FINAL JEE–MAIN EXAMINATION – APRIL, 2023

Held On Saturday 15th April, 2023

TIME : 09:00 AM to 12:00 PM

SECTION - A

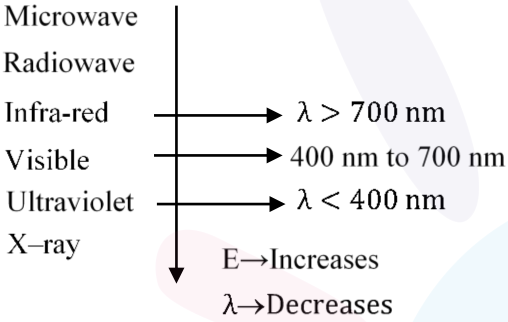
31. Match List I with List II of Electromagnetic waves with corresponding wavelength range :

List I	List II
(A) Microwave	(I) 400 nm to 1 nm
(B) Ultraviolet	(II) 1 nm to 10^{-3} nm
(C) X-Ray	(III) 1 mm to 700 nm
(D) Infra-rad	(IV) 0.1 m to 1 mm

Choose the correct answer from the options given below :

(1) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)
(2) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)
(3) (A)-(I), (B)-(IV), (C)-(II), (D)-(III)
(4) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

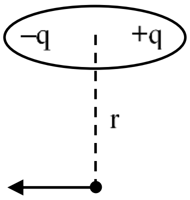
Sol. **(2)**



32. The electric field due to a short electric dipole at a large distance (r) from center of dipole on the equatorial plane varies with distance as :

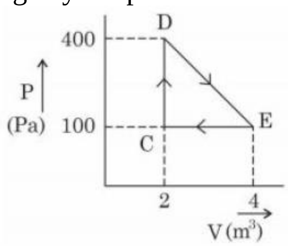
(1) r (2) $\frac{1}{r}$ (3) $\frac{1}{r^2}$ (4) $\frac{1}{r^3}$

Sol. **(4)**



$$E = \frac{KP}{r^3} \propto \frac{1}{r^3}$$

33. A thermodynamic system is taken through cyclic process. The total work done in the process is :



(1) 100 J (2) 300 J (3) 200 J (4) Zero

Sol. **(2)**

Work done = Area of graph

$$W = \frac{1}{2}(400-100)(4-2)$$

$$W = 300J$$

34. The half-life of a radioactive nucleus is 5 years. The fraction of the original sample that would decay in 15 years is :

(1) $\frac{3}{4}$ (2) $\frac{1}{8}$ (3) $\frac{7}{8}$ (4) $\frac{1}{4}$

Sol. **(3)**

$$N = N_0\left(\frac{1}{2}\right)^{t/T_{1/2}}$$

$$N = N_0\left(\frac{1}{2}\right)^{15/5}$$

$$N = \frac{N_0}{8}$$

Decayed nuclei $= N_0 - \frac{N_0}{8}$

$$= \frac{7N_0}{8}$$

35. The position vector of a particle related to time t is given by

$\vec{r} = (10t\hat{i} + 15t^2\hat{j} + 7\hat{k})m$

The direction of net force experienced by the particle is :

(1) Positive z-axis (2) In x – y plane (3) Positive y – axis (4) Positive x – axis

Sol. **(3)**

Given, $\vec{r} = 10t\hat{i} + 15t^2\hat{j} + 7\hat{k}$

$$\vec{v} = \frac{d\vec{r}}{dt} = 10\hat{j} + 30t\hat{j}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = 30\hat{j}$$

$\vec{F} = m\vec{a} \rightarrow$ along (+) y-axis

36. The height of transmitting antenna is 180 m and the height of the receiving antenna is 245 m. The maximum distance between them for satisfactory communication in line of sight will be:

(given R = 6400 km)

