



**FINAL JEE-MAIN EXAMINATION - JULY, 2022**  
**Held On Friday 29 July,2022**  
**TIME :9:00 AM to 12:00 NOON**

**SECTION-A**

1. Let R be a relation from the set {1, 2, 3.....,60} to itself such that  $R = \{(a, b) : b = pq, \text{ where } p, q \geq 3 \text{ are prime numbers}\}$ . Then, the number of elements in R is :

- (A) 600 (B) 660  
 (C) 540 (D) 720

**Official Ans. by NTA (B)**

**Ans. (B)**

**Sol.** Number of possible values of  $a = 60$ , for  $b = pq$ ,

If  $p = 3, q = 3, 5, 7, 11, 13, 17, 19$

If  $p = 5 \quad q = 5, 7, 11$

If  $p = 7 \quad q = 7$

Total cases =  $60 \times 11 = 660$

2. If  $z = 2 + 3i$ , then  $z^5 + (\bar{z})^5$  is equal to :

- (A) 244 (B) 224  
 (C) 245 (D) 265

**Official Ans. by NTA (A)**

**Ans. (A)**

**Sol.**  $z^5 + (\bar{z})^5 = (2 + 3i)^5 + (2 - 3i)^5$

$$= 2^5 C_0 2^5 + {}^5 C_2 2^3 (3i)^2 + {}^5 C_4 2^1 (3i)^4$$

$$= 2(32 + 10 \times 8(-9) + 5 \times 2 \times 81) = 244$$

3. Let A and B be two  $3 \times 3$  non-zero real matrices such that AB is a zero matrix. Then

- (A) The system of linear equations  $AX = 0$  has a unique solution  
 (B) The system of linear equations  $AX = 0$  has infinitely many solutions  
 (C) B is an invertible matrix  
 (D)  $\text{adj}(A)$  is an invertible matrix

**Official Ans. by NTA (B)**

**Ans. (B)**

**Sol.**  $AB = 0 \Rightarrow |AB| = 0$

$$\begin{array}{c} |A| |B| = 0 \\ \swarrow \quad \searrow \\ |A| = 0 \quad |B| = 0 \end{array}$$

If  $|A| \neq 0, B = 0$  (not possible)

If  $|B| \neq 0, A = 0$  (not possible)

Hence  $|A| = |B| = 0$

$\Rightarrow AX = 0$  has infinitely many solutions

4. If  $\frac{1}{(20-a)(40-a)} + \frac{1}{(40-a)(60-a)} + \dots + \frac{1}{(180-a)(200-a)} = \frac{1}{256}$ , then the maximum value of a is :

- (A) 198 (B) 202  
 (C) 212 (D) 218

**Official Ans. by NTA (C)**

**Ans. (C) Sol.**

By splitting

$$\frac{1}{20} \left[ \left( \frac{1}{20-a} - \frac{1}{40-a} \right) + \left( \frac{1}{40-a} - \frac{1}{60-a} \right) + \dots + \left( \frac{1}{180-a} - \frac{1}{200-a} \right) \right]$$

$$\Rightarrow \frac{1}{20} \left( \frac{1}{20-a} - \frac{1}{200-a} \right) = \frac{1}{256}$$

$$(20-a)(200-a) = 256 \times 9$$

$$a^2 - 220a + 1696 = 0$$

$$a = 8, 212$$

Hence maximum value of a is 212.

5. If  $\lim_{x \rightarrow 0} \frac{\alpha e^x + \beta e^{-x} + \gamma \sin x}{x \sin^2 x} = \frac{2}{3}$ ,

where  $\alpha, \beta, \gamma \in \mathbb{R}$ , then which of the following is NOT correct ?

- (A)  $\alpha^2 + \beta^2 + \gamma^2 = 6$   
 (B)  $\alpha\beta + \beta\gamma + \gamma\alpha + 1 = 0$   
 (C)  $\alpha\beta^2 + \beta\gamma^2 + \gamma\alpha^2 + 3 = 0$   
 (D)  $\alpha^2 - \beta^2 + \gamma^2 = 4$

**Official Ans. by NTA (C)**

**Ans. (C)**

Sol.

$$\lim_{x \rightarrow 0} \frac{\alpha \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots\right) + \beta \left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots\right) + \gamma \left(x - \frac{x^3}{3!} + \dots\right)}{x^3}$$

constant terms should be zero

$$\Rightarrow \alpha + \beta = 0$$

coeff of x should be zero

$$\Rightarrow \alpha - \beta + \gamma = 0$$

coeff of  $x^2$  should be zero

$$\lim_{x \rightarrow 0} \frac{x^3 \left(\frac{\alpha}{3!} - \frac{\beta}{3!} - \frac{\gamma}{3!}\right) + x^4 \left(\frac{\alpha}{3!} - \frac{\beta}{3!} - \frac{\gamma}{3!}\right)}{x^3} = \frac{2}{3}$$

$$\Rightarrow \frac{\alpha}{2} + \frac{\beta}{2} = 0$$

$$\frac{\alpha}{6} - \frac{\beta}{6} - \frac{\gamma}{6} = \frac{2}{3}$$

$$\Rightarrow \alpha = 1, \beta = -1, \gamma = -2$$

6. The integral  $\int_0^{\frac{\pi}{2}} \frac{1}{3 + 2 \sin x + \cos x} dx$  is equal to:

(A)  $\tan^{-1}(2)$                       (B)  $\tan^{-1}(2) - \frac{\pi}{4}$

(C)  $\frac{1}{2} \tan^{-1}(2) - \frac{\pi}{8}$                       (D)  $\frac{1}{2}$

**Official Ans. by NTA (B)**

**Ans. (B)**

Sol.

