



FINAL JEE–MAIN EXAMINATION – JUNE, 2022

Held On Monday 27th June, 2022

TIME: 9:00 AM to 12:00 PM

SECTION-A

1. The area of the polygon, whose vertices are the non-real roots of the equation $\bar{z} = iz^2$ is :

- (A) $\frac{3\sqrt{3}}{4}$ (B) $\frac{3\sqrt{3}}{2}$
 (C) $\frac{3}{2}$ (D) $\frac{3}{4}$

Official Ans. by NTA (A)

Ans. (A)

Sol. \Rightarrow Let $z = x + iy, x, y \in \mathbb{R}$

Now $\bar{z} = iz^2$

then $x - iy = i(x^2 - y^2 + 2xyi)$

$x - iy = i(x^2 - y^2) - 2xy$

$\Rightarrow x = -2xy$ & $-y = x^2 - y^2$

$\Rightarrow x(1 + 2y) = 0$

$x = 0$ or $y = -\frac{1}{2}$

Put $x = 0$ in $-y = x^2 - y^2$

We get $y = y^2$

$\Rightarrow y = 0, 1$

Similarly

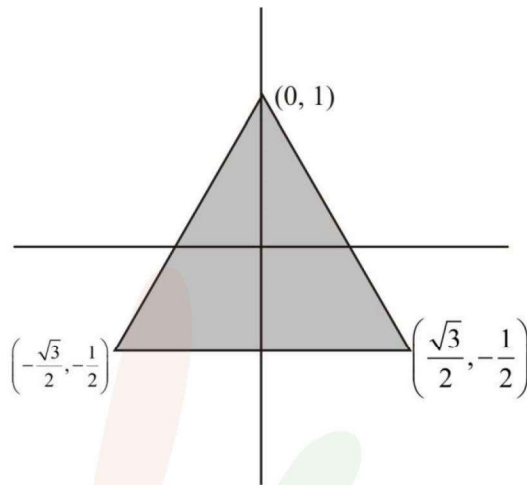
Put $y = -\frac{1}{2}$ in $-y = x^2 - y^2$

$\Rightarrow \frac{1}{2} = x^2 - \frac{1}{4}$

$\Rightarrow x^2 = \frac{3}{4}$

$x = \pm \frac{\sqrt{3}}{2}$

$z = \left(0, i, \frac{\sqrt{3}}{2} - \frac{1}{2}i, -\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$



$Area = \frac{1}{2} \cdot (\sqrt{3}) \left(\frac{3}{2}\right)$
 $= \frac{3\sqrt{3}}{4}$

2. Let the system of linear equations $x + 2y + z = 2$, $\alpha x + 3y - z = \alpha$, $-\alpha x + y + 2z = -\alpha$ be inconsistent.

Then α is equal to :

- (A) $\frac{5}{2}$ (B) $-\frac{5}{2}$
 (C) $\frac{7}{2}$ (D) $-\frac{7}{2}$

Official Ans. by NTA (D)

Ans. (D)

Sol. $\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ -2 & 1 & 2 \end{vmatrix}$

$= (6 + y) - 2((2\alpha - \alpha) + 1(\alpha + 3\alpha))$

$= 7 - 2\alpha + 4\alpha$

$= 7 + 2\alpha$

$\Delta = 0 \Rightarrow \alpha = -\frac{7}{2}$



$$\Delta_1 = \begin{vmatrix} 2 & 2 & 1 \\ \alpha & 3 & -1 \\ -\alpha & 1 & 2 \end{vmatrix}$$

$$= 14 + 2\alpha$$

$$\alpha = -x_2 = 7$$

$$\Delta_1 \neq 0$$

3. If $x = \sum_{n=0}^{\infty} a^n, y = \sum_{n=0}^{\infty} b^n, z = \sum_{n=0}^{\infty} c^n$, where a, b, c

are in A.P. and $|a| < 1, |b| < 1, |c| < 1, abc \neq 0$, then

(A) x, y, z are in A.P.

(B) x, y, z are in G.P.

(C) $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P.

(D) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1 - (a + b + c)$

Official Ans. by NTA (C)

Ans. (C)

Sol. $x = 1 + a + a^2 = \dots$

$$x = \frac{1}{1-a} \Rightarrow a = 1 - \frac{1}{x}$$

$$y = \frac{1}{1-b} \Rightarrow b = 1 - \frac{1}{y}$$

$$z = \frac{1}{1-c} \Rightarrow c = 1 - \frac{1}{z}$$

a, b, c are in A.P.

$$\Rightarrow 1 - \frac{1}{x}, 1 - \frac{1}{y}, 1 - \frac{1}{z} \text{ are in A.P.}$$

$$\Rightarrow -\frac{1}{x}, -\frac{1}{y}, -\frac{1}{z} \text{ are in A.P.}$$

$$\Rightarrow \frac{1}{x}, \frac{1}{y}, \frac{1}{z} \text{ are in A.P.}$$

4. Let $\frac{dy}{dx} = \frac{ax - by + a}{bx + cy + a}$, where a, b, c are constants,

represent a circle passing through the point (2, 5).