(1) 48 km (2) 104 km (3) 96 km (4) 56 km

Sol. **(2)**

$$d = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$

$$d = \sqrt{2R}\left(\sqrt{180} + \sqrt{245}\right)$$

$$d = \sqrt{2 \times 64 \times 10^5}\left(\sqrt{180} + \sqrt{245}\right)$$

$$d = 3577.7\ (13.416 + 15.652)\,m$$

$$d = 104\,km$$

37. A single slit of width a is illuminated by a monochromatic light of wavelength 600 nm. The value of 'a' for which first minimum appears at $\theta = 30^o$ on the screen will be :

(1) 0.6 μm (2) 3 μm (3) 1.8 μm (4) 1.2 μm

Sol. **(4)**

For minima, $a\sin\theta = \lambda$

$$a = \frac{\lambda}{\sin 30^\circ} = 2\lambda$$

$$a = 1200\,\text{nm}$$

$$a = 1.2\mu\text{m}$$

38. A 12 V battery connected to a coil of resistance 6 Ω through a switch, drives a constant current in the circuit. The switch is opened in 1 ms. The emf induced across the coil is 20 V. The inductance of the coil is :

(1) 8 mH (2) 10 mH (3) 12 mH (4) 5 mH

Sol. **(2)**

$$e = -L\frac{di}{dt}$$

$$i_1 = \frac{E}{R} = \frac{12}{6} = 2A$$

$$20 = -L\left(\frac{0-2}{10^{-3}}\right)$$

$$L = 10\text{mH}$$

39. Two identical particles each of mass 'm' go round a circle of radius a under the action of their mutual gravitational attraction. The angular speed of each particle will be :

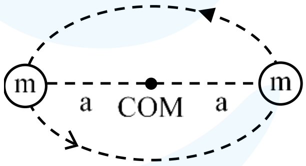
(1) $\sqrt{\frac{Gm}{a^3}}$ (2) $\sqrt{\frac{Gm}{4a^3}}$ (3) $\sqrt{\frac{Gm}{2a^3}}$ (4) $\sqrt{\frac{Gm}{8a^3}}$

Sol. **(2)**

$$F_C = m\omega^2 a$$

$$\frac{Gm^2}{(2a)^2} = m\omega^2 a$$

$$\omega = \sqrt{\frac{Gm}{4a^3}}$$



40. A body is released from a height equal to the radius (r) of the earth. The velocity of the body when it strikes the surface of the earth will be :

(Given g = acceleration due to gravity on the earth.)

(1) $\sqrt{gR}$ (2) $\sqrt{\frac{gR}{2}}$ (3) $\sqrt{4gR}$ (4) $\sqrt{2gR}$

Sol. **(1)**

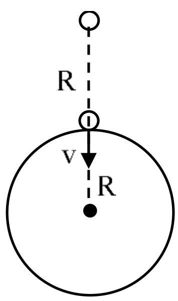
By energy conservation,

$$K_1 + U_1 = K_2 + U_2$$

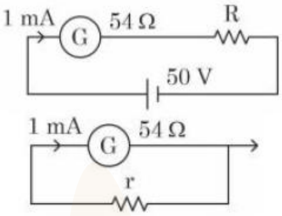
$$O - \frac{GMm}{2R} = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

$$V = \sqrt{\frac{GM}{R} \times \frac{R}{R}}$$

$$V = \sqrt{gR}$$



41. For designing a voltmeter of range 50 V and an ammeter of range 10 mA using a galvanometer which has a coil of resistance 54 Ω showing a full scale deflection for 1 mA as in figure.



(A) for voltmeter $R \approx 50k\Omega$
(B) for ammeter $r \approx 0.2\Omega$
(C) for ammeter $r \approx 6\Omega$
(D) for voltmeter $R \approx 5k\Omega$
(E) for voltmeter $R \approx 500\Omega$

Choose the correct answer from the options given below :

(1) (C) and (D) (2) (A) and (B) (3) (C) and (E) (4) (A) and (C)

Sol. **(4)**

For voltmeter,

$$I = \frac{50}{R+54} = 0.001A$$

$$R = 50k\Omega$$

For Ammeter,

$$I_r = 10 - 1 = 9mA$$

$$V_G = V_r$$

$$1mA \times 54 = 9mA \times r$$

$$r = 6\Omega$$

42. Given below are two statements :

Statement I: The equivalent resistance of resistors in a series combination is smaller than least resistance used in the combination.