$$I = \int_0^{\frac{\pi}{2}} \frac{dx}{3 + 2 \sin x + \cos x} = \int_0^{\frac{\pi}{2}} \frac{\sec^2 \frac{x}{2} dx}{2 \tan^2 \frac{x}{2} + 4 \tan \frac{x}{2} + 4}$$

Put  $\tan \frac{x}{2} = t$ , so

$$I = \int_0^1 \frac{dt}{(t+1)^2 + 1} = \tan^{-1}(t+1) \Big|_0^1 = \tan^{-1} 2 - \frac{\pi}{4}$$

7. Let the solution curve  $y = y(x)$  of the differential equation  $(1 + e^{2x}) \left(\frac{dy}{dx} + y\right) = 1$  pass through the point  $\left(0, \frac{\pi}{2}\right)$ . Then,  $\lim_{x \rightarrow \infty} e^x y(x)$  is equal to :

- (A)  $\frac{\pi}{4}$                                       (B)  $\frac{3\pi}{4}$   
 (C)  $\frac{\pi}{2}$                                       (D)  $\frac{3\pi}{2}$

**Official Ans. by NTA (B)**

**Ans. (B)**

Sol.  $\frac{dy}{dx} + y = \frac{1}{1 + e^{2x}}$

So integrating factor is  $e^{\int 1 \cdot dx} = e^x$

So solution is  $y \cdot e^x = \tan^{-1}(e^x) + c$

Now as curve is passing through  $\left(0, \frac{\pi}{2}\right)$  so

$$\Rightarrow c = \frac{\pi}{4}$$

$$\Rightarrow \lim_{x \rightarrow \infty} (y \cdot e^x) = \lim_{x \rightarrow \infty} \left(\tan^{-1}(e^x) + \frac{\pi}{4}\right) = \frac{3\pi}{4}$$

8. Let a line L pass through the point of intersection of the lines  $bx + 10y - 8 = 0$  and  $2x - 3y = 0$ ,  $b \in \mathbb{R} - \left\{\frac{4}{3}\right\}$ . If the line L also passes through the point  $(1, 1)$  and touches the circle  $17(x^2 + y^2) = 16$ , then the eccentricity of the ellipse  $\frac{x^2}{5} + \frac{y^2}{b^2} = 1$  is :

- (A)  $\frac{2}{\sqrt{5}}$                                       (B)  $\sqrt{\frac{3}{5}}$   
 (C)  $\frac{1}{\sqrt{5}}$                                       (D)  $\sqrt{\frac{2}{5}}$

**Official Ans. by NTA (B)**

**Ans. (B)**

Sol. Line is passing through intersection of  $bx + 10y - 8 = 0$  and  $2x - 3y = 0$  is  $(bx + 10y - 8) + \lambda(2x - 3y) = 0$ . As line is passing through  $(1, 1)$  so  $\lambda = b + 2$

Now line  $(3b+4)x - (3b-4)y - 8 = 0$  is tangent to circle  $17(x^2 + y^2) = 16$

$$\text{So } \frac{8}{\sqrt{(3b+4)^2 + (3b-4)^2}} = \frac{4}{\sqrt{17}}$$

$$\Rightarrow b^2 = 2 \Rightarrow e = \sqrt{\frac{3}{5}}$$

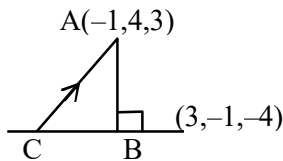
9. If the foot of the perpendicular from the point  $A(-1, 4, 3)$  on the plane  $P : 2x + my + nz = 4$ , is  $(-2, \frac{7}{2}, \frac{3}{2})$ , then the distance of the point A from the plane P, measured parallel to a line with direction ratios  $3, -1, -4$ , is equal to :

- (A) 1 (B)  $\sqrt{26}$   
 (C)  $2\sqrt{2}$  (D)  $\sqrt{14}$

Official Ans. by NTA (B)

Ans. (B)

Sol.



Let B be foot of  $\perp$  coordinates of  $B = (-2, \frac{7}{2}, \frac{3}{2})$

Direction ratio of line AB is  $\langle 2, 1, 3 \rangle$  so  $m = 1, n = 3$

$$\text{So equation of AC is } \frac{x+1}{3} = \frac{y-4}{-1} = \frac{z-3}{-4} = \lambda$$

So point C is  $(3\lambda - 1, -\lambda + 4, -4\lambda + 3)$ . But C lies on the plane, so

$$6\lambda - 2 - \lambda + 4 - 12\lambda + 9 = 4$$

$$\Rightarrow \lambda = 1 \Rightarrow C(2, 3, -1)$$

$$\Rightarrow AC = \sqrt{26}$$

10. Let  $\vec{a} = 3\hat{i} + \hat{j}$  and  $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ . Let  $\vec{c}$  be a vector satisfying  $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b} + \lambda\vec{c}$ . If  $\vec{b}$  and  $\vec{c}$  are non-parallel, then the value of  $\lambda$  is :

- (A) -5 (B) 5  
 (C) 1 (D) -1

Official Ans. by NTA (A)

Ans. (A)