Then the shortest distance of the point (11, 6) from this circle is :

(A) 10 (B) 8

(C) 7 (D) 5

Official Ans. by NTA (B)

Ans. (B)

Sol. Let equation of circle is

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(2x + 2g)}{(2y + 2f)}$$

Comparing with $\frac{dy}{dx} = \frac{ax - by + a}{bx + cy + a}$

$$\Rightarrow b = 0, a = -2, c = 2$$

$$\Rightarrow -2g = -2 \Rightarrow g = 1 \quad 2f = -2$$

$$f = -1$$

Now circle will be

$$x^2 + y^2 + 2x - 2y + c = 0$$

its passes through (2, 5)

which will give $c = -23$

so circle will be $x^2 + y^2 + 2x - 2y - 23 = 0$

centre $C = (-1, 1)$

and radius 5

Now P is (11, 6)

So minimum distance of P from circle will be

$$= \sqrt{(11+1)^2 + (6-1)^2} - 5$$

$$= 13 - 5$$

$$= 8$$

5. Let a be an integer such that $\lim_{x \rightarrow 7} \frac{18 - [1-x]}{[x-3a]}$

exists, where [t] is greatest integer $\leq t$. Then a is equal to :

(A) -6 (B) -2

(C) 2 (D) 6

Official Ans. by NTA (A)

Ans. (A)



Sol. $\lim_{x \rightarrow 7} \frac{18 - [1-x]}{[x] - 3a}$

L.H.L. $\lim_{x \rightarrow 7^-} \frac{18 - [1-x]}{[x] - 3a}$

$$= \frac{18 - (-6)}{6 - 3a}$$

$$= \frac{24}{6 - 3a}$$

R.H.L. $\lim_{x \rightarrow 7^+} \frac{18 - [1-x]}{[x] - 3a}$

$$= \frac{18 - (-7)}{7 - 3a}$$

$$= \frac{25}{7 - 3a}$$

Now L.H.L. = R.H.L.

$$\frac{24}{6 - 3a} = \frac{25}{7 - 3a}$$

$$\Rightarrow 168 - 72a = 150 - 75a$$

$$\Rightarrow 18 = -3a$$

$$\Rightarrow a = -6$$

6. The number of distinct real roots of $x^4 - 4x + 1 = 0$ is :

- (A) 4 (B) 2
(C) 1 (D) 0

Official Ans. by NTA (B)

Ans. (B)

Sol. Let $f(x) = x^4 - 4x + 1$

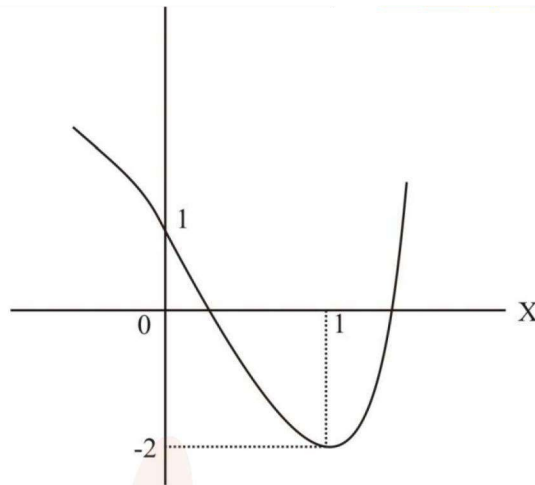
$$f(x) = 4x^3 - 4$$

$$f'(x) = 0 \Rightarrow x = 1$$

$x = 1$ is point of minima.

$$f(1) = -2$$

$$f(0) = 1$$



Hence 2 solutions.

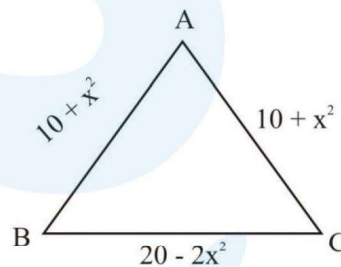
7. The lengths of the sides of a triangle are $10 + x^2$, $10 + x^2$ and $20 - 2x^2$. If for $x = k$, the area of the triangle is maximum, then $3k^2$ is equal to :

- (A) 5 (B) 8
(C) 10 (D) 12

Official Ans. by NTA (C)

Ans. (C)

Sol.



$$a = 20 - 2x^2, b = 10 + x^2, c = 10 + x^2$$

$$= \frac{a+b+c}{2}$$

$$= 20$$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{20(2x^2)(10-x^2)(10-x^2)}$$

$$= 2\sqrt{10} \sqrt{x^2(10-x^2)^2}$$

$$= 2\sqrt{10} |x(10-x^2)|$$



$$= 2\sqrt{10} |10x - x^3|$$

$$S = 10x - x^3$$

$$\frac{ds}{dx} = 10 - 3x^2$$

$$\frac{ds}{dx} = 0 \Rightarrow x^2 = \frac{10}{3}$$

$$3x^2 = 10$$

8. If $\cos^{-1}\left(\frac{y}{2}\right) = \log_e\left(\frac{x}{5}\right)^5$, $|y| < 2$, then :

(A) $x^2y'' + xy' - 25y = 0$

(B) $x^2y'' - xy' - 25y = 0$

(C) $x^2y'' - xy' + 25y = 0$

(D) $x^2y'' + xy' + 25y = 0$

Official Ans. by NTA (D)