Statement II: The resistivity of the material is independent of temperature.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both Statement I and Statement II are true
(2) Both Statement I and Statement II are false
(3) Statement I is false but Statement II is true
(4) Statement I is true but Statement II is false

Sol. **(2)**

In series,

$$R_{eq} = R_1 + R_2 + R_3 + ...$$

$$R_{eq} > R_{Greatest}$$

Hence, statement-I is false.

Resistivity of conductor increases with temperature.

Hence, statement-II is also false

43. The de Broglie wavelength of an electron having kinetic energy E is λ. If the kinetic energy of electron becomes $\frac{E}{4}$, then its de-Broglie wavelength will be :

(1) $\frac{\lambda}{\sqrt{2}}$ (2) 2λ (3) $\frac{\lambda}{2}$ (4) $\sqrt{2}\lambda$

Sol. **(2)**

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{E_2}{E_1}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

$$\lambda_2 = 2\lambda$$

44. A vector in x – y plane makes an angle of 30° with y-axis. The magnitude of y-component of vector is $2\sqrt{3}$. The magnitude of x-component of the vector will be :

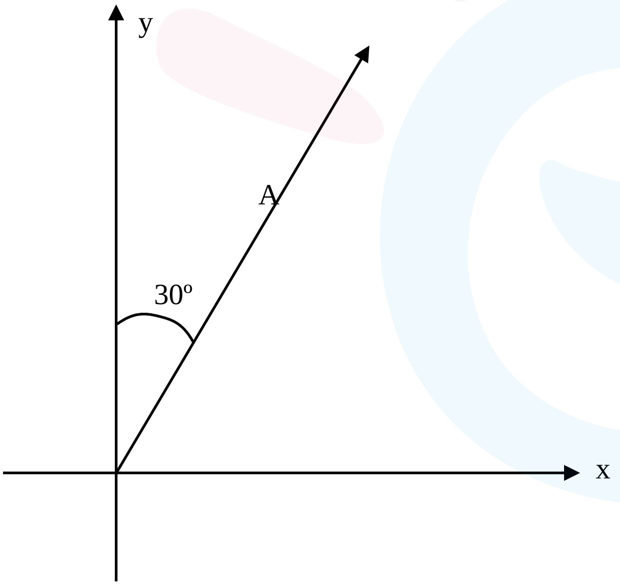
(1) 2 (2) $\sqrt{3}$ (3) $\frac{1}{\sqrt{3}}$ (4) 6

Sol. **(1)**

$$A_y = A\cos 30^\circ = 2\sqrt{3}$$

$$A = 4$$

$$A_x = A\sin 30^\circ = 2$$



45. The speed of a wave produced in water is given by $\upsilon = \lambda^a g^b \rho^c$. Where λ, g and ρ are wavelength of wave, acceleration due to gravity and density of water respectively. The values of a, b and c respectively, are :

(1) $\frac{1}{2}, 0, \frac{1}{2}$ (2) 1, -1, 0 (3) $\frac{1}{2}, \frac{1}{2}, 0$ (4) 1, 1, 0

Sol. **(3)**

By dimensional analysis,

$$V = \lambda^a g^b \rho^c$$

$$\left[L^1T^{-1}\right] = \left[L^1\right]^a \left[L^1T^{-2}\right]^b \left[M^1L^{-3}\right]^c$$

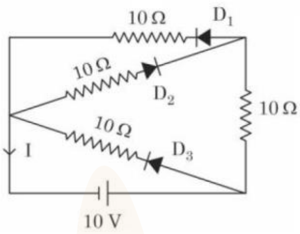
$$[L^1T^{-1}] = \left[M^cL^{a+b-3c}T^{-2b}\right]$$