Sol.  $\vec{a} = 3\hat{i} + \hat{j}, \vec{b} = \hat{i} + 2\hat{j} + \hat{k}$

$$\text{As } \vec{a} \times (\vec{b} \times \vec{c}) = \vec{b} + \lambda\vec{c}$$

$$\Rightarrow \vec{a} \cdot \vec{c} (\vec{b}) - (\vec{a} \cdot \vec{b}) \vec{c} = \vec{b} + \lambda\vec{c}$$

$$\Rightarrow \vec{a} \cdot \vec{c} = 1, \vec{a} \cdot \vec{b} = -\lambda$$

$$\Rightarrow (3\hat{i} + \hat{j}) \cdot (\hat{i} + 2\hat{j} + \hat{k}) = -\lambda$$

$$\Rightarrow \lambda = -5$$

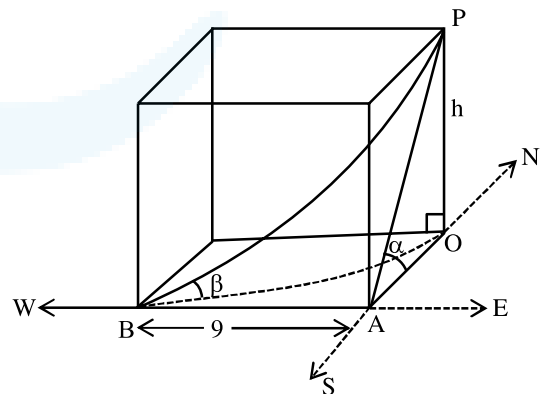
11. The angle of elevation of the top of a tower from a point A due north of it is  $\alpha$  and from a point B at a distance of 9 units due west of A is  $\cos^{-1}(\frac{3}{\sqrt{13}})$ . If the distance of the point B from the tower is 15 units, then  $\cot \alpha$  is equal to :

- (A)  $\frac{6}{5}$  (B)  $\frac{9}{5}$   
 (C)  $\frac{4}{3}$  (D)  $\frac{7}{3}$

Official Ans. by NTA (A)

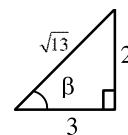
Ans. (A)

Sol.

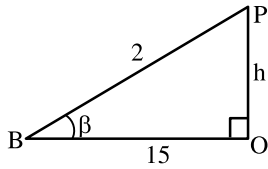


given  $OB = 15$

$$\cos \beta = \frac{3}{\sqrt{13}}$$



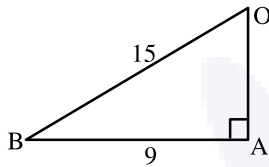
$$\tan \beta = \frac{2}{3}$$



$$\tan \beta = \frac{h}{15}$$

$$\frac{2}{3} = \frac{h}{15}$$

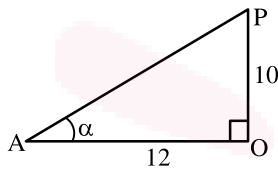
$$\boxed{10 = h}$$



$$OA^2 + AB^2 = 225$$

$$OA^2 + 81 = 225$$

$$\boxed{OA = 12}$$



$$\tan \alpha = \frac{10}{12}$$

$$\cot \alpha = \frac{12}{10} = \frac{6}{5}$$

12. The statement  $(p \wedge q) \Rightarrow (p \wedge r)$  is equivalent to :

- (A)  $q \Rightarrow (p \wedge r)$       (B)  $p \Rightarrow (p \wedge r)$   
 (C)  $(p \wedge r) \Rightarrow (p \wedge q)$       (D)  $(p \wedge q) \Rightarrow r$

**Official Ans. by NTA (D)**

**Ans. (D)**

**Sol.**  $(p \wedge q) \Rightarrow (p \wedge r)$

$$\sim (p \wedge q) \vee (p \wedge r)$$

$$(\sim p \vee \sim q) \vee (p \wedge r)$$

$$(\sim p \vee (p \wedge r)) \vee \sim q$$

$$(\sim p \vee p) \wedge (\sim p \vee r) \vee \sim q$$

$$(\sim p \vee r) \vee \sim q$$

$$(\sim p \vee \sim q) \vee r$$

$$\sim (p \wedge q) \vee r$$

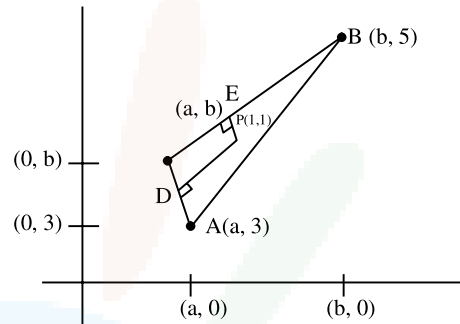
$$(p \wedge q) \Rightarrow r$$

13. Let the circumcentre of a triangle with vertices  $A(a, 3)$ ,  $B(b, 5)$  and  $C(a, b)$ ,  $ab > 0$  be  $P(1, 1)$ . If the line  $AP$  intersects the line  $BC$  at the point  $Q(k_1, k_2)$ , then  $k_1 + k_2$  is equal to :

- (A) 2      (B)  $\frac{4}{7}$       (C)  $\frac{2}{7}$       (D) 4

**Official Ans. by NTA (B)**

**Ans. (B)**



$$m_{AC} \rightarrow \infty$$

$$m_{PD} = 0$$

$$D\left(\frac{a+a}{2}, \frac{b+3}{2}\right)$$

$$D\left(a, \frac{b+3}{2}\right)$$

$$m_{PD} = 0$$

$$\frac{b+3}{2} - 1 = 0$$

$$b+3-2=0$$

$$\boxed{b = -1}$$

$$E\left(\frac{b+a}{2}, \frac{5+b}{2}\right) = \left(\frac{a-1}{2}, 2\right)$$

$$m_{CB} \cdot m_{EP} = -1$$

$$\left(\frac{5-b}{b-a}\right) = \left(\frac{2-1}{\frac{a-1}{2}-1}\right) = -1$$

$$\left(\frac{6}{-1-a}\right) = \left(\frac{2}{a-3}\right) = -1$$

$$12 = (1+a)(a-3)$$

$$12 = a^2 - 3a + a - 3$$

$$\Rightarrow a^2 - 2a - 15 = 0$$

$$(a-5)(a+3) = 0$$



$a = 5$  or  $a = -3$

Given  $ab > 0$

$a(-1) > 0$

$-a > 0$

$a < 0$

$a = -3$  Accept

AP line A  $(-3, 3)$  P(1, 1)

$$y - 1 = \left( \frac{3-1}{-3-1} \right) (x-1)$$

$-2y + 2 = x - 1$

$\Rightarrow x + 2y = 3$       Applying ....(1)