Ans. (D)

Sol. $\cos^{-1}\left(\frac{y}{2}\right) = \log_e\left(\frac{x}{5}\right)^5$

$$\cos^{-1}\left(\frac{y}{2}\right) = 5 \log_e\left(\frac{x}{5}\right)$$

$$\frac{-1}{\sqrt{1-\frac{y^2}{4}}} \cdot \frac{y'}{2} = 5 \cdot \frac{1}{x} \times \frac{1}{5}$$

$$\Rightarrow \frac{-y'}{\sqrt{4-y^2}} = \frac{5}{x}$$

$$-xy' = 5\sqrt{4-y^2}$$

$$-xy'' - y' = 5 \cdot \frac{1}{2\sqrt{4-y^2}} (-2y y')$$

$$\Rightarrow xy'' + y' = \frac{5y' \cdot y}{\sqrt{4-y^2}}$$

$$xy'' + y' = 5 \cdot \left(\frac{-5}{x}\right) y$$

$$x^2y'' + xy' = -25y$$

9. $\int \frac{(x^2+1)e^x}{(x+1)^2} dx = f(x)e^x + C$, Where C is a

constant, then $\frac{d^3 f}{dx^3}$ at $x = 1$ is equal to :

(A) $-\frac{3}{4}$ (B) $\frac{3}{4}$

(C) $-\frac{3}{2}$ (D) $\frac{3}{2}$

Official Ans. by NTA (B)

Ans. (B)

Sol. $\int \left(\frac{x^2+1}{(x+1)^2}\right) e^x \cdot dx$

$$= \int \left(\frac{x^2-1+2}{(x+1)^2}\right) e^x dx$$

$$= \int \left(\frac{x-1}{x+1} + \frac{2}{(x+1)^2}\right) e^x dx$$

$$= \int (f(x) + f'(x)) e^x dx$$

$$= f(x) e^x + c$$

Where $f(x) = \frac{x-1}{x+1}$

$$f'(x) = \frac{2}{(x+1)^2}$$

$$f''(x) = \frac{-4}{(x+1)^3}$$

$$= \frac{12}{(x+1)^4}$$

$$f''(1) = \frac{12}{16}$$

$$= \frac{3}{4}$$



10. The value of the integral $\int_{-2}^2 \frac{|x^3 + x|}{(e^{|x|} + 1)} dx$ is equal

to :

- (A) $5e^2$ (B) $3e^2$
 (C) 4 (D) 6

Official Ans. by NTA (D)

Ans. (D)

Sol. $f(x) = \frac{|x^3 + x|}{(e^{|x|} + 1)}$

$$\int_{-2}^2 f(x) dx = \int_0^2 (f(x) + f(-x)) dx$$

$$= \int_0^2 \left(\frac{|x^3 + x|}{(e^{|x|} + 1)} + \frac{|-x^3 - x|}{(e^{-x|-x|} + 1)} \right) dx$$

$$= \int_0^2 \left(\frac{|x^3 + x|}{(e^{|x|} + 1)} + \frac{|x^3 + x|}{(e^{-x|-x|} + 1)} \right) dx$$

$$= \int_0^2 \left(\frac{x^3 + x}{(e^{x^2} + 1)} + \frac{x^3 + x}{(e^{-x^2} + 1)} \right) dx$$

$$I = \int_0^2 \left(\frac{x^3 + x}{1 + e^{x^2}} + \frac{e^{x^2}(x^3 + x)}{1 + e^{x^2}} \right) dx$$

$$= \int_0^2 (x^3 + x) dx$$

$$= \left[\frac{x^4}{4} + \frac{x^2}{2} \right]_0^2$$

$$= 4 + 2 = 6$$

11. If $\frac{dy}{dx} + \frac{2^{x-y}(2^y - 1)}{2^x - 1} = 0, x, y > 0, y(1) = 1$, then

y(2) is equal to :

- (A) $2 + \log_2 3$ (B) $2 + \log_2 2$
 (C) $2 - \log_2 3$ (D) $2 - \log_2 3$

Official Ans. by NTA (D)

Ans. (D)