On comparing respective powers,

c = 0, – 2b = –1, a + b – 3c = 1,

$b = \frac{1}{2}$, $a = \frac{1}{2}$

46. In the given circuit, the current (I) through the battery will be



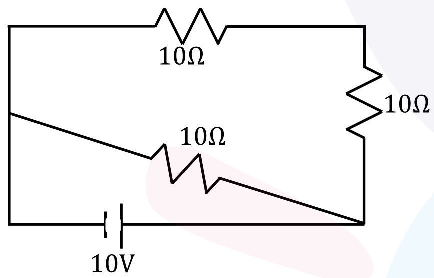
(1) 1A (2) 1.5A (3) 2A (4) 2.5 A

Sol. **(2)**

In given figure,

$D_1, D_3 \rightarrow$ Forward Biased

$D_2 \rightarrow$ Reversed Biased



$$R_{eq} = \frac{(20)(10)}{30} = \frac{20}{3}\Omega$$

$$I = \frac{V}{R} = \frac{10}{\left(\frac{20}{3}\right)} = 1.5A$$

47. In a linear Simple Harmonic Motion (SHM)

(A) Restoring force is directly proportional to the displacement.

(B) The acceleration and displacement are opposite in direction.

(C) The velocity is maximum at mean position.

(D) The acceleration is minimum at extreme points.

Choose the correct answer from the options given below :

(1) (C) and (D) only (2) (A), (C) and (D) only

(3) (A), (B) and (C) only (4) (A), (B) and (D) only

Sol. **(3)**

In SHM,

$F \propto -x \rightarrow A$

$a \propto -x \rightarrow B$

$V_{mean} \rightarrow \max imum \rightarrow c$

$a_{extreme} \rightarrow$ maximum

Hence, (A), (B) and (C) are true.

48. A wire of length 'L' and radius 'r' is clamped rigidly at one end. When the other end of the wire is pulled by a force f, its length increases by 'l'. Another wire of same material of length '2L' and radius '2r' is pulled by a force '2f'. Then the increase in its length will be :

(1) $l/2$ (2) $4l$ (3) l (4) $2l$

Sol. **(3)**

By hooke's law,

$$\gamma = \frac{Fl}{A\Delta l}$$

$$\Delta l \propto \frac{Fl}{A}$$

$$\frac{\Delta l_2}{\Delta l_1} = \frac{F_2 l_2}{F_1 l_1} \times \frac{A_1}{A_2}$$

$$= \frac{2f \times 2L}{f \times L} \times \frac{\pi(r)^2}{\pi(2r)^2}$$

$$\Delta l_1 = \Delta l_2$$

$$\Delta l_2 = l$$

49. A flask contains Hydrogen and Argon in the ratio 2 : 1 by mass. The temperature of the mixture is 30°C. The ratio of average kinetic energy per molecule of the two gases (K argon/K hydrogen) is :
(Given : Atomic Weight of Ar = 39.9)

(1) 2 (2) 39.9 (3) 1 (4) $\frac{39.9}{2}$

Sol. **(3)**

Kinetic energy per molecule = $\frac{3}{2}k_B T$

$\propto T$

$$\frac{k_{Ar}}{k_{H_2}} = \frac{T}{T} = 1$$

50. The position of a particle related to time is given by $x = (5t^2 - 4t + 5)$ m. The magnitude of velocity of the particle at t = 2s will be :

(1) 10 ms^{-1} (2) 06 ms^{-1} (3) 16 ms^{-1} (4) 14 ms^{-1}

Sol. **(3)**

Given, $x = (5t^2 - 4t + 5)m$

$$V = \frac{dx}{dt} = 10t - 4$$

$$t = 2, V = 20 - 4$$

$$V = 16m/s$$

Section - B

51. An election in a hydrogen atom revolves around its nucleus with a speed of 6.76 × 10^6 ms^{-1} in an orbit of radius 0.52 A°. The magnetic field produced at the nucleus of the hydrogen atom is _______ T.