Line BC B(-1, 5)

C(-3, -1)

$$(y - 5) = \frac{6}{2}(x + 1)$$

$y - 5 = 3x + 3$

$y = 3x + 8$       ....(2)

Solving (1) & (2)

$x + 2(3x + 8) = 3$

$\Rightarrow 7x + 16 = 3$

$7x = -13$

$x = -\frac{13}{7}$

$y = 3\left(-\frac{13}{7}\right) + 8$

$= \frac{-39 + 56}{7}$

$y = \frac{17}{7}$

$x + y = \frac{-13 + 17}{7} = \frac{4}{7}$

14. Let  $\hat{a}$  and  $\hat{b}$  be two unit vectors such that the angle between them is  $\frac{\pi}{4}$ . If  $\theta$  is the angle between

the vectors  $(\hat{a} + \hat{b})$  and  $(\hat{a} + 2\hat{b} + 2(\hat{a} \times \hat{b}))$ ,

then the value of  $164 \cos^2 \theta$  is equal to :

(A)  $90 + 27\sqrt{2}$       (B)  $45 + 18\sqrt{2}$

(C)  $90 + 3\sqrt{2}$       (D)  $54 + 90\sqrt{2}$

**Official Ans. by NTA (A)**

**Ans. (A)**

**Sol.**  $\hat{a} \cdot \hat{b} = \frac{\pi}{4} = \phi$

$\hat{a} \cdot \hat{b} = |\hat{a}| |\hat{b}| \cos \phi$

$\hat{a} \cdot \hat{b} = \cos \phi = \frac{1}{\sqrt{2}}$

$$\cos \theta = \frac{(\hat{a} + \hat{b}) \cdot (\hat{a} + 2\hat{b} + 2(\hat{a} \times \hat{b}))}{|\hat{a} + \hat{b}| |\hat{a} + 2\hat{b} + 2(\hat{a} \times \hat{b})|}$$

$|\hat{a} + \hat{b}|^2 = (\hat{a} + \hat{b}) \cdot (\hat{a} + \hat{b})$

$|\hat{a} + \hat{b}|^2 = 2 + 2\hat{a} \cdot \hat{b}$

$= 2 + \sqrt{2}$

$\hat{a} \times \hat{b} = |\hat{a}| |\hat{b}| \sin \phi \hat{n}$

$\hat{a} \times \hat{b} = \frac{\hat{n}}{\sqrt{2}}$       when  $\hat{n}$  is vector  $\perp$   $\hat{a}$  and  $\hat{b}$

let  $\vec{c} = \hat{a} \times \hat{b}$

We know.

$\vec{c} \cdot \vec{a} = 0$

$\vec{c} \cdot \vec{b} = 0$

$|\hat{a} + 2\hat{b} + 2\vec{c}|^2$

$= 1 + 4 + \frac{(4)}{2} + 4\hat{a} \cdot \hat{b} + 8\hat{b} \cdot \vec{c} + 4\vec{c} \cdot \hat{a}$

$= 7 + \frac{4}{\sqrt{2}} = 7 + 2\sqrt{2}$

Now

$(\hat{a} + \hat{b}) \cdot (\hat{a} + 2\hat{b} + 2\vec{c})$

$= |\hat{a}|^2 + 2\hat{a} \cdot \hat{b} + 0 + \hat{b} \cdot \hat{a} + 2|\hat{b}|^2 + 0$

$= 1 + \frac{2}{\sqrt{2}} + \frac{1}{\sqrt{2}} + 2$

$= 3 + \frac{3}{\sqrt{2}}$

$\cos \theta = \frac{3 + \frac{3}{\sqrt{2}}}{\sqrt{2 + \sqrt{2}} \sqrt{7 + 2\sqrt{2}}}$

$\cos^2 \theta = \frac{9(\sqrt{2} + 1)^2}{2(2 + \sqrt{2})(7 + 2\sqrt{2})}$

$$\cos^2 \theta = \left( \frac{9}{2\sqrt{2}} \right) \frac{(\sqrt{2}+1)}{(7+2\sqrt{2})}$$

$$164 \cos^2 \theta = \frac{(82)(9)}{\sqrt{2}} \frac{(\sqrt{2}+1)}{(7+2\sqrt{2})} \frac{(7-2\sqrt{2})}{(7-2\sqrt{2})}$$

$$= \frac{(82)(9)[7\sqrt{2}-4+7-2\sqrt{2}]}{\sqrt{2}(41)}$$

$$= (9\sqrt{2})[5\sqrt{2}+3]$$

$$= 90 + 27\sqrt{2}$$

15. If  $f(\alpha) = \int_1^\alpha \frac{\log_{10} t}{1+t} dt, \alpha > 0$ , then  $f(e^3) + f(e^{-3})$

is equal to :