Sol. $\frac{dy}{dx} + \frac{2^{x-y}(2^y - 1)}{2^x - 1} = 0,$

$x, y > 0, y(1) = 1, y(2) = ?$

$$\frac{dy}{dx} = -\frac{2^x(2^y - 1)}{2^y(2^x - 1)}$$

$$\int \frac{2^y}{2^y - 1} dy = -\int \frac{2^x}{2^x - 1} dx$$

$$\frac{1}{\ln 2} \int \frac{2^y \ln 2}{2^y - 1} dy = -\frac{1}{\ln 2} \int \frac{2^x \ln 2}{2^x - 1} dx$$

$$\frac{1}{\ln 2} \ln|2^y - 1| = \frac{-1}{\ln 2} \ln|2^x - 1| + C$$

At $x = 1, y = 1$

Putting this values in above relation we get $C = 0$

$$\ln|2^y - 1| + \ln|2^x - 1| = 0$$

$$(2^x - 1)(2^y - 1) = 1$$

$$2^y - 1 = \frac{1}{2^x + 1}$$

At $x = 2$

$$2^y = \frac{1}{3} + 1 = \frac{4}{3}$$

$$y = \log_2 \frac{4}{3} = \log_2 4 - \log_2 3 = 2 - \log_2 3$$

12. In an isosceles triangle ABC, the vertex A is (6, 1) and the equation of the base BC is $2x + y = 4$. Let the point B lie on the line $x + 3y = 7$. If (α, β) is the centroid ΔABC , then $15(\alpha + \beta)$ is equal to :

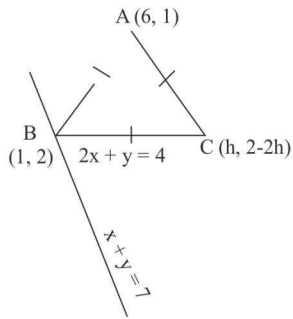
- (A) 39 (B) 41
 (C) 51 (D) 63

Official Ans. by NTA (C)

Ans. (C)



Sol.



Point B (1, 2)

Now let C be (h, 4 - 2h)

(As C lies on $2x + y = 4$)

$\therefore \Delta$ is isosceles with base BC

$\therefore AB = AC$

$$\sqrt{25+1} = \sqrt{(6-h)^2 + (2h-3)^2}$$

$$\sqrt{26} = \sqrt{36+h^2-12h+4h^2+9-12h}$$

$$26 = 5h^2 - 24h + 45 \Rightarrow 5h^2 - 24h + 19 = 0$$

$$\Rightarrow 5h^2 - 5h - 19h + 19 = 0$$

$$h = \frac{19}{5} \text{ or } h = 1$$

Thus $C\left(\frac{19}{5}, \frac{-18}{5}\right)$

$$\text{Centroid} \left(\frac{6+1+\frac{19}{5}}{3}, \frac{1+2-\frac{18}{5}}{3} \right)$$

$$\left(\frac{35+19}{15}, \frac{15-18}{15} \right)$$

$$\left(\frac{54}{15}, \frac{-3}{15} \right)$$

$$\alpha = \frac{54}{15}; \beta = \frac{-3}{15}$$

$$15(\alpha + \beta) = 51$$

13. Let the eccentricity of an ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b, \text{ be } \frac{1}{4}. \text{ If this ellipse passes}$$

through the point $\left(-4\sqrt{\frac{2}{5}}, 3\right)$, then $a^2 + b^2$ is equal

to :

(A) 29 (B) 31

(C) 32 (D) 34

Official Ans. by NTA (B)

Ans. (B)

Sol. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad a > b$

$$e^2 = 1 - \frac{b^2}{a^2}$$

$$\frac{1}{16} = 1 - \frac{b^2}{a^2}$$

$$\frac{b^2}{a^2} = 1 - \frac{1}{16} = \frac{15}{16} \Rightarrow b^2 = \frac{15}{16} a^2$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$16 \times \frac{2}{5} + \frac{9}{b^2} = 1$$

$$\frac{32}{5a^2} + \frac{9}{b^2} = 1$$

$$\frac{32}{5a^2} + \frac{9}{\frac{15}{16}a^2} = 1$$

$$\frac{80}{5a^2} = 1$$

$$16 = a^2$$

$$b^2 = 15$$



14. If two straight lines whose direction cosines are given by the relations $l + m - n = 0$, $3l^2 + m^2 + cnl = 0$ are parallel, then the positive value of c is :
- (A) 6 (B) 4
(C) 3 (D) 2

Official Ans. by NTA (A)

Ans. (A)

Sol. $l + m - n = 0$

$$3l^2 + m^2 + cl(1 + m) = 0$$

$$n = l + m$$

$$3l^2 + m^2 + cl^2 + clm = 0$$

$$(3 + c)l^2 + clm + m^2 = 0$$

$$(3 + c)\left(\frac{l}{m}\right)^2 + c\left(\frac{l}{m}\right) + 1 = 0 \dots\dots(1)$$

\therefore lines are parallel.

