FINAL JEE-MAIN EXAMINATION - JULY, 2022
Held On Monday 25 July, 2022
TIME :3:00 PM TO 06:00 PM

SECTION-A

1. For $z \in \mathbb{C}$ if the minimum value of $\left(|z-3\sqrt{2}|+|z-p\sqrt{2}i|\right)$ is $5\sqrt{2}$, then a value of p is _____

(A) 3 (B) $\frac{7}{2}$

(C) 4 (D) $\frac{9}{2}$

Official Ans. by NTA (C)

Ans. (C)

2. The number of real values λ, such that the system of linear equations

$2x - 3y + 5z = 9$

$x + 3y - z = -18$

$3x - y + (\lambda^2 - |\lambda|)z = 16$

has no solution, is :-

(A) 0 (B) 1

(C) 2 (D) 4

Official Ans. by NTA (C)

Ans. (C)

3. The number of bijective functions $f : \{1, 3, 5, 7,.....99\} \to \{2, 4, 6, 8,....., 100\}$, such that $f(3) \geq f(9) \geq f(15) \geq f(21) \geq ..... \geq f(99)$, is _____

(A) ${}^{50}P_{17}$ (B) ${}^{50}P_{33}$

(C) $33! \times 17!$ (D) $\frac{50!}{2}$

Official Ans. by NTA (B)

Ans. (B)

4. The remainder when $(11)^{1011} + (1011)^{11}$ is divided by 9 is

(A) 1 (B) 4

(C) 6 (D) 8

Official Ans. by NTA (D)

Ans. (D)

5. The sum $\sum_{n=1}^{21} \frac{3}{(4n-1)(4n+3)}$ is equal to

(A) $\frac{7}{87}$ (B) $\frac{7}{29}$

(C) $\frac{14}{87}$ (D) $\frac{21}{29}$

Official Ans. by NTA (B)

Ans. (B)

6. $\lim\limits_{x \to \frac{\pi}{4}} \frac{8\sqrt{2} - (\cos x + \sin x)^7}{\sqrt{2} - \sqrt{2}\sin 2x}$ is equal to

(A) 14 (B) 7

(C) $14\sqrt{2}$ (D) $7\sqrt{2}$

Official Ans. by NTA (A)

Ans. (A)

7. $\lim\limits_{n \to \infty} \frac{1}{2^n}\left(\frac{1}{\sqrt{1-\frac{1}{2^n}}} + \frac{1}{\sqrt{1-\frac{2}{2^n}}} + \frac{1}{\sqrt{1-\frac{3}{2^n}}} + ... + \frac{1}{\sqrt{1-\frac{2^n-1}{2^n}}}\right)$

is equal to

(A) $\frac{1}{2}$ (B) 1

(C) 2 (D) –2

Official Ans. by NTA (C)

Ans. (C)

8. If A and B are two events such that $P(A) = \frac{1}{3}, P(B) = \frac{1}{5}$ and $P(A \cup B) = \frac{1}{2}$, then $P(A \mid B') + P(B \mid A')$ is equal to

(A) $\frac{3}{4}$ (B) $\frac{5}{8}$

(C) $\frac{5}{4}$ (D) $\frac{7}{8}$

Official Ans. by NTA (B)

Ans. (B)

9. Let [t] denote the greatest integer less than or equal to t. Then the value of the integral $\int_{-3}^{101}([\sin(\pi x)]+e^{[\cos(2\pi x)]})dx$ is equal to

(A) $\frac{52(1-e)}{e}$ (B) $\frac{52}{e}$

(C) $\frac{52(2+e)}{e}$ (D) $\frac{104}{e}$

Official Ans. by NTA (B)

Ans. (B)

10. Let the point P (α, β) be at a unit distance from each of the two lines L_1 : 3x – 4y + 12 = 0, and L_2 : 8x + 6y + 11 = 0. If P lies below L_1 and above L_2, then 100 (α + β) is equal to

(A) –14 (B) 42

(C) –22 (D) 14

Official Ans. by NTA (D)