(A) 9 (B)  $\frac{9}{2}$

(C)  $\frac{9}{\log_e(10)}$  (D)  $\frac{9}{2 \log_e(10)}$

**Official Ans. by NTA (D)**

**Ans. (D)**

**Sol.**  $f(e^3) = \int_1^{e^3} \frac{\ln t}{\ln 10(1+t)} dt \dots (1)$

$$f(\alpha) = \int_1^\alpha \frac{\ln t}{(\ln 10)(1+t)} dt$$

$$t = \frac{1}{x} \Rightarrow x = \frac{1}{t}$$

$$dt = \frac{-1}{x^2} dx$$

$$= \int_1^\alpha \frac{-\ln x}{(\ln 10)\left(1+\frac{1}{x}\right)} \left(-\frac{1}{x^2}\right) dx$$

$$f(\alpha) = \frac{1}{\ln 10} \int_1^\alpha \frac{\ln x}{x(x+1)} dx$$

$$f(e^{-3}) = \frac{1}{\ln 10} \int_1^{e^3} \frac{\ln t}{t(t+1)} dt \dots (2)$$

Add (1) & (2)

$$f(e^3) + f(e^{-3}) = \left( \frac{1}{\ln 10} \right) \int_1^{e^3} \frac{\ln t}{(1+t)} \left[ 1 + \frac{1}{t} \right] dt$$

$$= \left( \frac{1}{\ln 10} \right) \int_1^{e^3} \frac{\ln t}{t} dt$$

$$\ln t = r$$

$$\frac{dt}{t} = dr$$

$$= \frac{1}{\ln 10} \int_0^3 r dr$$

$$= \left( \frac{1}{\ln 10} \right) \left( \frac{r^2}{2} \right) \Big|_0^3$$

$$= \left( \frac{1}{\log 10} \right) \left( \frac{9}{2} \right)$$

$$= \frac{9}{2 \log_e 10}$$

16. The area of the region

$\left\{ (x, y) : |x-1| \leq y \leq \sqrt{5-x^2} \right\}$  is equal to :

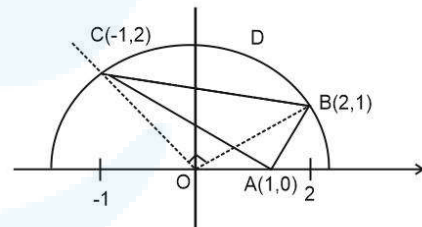
(A)  $\frac{5}{2} \sin^{-1} \left( \frac{3}{5} \right) - \frac{1}{2}$  (B)  $\frac{5\pi}{4} - \frac{3}{2}$

(C)  $\frac{3\pi}{4} + \frac{3}{2}$  (D)  $\frac{5\pi}{4} - \frac{1}{2}$

**Official Ans. by NTA (D)**

**Ans. (D)**

**Sol.**



$$|x-1| < y < \sqrt{5-x^2}$$

When  $|x-1| = \sqrt{5-x^2}$

$$\Rightarrow (x-1)^2 = 5-x^2$$

$$\Rightarrow x^2 - x - 2 = 0$$

$$\Rightarrow x = 2, -1$$

Required Area = Area of  $\Delta ABC$  + Area of region

BCD

$$= \frac{1}{2} \begin{vmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ -1 & 2 & 1 \end{vmatrix} + \frac{\pi}{4} (\sqrt{5})^2 - \frac{1}{2} (\sqrt{5})^2$$

$$= \frac{5\pi}{4} - \frac{1}{2}$$

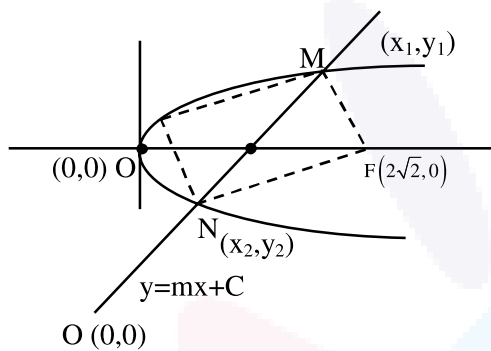


17. Let the focal chord of the parabola  $P : y^2 = 4x$  along the line  $L : y = mx + c, m > 0$  meet the parabola at the points  $M$  and  $N$ . Let the line  $L$  be a tangent to the hyperbola  $H : x^2 - y^2 = 4$ . If  $O$  is the vertex of  $P$  and  $F$  is the focus of  $H$  on the positive  $x$ -axis, then the area of the quadrilateral  $OMFN$  is :

- (A)  $2\sqrt{6}$                       (B)  $2\sqrt{14}$   
 (C)  $4\sqrt{6}$                       (D)  $4\sqrt{14}$

**Official Ans. by NTA (B)**

**Ans. (B)**



**Sol.**

$$H : \frac{x^2}{4} - \frac{y^2}{4} = 1$$

Focus  $(ae, 0)$

$$F(2\sqrt{2}, 0)$$

Line  $L : y = mx + c$  pass  $(1,0)$

$$0 = m + c \dots\dots(1)$$

Line  $L$  is tangent to Hyperbola.  $\frac{x^2}{4} - \frac{y^2}{4} = 1$

$$C = \pm\sqrt{a^2m^2 - \ell^2}$$

$$C = \pm\sqrt{4m^2 - 4}$$

From (1)

$$-m = \pm\sqrt{4m^2 - 4}$$

Squaring

$$m^2 = 4m^2 - 4$$

$$4 = 3m^2$$

$$\frac{2}{\sqrt{3}} = m \text{ (as } m > 0)$$

$$C = -m$$

$$C = \frac{-2}{\sqrt{3}}$$

$$y = \frac{2x}{\sqrt{3}} - \frac{2}{\sqrt{3}}$$

$$y^2 = 4x$$

$$\Rightarrow \left(\frac{2x-2}{\sqrt{3}}\right)^2 = 4x$$

$$\Rightarrow x^2 + 1 - 2x = 3x$$

$$\Rightarrow x^2 - 5x + 1 = 0$$

$$y^2 = 4\left(\frac{\sqrt{3}y + 2}{2}\right)$$

$$y^2 = 2\sqrt{3}y + 4$$

$$\Rightarrow y^2 - 2\sqrt{3}y - 4 = 0$$

Area

$$\frac{1}{2} \begin{vmatrix} 0 & x_1 & 2\sqrt{2} & x_2 & 0 \\ 0 & y_1 & 0 & y_2 & 0 \end{vmatrix}$$