Roots of (1) must be equal

$$\Rightarrow D = 0$$

$$c^2 - 4(3 + c) = 0$$

$$c^2 - 4c - 12 = 0$$

$$(c - 6)(c + 2) = 0$$

$$c = 6 \text{ or } c = -2$$

+ve value of $c = 6$

15. Let $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} - 3\hat{j} + 2\hat{k}$. Then the number of vectors \vec{b} such that $\vec{b} \times \vec{c} = \vec{a}$ and $|\vec{b}| \in \{1, 2, \dots, 10\}$ is :
- (A) 0 (B) 1
(C) 2 (D) 3

Official Ans. by NTA (A)

Ans. (A)

Sol. $\vec{a} = i + j - k$

$$\vec{c} = 2i - 3j + 2k$$

$$\vec{b} \times \vec{c} = \vec{a}$$

$$|\vec{b}| \in \{1, 2, \dots, 10\}$$

$$\therefore \vec{b} \times \vec{c} = \vec{a}$$

$\Rightarrow \vec{a}$ is perpendicular to \vec{b} as well as \vec{a} is perpendicular to \vec{c}

$$\text{Now } \vec{a} \cdot \vec{c} = 2 - 3 - 2 = -3 \neq 0$$

This $\vec{b} \times \vec{c} = \vec{a}$ is not possible.

No. of vectors $\vec{b} = 0$

16. Five numbers x_1, x_2, x_3, x_4, x_5 are randomly selected from the numbers 1, 2, 3, ..., 18 and are arranged in the increasing order ($x_1 < x_2 < x_3 < x_4 < x_5$). The probability that $x_2 = 7$ and $x_4 = 11$ is :

(A) $\frac{1}{136}$ (B) $\frac{1}{72}$

(C) $\frac{1}{68}$ (D) $\frac{1}{34}$

Official Ans. by NTA (C)

Ans. (C)

Sol. No. of ways to select and arrange x_1, x_2, x_3, x_4, x_5 from 1, 2, 3, ..., 18

$$n(s) = {}^{18}C_5$$

$$x_1 \quad (x_2) \quad x_3 \quad (x_4) \quad x_5$$

$$7 \qquad \qquad \qquad 11$$

$$n(E) = {}^6C_1 \times {}^3C_1 \times {}^7C_1$$

$$P(E) = \frac{6 \times 3 \times 7}{{}^{18}C_5}$$

$$\frac{1}{17 \times 4} = \frac{1}{68}$$

17. Let X be a random variable having binomial distribution $B(7, p)$. If $P(X = 3) = 5P(X = 4)$, then the sum of the mean and the variance of X is :

(A) $\frac{105}{16}$ (B) $\frac{7}{16}$

(C) $\frac{77}{36}$ (D) $\frac{49}{16}$

Official Ans. by NTA (C)

Ans. (C)

Sol. $B(7, p)$

$$n = 7 \quad p = p$$

given

$$P(x = 3) = 5P(x = 4)$$



$${}^7C_3 \times p^3 (1-p)^4 = 5 \cdot {}^7C_4 p^4 (1-p)^3$$

$$\frac{{}^7C_3}{5 \times {}^7C_4} = \frac{p}{1-p}$$

$$1-p = 5p$$

$$6p = 1$$

$$p = \frac{1}{6} \Rightarrow q = \frac{5}{6}$$

$$n = 7$$

$$\text{Mean} = np = 7 \times \frac{1}{6} = \frac{7}{6}$$

$$\text{Var} = npq = 7 \times \frac{1}{6} \times \frac{5}{6} = \frac{35}{36}$$

Sum

$$= \frac{7}{6} + \frac{35}{36}$$

$$= \frac{42+35}{36}$$

$$= \frac{77}{36}$$

18. The value of $\cos\left(\frac{2\pi}{7}\right) + \cos\left(\frac{4\pi}{7}\right) + \cos\left(\frac{6\pi}{7}\right)$

is equal to :

(A) -1 (B) $-\frac{1}{2}$

(C) $-\frac{1}{3}$ (D) $-\frac{1}{4}$

Official Ans. by NTA (B)

Ans. (B)

Sol. $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}$

$$= \frac{\sin\left(3 \times \frac{\pi}{7}\right)}{\sin\frac{\pi}{7}} \times \cos\left(\frac{2\pi}{7} + \frac{6\pi}{7}\right)$$

$$= \frac{2 \sin\left(\frac{3\pi}{7}\right)}{2 \sin\frac{\pi}{7}} \times \cos\left(\frac{4\pi}{7}\right)$$

$$= \frac{\sin\left(\frac{7\pi}{7}\right) + \sin\left(\frac{-\pi}{7}\right)}{2 \sin\frac{\pi}{7}}$$

$$= \frac{-\sin\frac{\pi}{7}}{2 \sin\frac{\pi}{7}}$$

$$= -\frac{1}{2}$$

19. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\left(\cos\frac{7\pi}{6}\right) + \tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ is

equal to :

(A) $\frac{11\pi}{12}$

(B) $\frac{17\pi}{12}$

(C) $\frac{31\pi}{12}$

(D) $-\frac{3\pi}{4}$

Official Ans. by NTA (A)

Ans. (A)