Ans. (D)

11. Let a smooth curve y = f(x) be such that the slope of the tangent at any point (x, y) on it is directly proportional to $\left(\frac{-y}{x}\right)$. If the curve passes through the point (1, 2) and (8, 1), then $\left|y\left(\frac{1}{8}\right)\right|$ is equal to

(A) $2\log_e 2$ (B) 4

(C) 1 (D) $4\log_e 2$

Official Ans. by NTA (B)

Ans. (B)

12. If the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the line $\frac{x}{7}+\frac{y}{2\sqrt{6}}=1$ on the x-axis and the line $\frac{x}{7}-\frac{y}{2\sqrt{6}}=1$ on the y-axis, then the eccentricity of the ellipse is

(A) $\frac{5}{7}$ (B) $\frac{2\sqrt{6}}{7}$

(C) $\frac{3}{7}$ (D) $\frac{2\sqrt{5}}{7}$

Official Ans. by NTA (A)

Ans. (A)

13. The tangents at the point A(1, 3) and B(1, –1) on the parabola y^2 – 2x – 2y = 1 meet at the point P. Then the area (in unit^2) of the triangle PAB is :-

(A) 4 (B) 6

(C) 7 (D) 8

Official Ans. by NTA (D)

Ans. (D)

14. Let the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{7}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{\alpha}=\frac{1}{25}$ coincide. Then the length of the latus rectum of the hyperbola is:-

(A) $\frac{32}{9}$ (B) $\frac{18}{5}$

(C) $\frac{27}{4}$ (D) $\frac{27}{10}$

Official Ans. by NTA (D)

Ans. (D)

15. A plane E is perpendicular to the two planes 2x – 2y + z = 0 and x – y + 2z = 4, and passes through the point P(1, –1, 1). If the distance of the plane E from the point Q(a, a, 2) is $3\sqrt{2}$, then (PQ)^2 is equal to

(A) 9 (B) 12

(C) 21 (D) 33

Official Ans. by NTA (C)

Ans. (C)

16. The shortest distance between the lines $\frac{x+7}{-6}=\frac{y-6}{7}=z$ and $\frac{7-x}{2}=y-2=z-6$ is

(A) $2\sqrt{29}$ (B) 1

(C) $\sqrt{\frac{37}{29}}$ (D) $\frac{\sqrt{29}}{2}$

Official Ans. by NTA (A)

Ans. (A)

17. Let $\vec{a}=\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b}$ be a vector such that $\vec{a}\times\vec{b}=2\hat{i}-\hat{k}$ and $\vec{a}.\vec{b}=3$. Then the projection of $\vec{b}$ on the vector $\vec{a}-\vec{b}$ is :-

(A) $\frac{2}{\sqrt{21}}$ (B) $2\sqrt{\frac{3}{7}}$

(C) $\frac{2}{3}\sqrt{\frac{7}{3}}$ (D) $\frac{2}{3}$

Official Ans. by NTA (A)

Ans. (A)

18. If the mean deviation about median for the number 3, 5, 7, 2k, 12, 16, 21, 24 arranged in the ascending order, is 6 then the median is

(A) 11.5 (B) 10.5

(C) 12 (D) 11

Official Ans. by NTA (D)

Ans. (D)

19. $2\sin\left(\frac{\pi}{22}\right)\sin\left(\frac{3\pi}{22}\right)\sin\left(\frac{5\pi}{22}\right)\sin\left(\frac{7\pi}{22}\right)\sin\left(\frac{9\pi}{22}\right)$ is equal to

(A) $\frac{3}{16}$ (B) $\frac{1}{16}$

(C) $\frac{1}{32}$ (D) $\frac{9}{32}$

Official Ans. by NTA (B)

Ans. (B)

20. Consider the following statements :

P : Ramu is intelligent

Q : Ramu is rich

R : Ramu is not honest

The negation of the statement "Ramu is intelligent and honest if and only if Ramu is not rich" can be expressed as :