$$= \frac{1}{2} [-2\sqrt{2}y_1 + 2\sqrt{2}y_2]$$

$$= \sqrt{2} |y_2 - y_1| = \frac{(\sqrt{2})\sqrt{12+16}}{111}$$

$$= \sqrt{56}$$

$$= 2\sqrt{14}$$

18. The number of points, where the function  $f : \mathbf{R} \rightarrow \mathbf{R}, f(x) = |x - 1| \cos |x - 2| \sin |x - 1| + (x - 3) |x^2 - 5x + 4|$ , is **NOT** differentiable, is :

- (A) 1                      (B) 2                      (C) 3                      (D) 4

**Official Ans. by NTA (B)**

**Ans. (B)**

**Sol.**  $f(x) = |x - 1| \cos |x - 2| \sin |x - 1| + (x - 3) |x^2 - 5x + 4|$   
 $= |x - 1| \cos |x - 2| \sin |x - 1| + (x - 3) |x - 1| |x - 4|$   
 $= |x - 1| [\cos |x - 2| \sin |x - 1| + (x - 3) |x - 4|]$   
 Non differentiable at  $x = 1$  and  $x = 4$ .

19. Let  $S = \{1, 2, 3, \dots, 2022\}$ . Then the probability, that a randomly chosen number  $n$  from the set  $S$  such that  $\text{HCF}(n, 2022) = 1$ , is :

- (A)  $\frac{128}{1011}$                       (B)  $\frac{166}{1011}$   
 (C)  $\frac{127}{337}$                       (D)  $\frac{112}{337}$

**Official Ans. by NTA (D)**

**Ans. (D)**

**Sol.** Total number of elements = 2022

$$2022 = 2 \times 3 \times 337$$

$$\text{HCF}(n, 2022) = 1$$

is feasible when the value of 'n' and 2022 has no common factor.

A = Number which are divisible by 2 from {1,2,3.....2022}

$$n(A) = 1011$$

B = Number which are divisible by 3 from {1,2,3.....2022}

$$n(B) = 674$$

$A \cap B$  = Number which are divisible by 6

from {1,2,3.....2022}

$$6, 12, 18, \dots, 2022$$

$$\boxed{337 = n(A \cap B)}$$

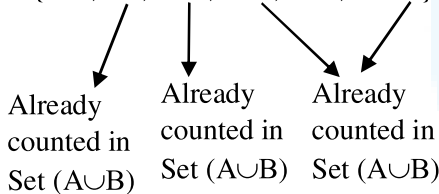
$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$= 1011 + 674 - 337$$

$$= 1348$$

C = Number which divisible by 337 from {1,.....1022}

$$C = \{337, 674, 1011, 1348, 1685, 2022\}$$



Total elements which are divisible by 2 or 3 or 337

$$= 1348 + 2 = 1350$$

Favourable cases = Element which are neither divisible by 2, 3 or 337

$$= 2022 - 1350$$

$$= 672$$

$$\text{Required probability} = \frac{672}{2022} = \frac{112}{337}$$

**20.** Let  $f(x) = 3^{(x^2-2)^3+4}$ ,  $x \in \mathbf{R}$ . Then which of the following statements are true ?

P :  $x = 0$  is a point of local minima of f

Q :  $x = \sqrt{2}$  is a point of inflection of f

R : f' is increasing for  $x > \sqrt{2}$

- (A) Only P and Q                      (B) Only P and R  
(C) Only Q and R                      (D) All, P, Q and R

**Official Ans. by NTA (D)**

**Ans. (D)**

**Sol.**  $f(x) = 81 \cdot 3^{(x^2-2)^3}$

$$f'(x) = 81 \cdot 3^{(x^2-2)^3} \cdot \ln 3 \cdot 3(x^2-2)^2 \cdot 2x$$

$$= (81 \times 6) 3^{(x^2-2)^3} x (x^2-2)^2 \ln 3$$

$$\begin{array}{cccc} + & & - & & + & & + \\ | & & | & & | & & | \\ -\sqrt{2} & & 0 & & \sqrt{2} & & \end{array}$$

$x = 6$  is point of local min

$$f'(x) = \underbrace{(486 \cdot \ln 3)}_k 3^{(x^2-2)^3} \underbrace{x(x^2-2)^2}_{g(x)}$$

$$g'(x) = 3^{(x^2-2)^3} (x^2-2)^2 + x \cdot 3^{(x^2-2)^3} \cdot 4x \cdot (x^2-2)$$

$$+ x \cdot (x^2-2)^2 \cdot 3^{(x^2-2)^3} \ln 3 \cdot 3(x^2-2)^2 \cdot 2x$$

$$= 3^{(x^2-2)^3} (x^2-2) [x^2-2 + 4x^2 + 6x^2 \ln 3 (x^2-2)^3]$$

$$g'(x) = 3^{(x^2-2)^3} (x^2-2) [5x^2 - 2 + 6x^2 \ln 3 (x^2-2)^3]$$

$$f''(x) = k \cdot g'(x)$$

$$f''(\sqrt{2}) = 0, f''(\sqrt{2}^+) > 0, f''(\sqrt{2}^-) < 0$$

$x = \sqrt{2}$  is point of inflection

$f''(x) > 0$  for  $x > \sqrt{2}$  so f(x) is increasing

**SECTION-B**

**1.** Let  $S = \{\theta \in (0, 2\pi) : 7 \cos^2\theta - 3 \sin^2\theta - 2 \cos^2 2\theta = 2\}$ . Then, the sum of roots of all the equations  $x^2 - 2(\tan^2\theta + \cot^2\theta)x + 6 \sin^2\theta = 0$   $\theta \in S$ , is \_\_\_\_\_.