Sol. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\left(\cos\frac{7\pi}{6}\right) + \tan^{-1}\tan\left(\frac{3\pi}{4}\right)$

$$\sin^{-1}\sin\left(\frac{2\pi}{3}\right) = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$$

$$\cos^{-1}\left(\cos\frac{2\pi}{6}\right) = 2\pi - \frac{7\pi}{6} = \frac{5\pi}{6}$$

$$\tan^{-1}\tan\left(\frac{3\pi}{4}\right) = \frac{3\pi}{4} - \pi = \frac{-\pi}{4}$$

$$\sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\cos\frac{7\pi}{6} + \tan^{-1}\tan\frac{3\pi}{4}$$

$$= \frac{11\pi}{12}$$



20. The Boolean expression $(\sim(p \wedge q)) \vee q$ is equivalent to :

- (A) $q \rightarrow (p \wedge q)$ (B) $p \rightarrow q$
 (C) $p \rightarrow (p \rightarrow q)$ (D) $p \rightarrow (p \vee q)$

Official Ans. by NTA (D)

Ans. (D)

Sol. $(\sim(p \wedge q)) \vee q$

$$\begin{aligned} &= (\sim p \vee \sim q) \vee q \\ &= \sim p \vee \sim q \vee q \\ &= \sim p \vee t \\ &= \text{this statement is a tautology option D} \end{aligned}$$

$p \Rightarrow (p \vee q)$ is also a tautology.

OR

p	q	$P \wedge q$	$\sim(p \wedge q)$	$\sim(p \wedge q) \vee q$	$p \vee q$	$p \rightarrow (p \vee q)$
T	T	T	F	T	T	T
T	F	F	T	T	T	T
F	T	F	T	T	T	T
F	F	F	T	T	F	T

SECTION-B

1. Let $f : R \rightarrow R$ be a function defined $f(x) = \frac{2e^{2x}}{e^{2x} + e}$.

Then $f\left(\frac{1}{100}\right) + f\left(\frac{2}{100}\right) + f\left(\frac{3}{100}\right) + \dots + f\left(\frac{99}{100}\right)$ is equal to _____.

Official Ans. by NTA (99)

Ans. (99)

Sol.

$$f(x) + f(1-x) = \frac{2e^{2x}}{e^{2x} + e} + \frac{2e^{2-2x}}{e^{2-2x} + e} = \left[\frac{e^{2x}}{e^{2x} + e} + \frac{e^2}{e^2 + e^{2x+1}} \right]$$

$$= 2 \left[\frac{e^{2x-1}}{e^{2x-1} + 1} + \frac{1}{1 + e^{2x-1}} \right] = 2$$

$$f\left(\frac{1}{100}\right) + f\left(\frac{2}{100}\right) + f\left(\frac{3}{100}\right) + \dots + f\left(\frac{99}{100}\right)$$

$$= \left\{ f\left(\frac{1}{100}\right) + f\left(\frac{99}{100}\right) \right\} + \left\{ f\left(\frac{2}{100}\right) + f\left(\frac{98}{100}\right) \right\} + \dots + \left\{ f\left(\frac{49}{100}\right) + f\left(\frac{51}{100}\right) \right\} + f\left(\frac{1}{2}\right)$$

$$= (2 + 2 + 2 + \dots + 49 \text{ times}) + \frac{2e}{e + e}$$

$$= 98 + 1 = 99$$

2. If the sum of all the roots of the equation

$$e^{2x} - 11e^x - 45e^{-x} + \frac{81}{2} = 0 \text{ is } \log_e P, \text{ then } p \text{ is}$$

equal to _____.

Official Ans. by NTA (45)

Ans. (45)

Sol. $e^{2x} - 11e^x - 45e^{-x} + \frac{81}{2} = 0$]

$$\left(e^x \right)^3 - 11 \left(e^x \right)^2 - 45 + \frac{81e^x}{2} = 0$$

$$e^x = t$$

$$2t^3 - 22t^2 + 81t - 90 = 0$$

$$t_1 t_2 t_3 = 45$$

$$e^{x_1} \cdot e^{x_2} \cdot e^{x_3} = 45$$

$$e^{x_1 + x_2 + x_3} = 45$$

$$\log_e e^{x_1 + x_2 + x_3} = \log_e 45$$

$$x_1 + x_2 + x_3 = \log_e 45$$

$$\log_e P = \log_e 45$$

$$P = 45$$



3. The positive value of the determinant of the matrix

$$A, \text{ whose } Adj(Adj(A)) = \begin{pmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{pmatrix},$$

is _____.

Official Ans. by NTA (14)

Ans. (14)

Sol. $Adj(AdjA) = \begin{bmatrix} 14 & 18 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{bmatrix}$

$$|Adj(AdjA)| = \begin{vmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{vmatrix} = 14 \times 14 \times 14 \begin{vmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix}$$

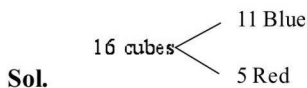
$$= (14)^3 [3 - 2(-5) - 1(-1)] = (14)^3 [14] = (14)^4$$

$$|A|^4 = (14)^4 \Rightarrow |A| = 14$$

4. The number of ways, 16 identical cubes, of which 11 are blue and rest are red, can be placed in a row so that between any two red cubes there should be at least 2 blue cubes, is _____.