(A) $((P\wedge(\sim R))\wedge Q)\wedge((\sim Q)\wedge((\sim P)\vee R))$

(B) $((P\wedge R)\wedge Q)\vee((\sim Q)\wedge((\sim P)\vee(\sim R)))$

(C) $((P\wedge R)\wedge Q)\wedge((\sim Q)\wedge((\sim P)\vee(\sim R)))$

(D) $((P\wedge(\sim R))\wedge Q)\vee((\sim Q)\wedge((\sim P)\vee R))$

Official Ans. by NTA (D)

Ans. (D)

SECTION-B

1. Let A : {1, 2, 3, 4, 5, 6, 7}. Define B = {T ⊆ A : either $1\notin T$ or $2\in T$} and C = T ⊆ A : T the sum of all the elements of T is a prime number}. Then the number of elements in the set $B\cup C$ is _____

Official Ans. by NTA (107)

Ans. (107)

2. Let f(x) be a quadratic polynomial with leading coefficient 1 such that f(0) = p, $p\neq 0$ and $f(1)=\frac{1}{3}$. If the equation f(x) = 0 and fofofof(x) = 0 have a common real root, then f(–3) is equal to.....

Official Ans. by NTA (25)

Ans. (25)

3. Let $A=\begin{bmatrix}1 & a & a\\ 0 & 1 & b\\ 0 & 0 & 1\end{bmatrix}, a,b\in\mathbb{R}$. If for some $n\in N$, $A^n=\begin{bmatrix}1 & 48 & 2160\\ 0 & 1 & 96\\ 0 & 0 & 1\end{bmatrix}$ then n + a + b is equal to _____

Official Ans. by NTA (24)

Ans. (24)

4. The sum of the maximum and minimum values of the function f(x) = |5x – 7| + [x² + 2x] is the interval $\left[\frac{5}{4},2\right]$, where [t] is the greatest integer $\le t$ is _____

Official Ans. by NTA (15)

Ans. (15)

5. Let y = y(x) be the solution of the differential equation $\frac{dy}{dx}=\frac{4y^3+2yx^2}{3xy^2+x^3}, y(1)=1$. If for some $n\in N, y(2)\in[n-1,n)$, then n is equal to _____

Official Ans. by NTA (3)

Ans. (3)

6. Let f be a twice differentiable function on R. If $f'(0)=4$ and

$$f(x)+\int_0^x (x-t)f'(t)dt=(e^{2x}+e^{-2x})\cos 2x+\frac{2}{a}x,$$

then $(2a+1)^5 a^2$ is equal to _____

Official Ans. by NTA (8)

Ans. (8)

7. Let $a_n=\int_{-1}^{n}\left(1+\frac{x}{2}+\frac{x^2}{2}+\frac{x^3}{3}+.....+\frac{x^{n-1}}{n}\right)dx$ for $n\in N$. Then the sum of all the elements of the set $\{n\in N: a_n\in(2,30)\}$ is _____

Official Ans. by NTA (5)

Ans. (5)

8. If the circles $x^2+y^2+6x+8y+16=0$ and $x^2+y^2+2(3-\sqrt{3})x+x+2(4-\sqrt{6})y=k+6\sqrt{3}+8\sqrt{6}, k>0$, touch internally at the point P(α, β), then $(\alpha+\sqrt{3})^2+(\beta+\sqrt{6})^2$ is equal to _____

Official Ans. by NTA (25)

Ans. (25)

9. Let the area enclosed by the x-axis, and the tangent and normal drawn to the curve $4x^3-3xy^2+6x^2-5xy-8y^2+9x+14=0$ at the point (–2, 3) be A. Then 8A is equal to _____

Official Ans. by NTA (170)

Ans. (170)

10. Let $x=\sin(2\tan^{-1}\alpha)$ and $y=\sin\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right)$. If $S=\{\alpha\in R: y^2=1-x\}$, then $\sum_{\alpha\in S}16\alpha^3$ is equal to _____

Official Ans. by NTA (130)

Ans. (130)