**Official Ans. by NTA (16)**

**Ans. (16)**

**Sol.**  $7 \cos^2\theta - 3 \sin^2\theta - 2 \cos^2 2\theta = 2$   
 $4 \cos^2\theta + 3 \cos 2\theta - 2 \cos^2 2\theta = 2$   
 $2(1 + \cos 2\theta) + 3 \cos 2\theta - 2 \cos^2 2\theta = 2$   
 $2 \cos^2 2\theta - 5 \cos 2\theta = 0$   
 $\cos 2\theta (2 \cos 2\theta - 5) = 0$   
 $\cos 2\theta = 0$





$$2\theta = (2n + 1) \frac{\pi}{2}$$

$$\theta = (2n + 1) \frac{\pi}{4}$$

$$S = \left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \right\}$$

For all four values of  $\theta$

$$x^2 - 2(\tan^2\theta + \cot^2\theta)x + 6\sin^2\theta = 0$$

$$\Rightarrow x^2 - 4x + 3 = 0$$

Sum of roots of all four equations =  $4 \times 4 = 16$ .

2. Let the mean and the variance of 20 observations  $x_1, x_2, \dots, x_{20}$  be 15 and 9, respectively. For  $\alpha \in \mathbb{R}$ , if the mean of  $(x_1 + \alpha)^2, (x_2 + \alpha)^2, \dots, (x_{20} + \alpha)^2$  is 178, then the square of the maximum value of  $\alpha$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (4)**

**Ans. (4)**

**Sol.**  $\sum x_1 = 15 \times 20 = 300 \quad \dots(i)$

$$\frac{\sum x_1^2}{20} - (15)^2 = 9 \quad \dots(ii)$$

$$\sum x_1^2 = 234 \times 20 = 4680$$

$$\frac{\sum (x_1 + \alpha)^2}{20} = 178 \Rightarrow \sum (x_1 + \alpha)^2 = 3560$$

$$\Rightarrow \sum x_1^2 + 2\alpha \sum x_1 + \sum \alpha^2 = 3560$$

$$4680 + 600\alpha + 20\alpha^2 = 3560$$

$$\Rightarrow \alpha^2 + 30\alpha + 56 = 0$$

$$\Rightarrow (\alpha + 28)(\alpha + 2) = 0$$

$$\alpha = -2, -28$$

Square of maximum value of  $\alpha$  is 4

3. Let a line with direction ratios  $a, -4a, -7$  be perpendicular to the lines with direction ratios  $3, -1, 2b$  and  $b, a, -2$ . If the point of intersection of the line  $\frac{x+1}{a^2+b^2} = \frac{y-2}{a^2-b^2} = \frac{z}{1}$  and the plane  $x - y + z = 0$  is  $(\alpha, \beta, \gamma)$ , then  $\alpha + \beta + \gamma$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (10)**

**Ans. (10)**

**Sol.**  $(a, -4a, -7) \perp (3, -1, 2b)$   
 $a = 2b \quad \dots(i)$

$(a, -4a, -7) \perp (b, a, -2)$   
 $3a + 4a - 14b = 0$   
 $ab - 4a^2 + 14 = 0 \quad \dots(ii)$

From Equations (i) and (ii)

$$2b^2 - 16b^2 + 14 = 0$$

$$b^2 = 1$$

$$a^2 = 4b^2 = 4$$

$$\frac{x+1}{5} = \frac{y-2}{3} = \frac{z}{1} = k$$

$$\alpha = 5k - 1, \beta = 3k + 2, \gamma = k$$

As  $(\alpha, \beta, \gamma)$  satisfies  $x - y + z = 0$

$$5k - 1 - (3k + 2) + k = 0$$

$$k = 1$$

$$\therefore \alpha + \beta + \gamma = 9k + 1 = 10$$

4. Let  $a_1, a_2, a_3, \dots$  be an A.P. If  $\sum_{r=1}^{\infty} \frac{a_r}{2^r} = 4$ , then  $4a_2$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (16)**

**Ans. (16)**

**Sol.**  $S = \frac{a_1}{2} + \frac{a_2}{2^2} + \frac{a_3}{2^3} + \dots$

$$\frac{S}{2} = \frac{a_1}{2^2} + \frac{a_2}{2^3} + \dots$$

---


$$\frac{S}{2} = \frac{a_1}{2} + d \left( \frac{1}{2^2} + \frac{1}{2^3} + \dots \right)$$

$$\frac{S}{2} = \frac{a_1}{2} + d \left( \frac{1}{4} \right)$$

$$\therefore S = a_1 + d = a_2 = 4$$

Or  $4a_2 = 16$

5. Let the ratio of the fifth term from the beginning to the fifth term from the end in the binomial expansion of  $\left( \sqrt[4]{2} + \frac{1}{\sqrt[4]{3}} \right)^n$ , in the increasing powers of  $\frac{1}{\sqrt[4]{3}}$  be  $\sqrt[4]{6} : 1$ . If the sixth term from the beginning is  $\frac{\alpha}{\sqrt[4]{3}}$ , then  $\alpha$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (84)**

**Ans. (84)**



Sol.  $\frac{T_5}{T_{n-3}} = \frac{{}^n C_4 (2^{1/4})^{n-4} (3^{-1/4})^4}{{}^n C_{n-4} (2^{1/4})^4 (3^{-1/4})^{n-4}} = \frac{\sqrt[4]{6}}{1}$

$\Rightarrow 2^{\frac{n-8}{4}} 3^{\frac{n-8}{4}} = 6^{1/4}$

$\Rightarrow 6^{n-8} = 6$

$\Rightarrow n-8=1 \Rightarrow n=9$

$T_6 = {}^9 C_5 (2^{1/4})^4 (3^{-1/4})^5 = \frac{84}{\sqrt[4]{3}}$

$\therefore \alpha = 84$

6. The number of matrices of order  $3 \times 3$ , whose entries are either 0 or 1 and the sum of all the entries is a prime number, is \_\_\_\_\_.

