{1}{2}$

Maximum power,

$P = LI\frac{dI}{dt}$

$P = LI_0\left(1 - \frac{1}{2}\right)\left(\frac{I_0}{\tau} \times \frac{1}{2}\right)$

$$P = \frac{L I_0^2}{4 \times \frac{L}{R}} = \frac{I_0^2 R}{4}$$

$$P = \frac{E^2}{4R}$$

$$a = 2,\ 2b = 4R$$

$$b = 2R = 50$$

$$\frac{b}{a} = 25$$

58. A potential V_o is applied across a uniform wire of resistance R. The power dissipation is P_1. The wire is then cut into two equal halves and a potential of V_o is applied across the length of each half. The total power dissipation across two wires is P_2. The ratio $P_2 : P_1$ is $\sqrt{x} : 1$. The value of x is ________________

Sol. **(16)**

$$P_1 = \frac{v_0^2}{R}$$

$$P_2 = \frac{v_0^2}{\left(\frac{R}{2}\right)} + \frac{v_0^2}{\left(\frac{R}{2}\right)}$$

$$P_2 = 4P_1$$

$$\frac{P_2}{P_1} = \frac{4}{1} = \frac{\sqrt{x}}{1}$$

$$x = 16$$

59. At a given point of time the value of displacement of a simple harmonic oscillator is given as y = A cos (30°). If amplitude is 40 cm and kinetic energy at that time is 200 J, the value of force constant is $1.0 \times 10^x\ Nm^{-1}$. The value of x is ____________

Sol. **(4)**

$$v = \omega\sqrt{A^2 - x^2}$$

$$y = A \times \frac{\sqrt{3}}{2}$$

$$v = \omega\sqrt{A^2 - \frac{3A^2}{4}} = \frac{\omega A}{2}$$

Given, $KE = 200\,J$

$$\frac{1}{2} m \frac{\omega^2 A^2}{4} = 200$$

$$KA^2 = 1600 \quad (K = m\omega^2)$$

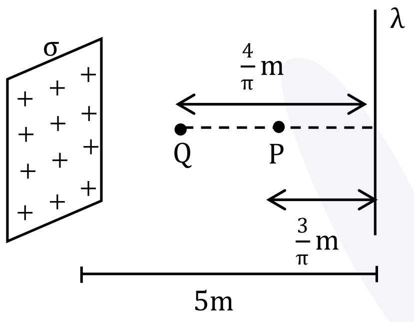
$$K = \frac{1600}{(40 \times 10^{-2})^2}$$

$$K = 10^4\ N/m$$

$$x = 4$$

60. A thin infinite sheet charge and an infinite line charge of respective charge densities $+\sigma$ and $+\lambda$ are placed parallel at 5 m distance from each other. Points 'P' and 'Q' are at $\frac{3}{\pi}$m and $\frac{4}{\pi}$m perpendicular distances from line charge towards sheet charge, respectively. 'E_P' and 'E_Q' are the magnitudes of resultant electric field intensities at point 'P' and 'Q' respectively. If $\frac{E_P}{E_Q} = \frac{4}{a}$ for $2|\sigma| = |\lambda|$, then the value of a is ___________

Sol. **(6)**



$$E_P = \frac{2K\lambda}{r} - \frac{\sigma}{2\varepsilon_0}$$

$$E_P = \frac{\sigma}{2\varepsilon_0} - \frac{\lambda}{2\pi\varepsilon_0\left(\frac{3}{\pi}\right)}$$

$$E_P = \frac{2\sigma}{2\varepsilon_0} - \frac{2\sigma}{6\varepsilon_0} = \frac{\sigma}{6\varepsilon_0}$$

Similarly, $E_Q = \frac{\sigma}{2\varepsilon_0} - \frac{2\sigma}{2\pi\varepsilon_0\left(\frac{4}{\pi}\right)} = \frac{\sigma}{4\varepsilon_0}$

$$\frac{E_P}{E_Q} = \frac{4}{6}$$

a = 6