Sol. **(40)**

Biot-Savart law,

$$B = \frac{\mu_0}{4\pi} \times \frac{qV}{r^2}$$

$$B = 10^{-7} \times \frac{1.6 \times 10^{-19} \times 6.76 \times 10^6}{(0.52 \times 10^{-10})^2}$$

$$B = 40T$$

52. A 20 cm long metallic rod is rotated with 210 rpm about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant and uniform magnetic field 0.2 T parallel to the axis exists everywhere. The emf developed between the centre and the ring is _______ mV. (Take $\pi = 22/7$)

Sol. **(88)**

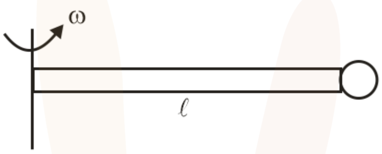
$\omega = 210 \times \frac{\pi}{30}$

$= 7\pi = 22 \text{rad/s}$

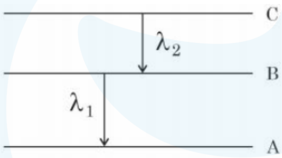
$\text{emf} = e = \frac{1}{2}B\omega(\ell)^2$

$= \frac{1}{2} \times 0.2 \times 22 \times (0.2)^2$

$= \frac{11}{125}\text{V} = 88\text{mV}$



53. As per given figure A, B and C are the first, second and third excited energy levels of hydrogen atom respectively. If the ratio of the two wavelengths $\left(\text{i.e.} \frac{\lambda_1}{\lambda_2}\right)$ is $\frac{7}{4n}$, then the value of n will be _______.



Sol. **(5)**

For A, n = 2

B, n = 3

C, n = 4

$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$\frac{1}{\lambda_2} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$

$\frac{1}{\lambda_2} = \frac{7R}{144}$ ....(1)

$\frac{1}{\lambda_1} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$

$\frac{1}{\lambda_1} = \frac{5R}{36}$ ....(2)

(1) and (2)

$\frac{\lambda_1}{\lambda_2} = \frac{7}{20} = \frac{7}{4 \times 5}$

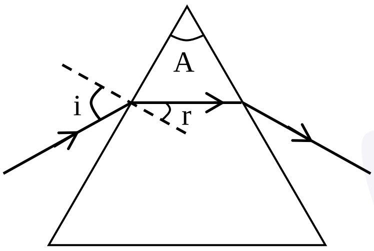
$n = 5$

54. The refractive index of a transparent liquid filled in an equilateral hollow prism is $\sqrt{2}$. The angle of minimum deviation for the liquid will be _________.

Sol. (30)

For minimum deviation

$$r = \frac{A}{2} = \frac{60}{2} = 30^o$$



$$1 \sin i = \sqrt{2} \sin r$$

$$\sin i = \sqrt{2} \times \sin 30^o$$

$$\sin i = \frac{1}{\sqrt{2}}$$

$$i = 45^o$$

$$\delta_{min} = 2i - A = 90 - 60 = 30^o$$

$$\delta_{min} = 30^o$$

55. A block of mass 10 kg is moving along x-axis under the action of force F = 5x N. The work done by the force in moving the block from x = 2m to 4m will be ________J.

Sol. (30)

Work done, $w = \int F dx$

$$w = \int_2^4 5x dx$$

$$w = \frac{5}{2}\left[x^2\right]_2^4$$

$$w = \frac{5}{2}(16 - 4)$$

$$w = 30J$$

56. The fundamental frequency of vibration of a string stretched between two rigid support is 50 Hz. The mass of the string is 18g and its linear mass density is 20 g/m. The speed of the transverse waves so produced in the string is _____ ms^{-1}

Sol. (90)

$$\mu = \frac{m}{\ell} = 20 gm / m$$

$$\frac{18gm}{\ell} = 20 gm / m$$

$$\ell = \frac{9}{10} m$$

For fundamental mode,

$$f = \frac{V}{2\ell}$$

$$V = 50 \times \frac{18}{10} = 90 m / s$$

57. A solid sphere and a solid cylinder of same mass and radius are rolling on a horizontal surface without slipping. The ratio of their radius of gyrations respectively (k_{sph} : k_{cyl}) is $2 : \sqrt{x}$. The value of x is ______