Official Ans. by NTA (56)

Ans. (56)



$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 11$$

$$x_1, x_6 \geq 0, \quad x_2, x_3, x_4, x_5 \geq 2$$

$$x_2 = t_1 + 2$$

$$x_3 = t_3 + 2$$

$$x_4 = t_4 + 2$$

$$x_5 = t_5 + 2$$

$$x_1, t_2, t_3, t_4, t_5, x_6 \geq 0$$

$$\text{No. of solutions} = {}^{6+3-1}C_3 = {}^8C_3 = 56$$

5. If the coefficient of x^{10} in the binomial expansion

$$\text{of } \left(\frac{\sqrt{x}}{5^4} + \frac{\sqrt{5}}{x^3} \right)^{60} \text{ is } 5^k l, \text{ where } l, k \in \mathbb{N} \text{ and } l \text{ is co-}$$

prime to 5, then k is equal to _____.

Official Ans. by NTA (5)

Ans. (5)

Sol. $\left(\frac{\sqrt{x}}{5^{1/4}} + \frac{\sqrt{5}}{x^{1/3}} \right)^{60}$

$$T_{r+1} = {}^{60}C_r \left(\frac{x^{1/2}}{5^{1/4}} \right)^{60-r} \left(\frac{5^{1/2}}{x^{1/3}} \right)^r$$

$$= {}^{60}C_r 5^{\frac{3r-60}{4}} \cdot x^{\frac{180-5r}{6}}$$

$$\frac{180-5r}{6} = 10 \Rightarrow r = 24$$

$$\text{Coeff. of } x^{10} = {}^{60}C_{24} 5^3 = \frac{60}{24 \cdot 36} 5^3$$

$$\text{Powers of 5 in } = {}^{60}C_{24} \cdot 5^3 = \frac{5^{14}}{5^4 \times 5^8} \times 5^3 = 5^5$$

6. Let

$$A_1 = \{(x, y) : |x| \leq y^2, |x| + 2y \leq 8\} \text{ and}$$

$$A_2 = \{(x, y) : |x| + |y| \leq k\}. \text{ If } 27 \text{ (Area } A_1) = 5$$

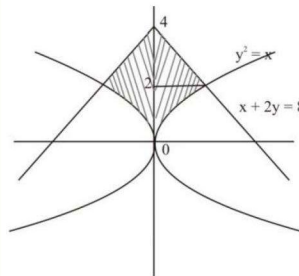
(Area A_2), then k is equal to :

Official Ans. by NTA (6)

Ans. (6)

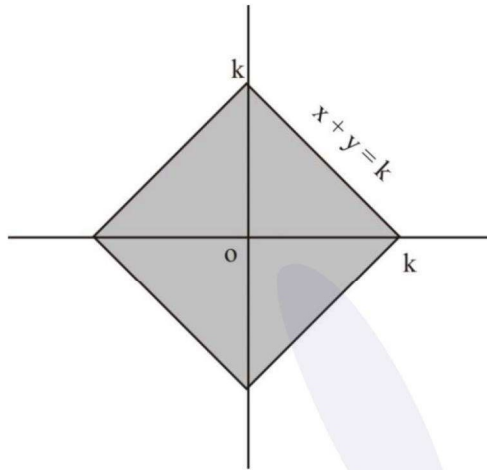
Sol. $A_1 = \{(x, y) : |x| \leq y^2, |x| + 2y \leq 8\} \text{ and}$

$$A_2 = \{(x, y) : |x| + |y| \leq k\}.$$





$$\begin{aligned} \text{area}(A_1) &= 2 \left[\int_0^2 y^2 dy + \int_2^4 (8-2y) dy \right] \\ &= 2 \left[\left(\frac{y^3}{3} \right)_0^2 + (8y - y^2)_2^4 \right] \end{aligned}$$



$$\text{area}(A_1) = 2 \times \frac{20}{3} = \frac{40}{3}$$

$$\text{Area}(A_2) = 4 \times \frac{1}{2} k^2$$

$$\text{Area}(A_2) = 2k^2$$

Now

$$27(\text{Area } A_1) = 5(\text{Area } A_2)$$

$$9 \times 4 = k^2$$

$$k = 6$$

7. If the sum of the first ten terms of the series

$$\frac{1}{5} + \frac{2}{65} + \frac{3}{325} + \frac{4}{1025} + \frac{5}{2501} + \dots \text{ is } \frac{m}{n}, \text{ where}$$

m and n are co-prime numbers, then m + n is equal to _____.