Official Ans. by NTA (282)

Ans. (282)

Sol.  $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} a_{ij} \in \{0,1\}$

$\sum a_{ij} = 2,3,5,7$

Total matrix =  ${}^9 C_2 + {}^9 C_3 + {}^9 C_5 + {}^9 C_7$

= 282

7. Let  $p$  and  $p + 2$  be prime numbers and let

$\Delta = \begin{vmatrix} p! & (p+1)! & (p+2)! \\ (p+1)! & (p+2)! & (p+3)! \\ (p+2)! & (p+3)! & (p+4)! \end{vmatrix}$

Then the sum of the maximum values of  $\alpha$  and  $\beta$ , such that  $p^\alpha$  and  $(p + 2)^\beta$  divide  $\Delta$ , is \_\_\_\_\_.

Official Ans. by NTA (4)

Ans. (4)

Sol.  $\Delta = \begin{vmatrix} P! & (P+1)! & (P+2)! \\ (P+1)! & (P+2)! & (P+3)! \\ (P+2)! & (P+3)! & (P+4)! \end{vmatrix}$

$\Delta = P!(P+1)!(P+2)! \begin{vmatrix} 1 & 1 & 1 \\ P+1 & P+2 & P+3 \\ (P+2)(P+1) & (P+3)(P+2) & (P+4)(P+3) \end{vmatrix}$

$\Delta = 2P!(P+1)!(P+2)!$

Which is divisible by  $P^\alpha$  &  $(P + 2)^\beta$

$\therefore \alpha = 3, \beta = 1$

Ans. 4

8. If  $\frac{1}{2 \times 3 \times 4} + \frac{1}{3 \times 4 \times 5} + \frac{1}{4 \times 5 \times 6} + \dots +$

$\frac{1}{100 \times 101 \times 102} = \frac{k}{101}$ , then  $34k$  is equal to

\_\_\_\_\_.

Official Ans. by NTA (286)

Ans. (286)

Sol.  $\frac{1}{2.3.4} + \frac{1}{3.4.5} + \dots + \frac{1}{100.101.102} = \frac{k}{101}$

$\frac{4-2}{2.3.4} + \frac{5-3}{3.4.5} + \dots + \frac{102-100}{100.101.102} = \frac{2k}{101}$

$\frac{1}{2.3} - \frac{1}{3.4} + \frac{1}{3.4} - \frac{1}{4.5} + \dots + \frac{1}{100.101} - \frac{1}{101.102} = \frac{2k}{101}$

$\frac{1}{2.3} - \frac{1}{101.102} = \frac{2k}{101}$

$\therefore 2k = \frac{101}{6} - \frac{1}{102}$

$\therefore 34k = 286$

9. Let  $S = \{4, 6, 9\}$  and  $T = \{9, 10, 11, \dots, 1000\}$ . If

$A = \{a_1 + a_2 + \dots + a_k : k \in \mathbb{N}, a_1, a_2, a_3, \dots, a_k \in S\}$ ,

then the sum of all the elements in the set  $T - A$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (11)**

**Ans. (11)**

**Sol.**  $S = \{4, 6, 9\}$   $T = \{9, 10, 11, \dots, 1000\}$

$A = \{a_1 + a_2 + \dots + a_k : k \in \mathbb{N}\} \ \& \ a_i \in S$

Here by the definition of set 'A'

$A = \{a : a = 4x + 6y + 9z\}$

Except the element 11, every element of set T is of the form  $4x + 6y + 9z$  for some  $x, y, z \in \mathbb{W}$

$\therefore T - A = \{11\}$

**Ans. 11**

**10.** Let the mirror image of a circle  $c_1 : x^2 + y^2 - 2x - 6y + \alpha = 0$  in line  $y = x + 1$  be  $c_2 : 5x^2 + 5y^2 + 10gx + 10fy + 38 = 0$ . If  $r$  is the radius of circle  $c_2$ , then  $\alpha + 6r^2$  is equal to \_\_\_\_\_

**Official Ans. by NTA (12)**

**Ans. (12)**

**Sol.** Image of centre  $c_1 \equiv (1, 3)$  in  $x - y + 1 = 0$  is given

by

$$\frac{x_1 - 1}{1} = \frac{y_1 - 3}{-1} = \frac{-2(1 - 3 + 1)}{1^2 + 1^2}$$

$$\Rightarrow x_1 = 2, y_1 = 2$$

$\therefore$  Centre of circle  $c_2 \equiv (2, 2)$

$$\therefore \text{Equation of } c_2 \text{ be } x^2 + y^2 - 4x - 4y + \frac{38}{5} = 0$$

$$\text{Now radius of } c_2 \text{ is } \sqrt{4 + 4 - \frac{38}{5}} = \sqrt{\frac{2}{5}} = r$$

$$(\text{radius of } c_1)^2 = (\text{radius of } c_2)^2$$

$$\Rightarrow 10 - \alpha = \frac{2}{5} \Rightarrow \alpha = \frac{48}{5}$$

$$\therefore \alpha + 6r^2 = \frac{48}{5} + \frac{12}{5} = 12$$