Sol. **(5)**

$$I_{sphere} = \frac{2}{5}MR^2 = Mk^2$$

$$k_{sph} = \sqrt{\frac{2}{5}}R$$

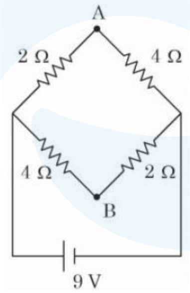
$$I_{cylinder} = \frac{MR^2}{2} = Mk^2$$

$$k_{cyl} = \frac{R}{\sqrt{2}}$$

$$\frac{k_{sph}}{k_{cyl}} = \frac{2}{\sqrt{5}}$$

$$x = 5$$

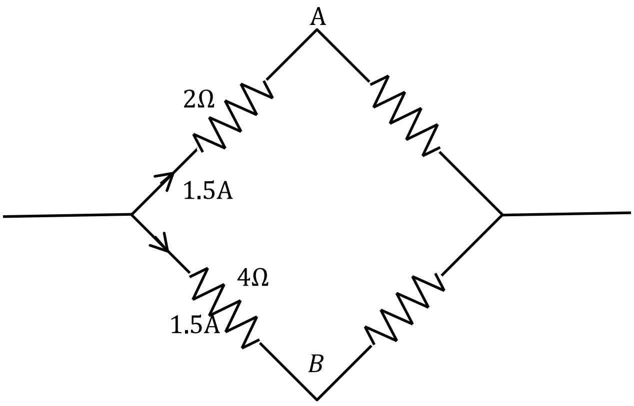
58. A network of four resistances is connected to 9 V battery, as shown in figure. The magnitude of voltage difference between the points A and B is ______________ V.



Sol. **(3)**

$$R_{eq} = \frac{6}{2} = 3\Omega$$

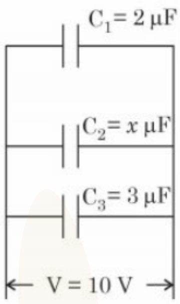
$$I = \frac{V}{R} = \frac{9}{3} = 3A$$



$$V_A + (1.5)(2) - (1.5)4 = V_B$$

$$V_A - V_B = 3V$$

59. In the given figure the total charge stored in the combination of capacitors is 100 μ C. The value of 'x' is _______.



Sol. **(5)**

All the capacitors are in parallel

$C_{eq} = (5 + x)\mu F$

$Q = C_{aq} V$

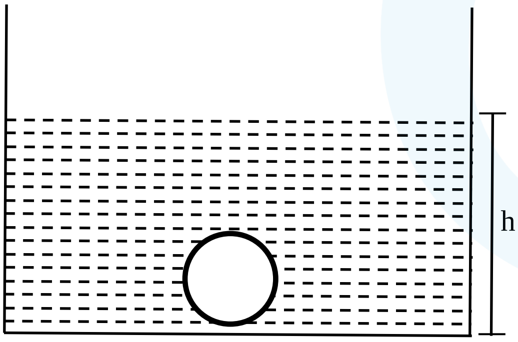
$100 = (5 + x)(10)$

$x = 5$

60. There is an air bubble of radius 1.0 mm in a liquid of surface tension 0.075 nm^{-1} and density 1000 kg m^{-3} at a depth of 10 cm below the free surface. The amount by which the pressure inside the bubble is greater than the atmospheric pressure is _________ Pa (g = 10 ms^{-2})

Sol. **(1150)**

$P_{in} = P_0 + \rho gh + \frac{2T}{r}$



$P_{in} - P_0 = 1000 \times 10 \times 0.1 + \frac{2 \times 0.075}{0.001}$

$= 1000 + 150$

$P_{in} - P_0 = 1150 Pa$