Official Ans. by NTA (276)

Ans. (276)

Sol.
$$\frac{1}{5} + \frac{2}{65} + \frac{3}{325} + \frac{4}{1025} + \frac{5}{2501} + \dots$$

$$T_n = \frac{n}{4n^4 + 1}$$

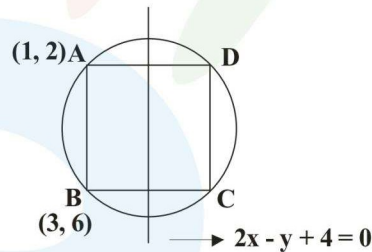
$$\begin{aligned} &= \frac{n}{(2n^2 + 1)^2 - (2n)^2} = \frac{n}{(2n^2 + 2n + 1)(2n^2 - 2n + 1)} \\ &= \frac{1}{4} \left[\frac{1}{2n^2 - 2n + 1} - \frac{1}{2n^2 + 2n + 1} \right] \\ S_{10} &= \sum_{n=1}^{10} T_n = \frac{1}{4} \left[\frac{1}{1} - \frac{1}{5} + \frac{1}{5} - \frac{1}{13} + \dots + \frac{1}{200 + 20 + 1} \right] \\ &= \frac{1}{4} \left[1 - \frac{1}{221} \right] = \frac{1}{4} \times \frac{220}{221} = \frac{55}{221} = \frac{m}{n} \end{aligned}$$

$$m + n = 55 + 221 = 276$$

8. A rectangle R with end points of the one of its sides as (1, 2) and (3, 6) is inscribed in a circle. If the equation of a diameter of the circle is $2x - y + 4 = 0$, then the area of R is _____.

Official Ans. by NTA (16)

Ans. (16)



Sol.

Eq. of line AB

$$y = 2x$$

$$\text{Slope of AB} = 2$$

$$\text{Slope of given diameter} = 2$$

So the diameter is parallel to AB

Distance between diameter and line AB

$$= \left(\frac{4}{\sqrt{2^2 + 12}} \right) = \frac{4}{\sqrt{5}}$$

$$\text{Thus BC} = 2 \times \frac{4}{\sqrt{5}} = \frac{8}{\sqrt{5}}$$

$$AB = \sqrt{(1-3)^2 + (2-6)^2} = \sqrt{20} = 2\sqrt{5}$$

$$\text{Area} = AB \times BC = \frac{8}{\sqrt{5}} \times 2\sqrt{5} = 16 \text{ Ans.}$$



9. A circle of radius 2 unit passes through the vertex and the focus of the parabola $y^2 = 2x$ and touches the parabola $y = \left(x - \frac{1}{4}\right)^2 + \alpha$, where $\alpha > 0$.
Then $(4\alpha - 8)^2$ is equal to _____.

Official Ans. by NTA (63)

Ans. (63)

Sol. Vertex and focus of parabola $y^2 = 2x$

are V (0, 0) and S $\left(\frac{1}{2}, 0\right)$ resp.

Let equation of circle be

$$(x - h)^2 + (y - k)^2 = 4$$

\therefore Circle passes through (0, 0)

$$\Rightarrow h^2 + k^2 = 4 \dots\dots(1)$$

\therefore Circle passes through $\left(\frac{1}{2}, 0\right)$

$$\left(\frac{1}{2} - h\right)^2 + k^2 = 4$$

$$\Rightarrow h^2 + k^2 - h = \frac{15}{4} \dots\dots(2)$$

On solving (1) and (2)

$$4 - h = \frac{15}{4}$$

$$h = 4 - \frac{15}{4} = \frac{1}{4}$$

$$k = +\frac{\sqrt{63}}{4}$$

$k = -\frac{\sqrt{63}}{4}$ is rejected as circle with centre

$\left(\frac{1}{4}, -\frac{\sqrt{63}}{4}\right)$ can't touch given parabola.

Equation of circle is

$$\left(x - \frac{1}{4}\right)^2 + \left(k - \frac{\sqrt{63}}{4}\right)^2 = 4$$

From figure

$$\alpha = 2 + \frac{\sqrt{63}}{4} = \frac{8 + \sqrt{63}}{4}$$

$$4\alpha - 8 = \sqrt{63}$$

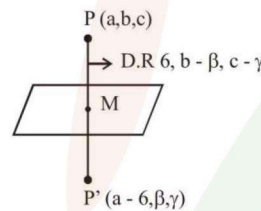
$$(4\alpha - 8)^2 = 63$$

10. Let the mirror image of the point (a, b, c) with respect to the plane $3x - 4y + 12z + 19 = 0$ be (a - 6, β , γ). If a + b + c = 5, then $7\beta - 9\gamma$ is equal to _____.

Official Ans. by NTA (137)

Ans. (137)

Sol.



$$M = \left(a - 3, \frac{\beta + b}{2}, \frac{\gamma + c}{2}\right)$$

Since M lies on $3x + 4y + 12z + 19 = 0$

$$\Rightarrow 6a - 4b + 12c - 4\beta + 12\gamma + 20 = 0 \dots\dots(1)$$

Since PP' is parallel to normal of the plane then

$$\frac{6}{3} = \frac{b - \beta}{-4} = \frac{c - \gamma}{12}$$

$$\Rightarrow \beta = b + 8, \quad \gamma = c - 24$$

$$a + b + c = 5 \Rightarrow a + \beta - 8 + \gamma + 24 = 5$$

$$\Rightarrow a = -\beta - \gamma - 11$$

Now putting these values in (1) we get

$$6(-\beta - \gamma - 11) - 4(\beta - 8) + 12(\gamma + 24) - 4\beta + 12\gamma + 20 = 0$$

$$\Rightarrow 7\beta - 9\gamma = 170 - 33 = 137$$