

FINAL JEE-MAIN EXAMINATION - JANUARY, 2020

Held On Tuesday, 7 January 2020

TIME : 2 : 30 PM to 05 : 30 PM

1. Let  $A = [a_{ij}]$ ,  $B = [b_{ij}]$  are two  $3 \times 3$  matrices such that  $b_{ij} = \lambda^{i+j-2} a_{ij}$  &  $|B| = 81$ . Find  $|A|$  if  $\lambda = 3$ .

माना  $A = [a_{ij}]$ ,  $B = [b_{ij}]$ ,  $3 \times 3$  क्रम की दो आव्यूह इस प्रकार है कि  $b_{ij} = \lambda^{i+j-2} a_{ij}$  तथा  $|B| = 81$ . यदि  $\lambda = 3$  है तब  $|A|$  का मान है—

- (1)  $\frac{1}{9}$                                       (2) 3                                      (3)  $\frac{1}{81}$                                       (4)  $\frac{1}{27}$

Ans. (1)

Sol.  $|B| = \begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix} = \begin{vmatrix} 3^0 a_{11} & 3^1 a_{12} & 3^2 a_{13} \\ 3^1 a_{21} & 3^2 a_{22} & 3^3 a_{23} \\ 3^2 a_{31} & 3^3 a_{32} & 3^4 a_{33} \end{vmatrix} \Rightarrow 81 = 3^3 \cdot 3 \cdot 3^2 |A| \Rightarrow 3^4 = 3^6 |A| \Rightarrow |A| = \frac{1}{9}$

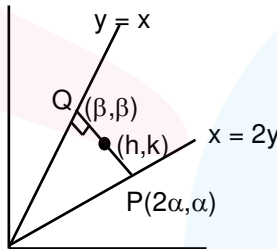
2. From any point P on the line  $x = 2y$  perpendicular is drawn on  $y = x$ . Let foot of perpendicular is Q. Find the locus of mid point of PQ.

रेखा  $x = 2y$  पर स्थित किसी बिन्दु P से रेखा  $y = x$  पर लम्ब खींचा जाता है। माना Q लम्बपाद है तब PQ के मध्यबिन्दु का बिन्दुपथ ज्ञात करो—

- (1)  $2x = 3y$                                       (2)  $5x = 7y$                                       (3)  $3x = 2y$                                       (4)  $7x = 5y$

Ans. (2)

Sol.



slope of PQ की प्रवणता =  $\frac{k - \alpha}{h - 2\alpha} = -1$

$\Rightarrow k - \alpha = -h + 2\alpha$

$\Rightarrow \alpha = \frac{h+k}{3}$  .....(1)

Also तथा  $2h = 2\alpha + \beta$

$2k = \alpha + \beta$

$\Rightarrow 2h = \alpha + 2k$

$\Rightarrow \alpha = 2h - 2k$  .....(2)

from (1) और & (2) से

$\frac{h+k}{3} = 2(h-k)$

so locus is अतः बिन्दुपथ  $6x - 6y = x + y \Rightarrow 5x = 7y$



3. Pair of tangents are drawn from origin to the circle  $x^2 + y^2 - 8x - 4y + 16 = 0$  then square of length of chord of contact is

मूलबिन्दु से वृत्त  $x^2 + y^2 - 8x - 4y + 16 = 0$  पर स्पर्श रेखाएँ खींची जाती है तब स्पर्शजीवा की लम्बाई का वर्ग है-

- (1)  $\frac{64}{5}$                       (2)  $\frac{24}{5}$                       (3)  $\frac{8}{5}$                       (4)  $\frac{8}{13}$

Ans. (1)

Sol.  $L = \sqrt{S_1} = \sqrt{16} = 4$

$R = \sqrt{16 + 4 - 16} = 2$

Length of Chord of contact स्पर्शजीवा की लम्बाई  $= \frac{2LR}{\sqrt{L^2 + R^2}} = \frac{2 \times 4 \times 2}{\sqrt{16 + 4}} = \frac{16}{\sqrt{20}}$

square of length of chord of contact स्पर्शजीवा की लम्बाई का वर्ग  $= \frac{64}{5}$

4. Contrapositive of if  $A \subset B$  and  $B \subset C$  then  $C \subset D$

- (1)  $C \not\subset D$  or  $A \not\subset B$  or  $B \not\subset C$                       (2)  $C \subset D$  and  $A \not\subset B$  or  $B \not\subset C$   
 (3)  $C \subset D$  or  $A \not\subset B$  and  $B \not\subset C$                       (4)  $C \subset D$  or  $A \not\subset B$  or  $B \not\subset C$

यदि  $A \subset B$  और  $B \subset C$  तब  $C \subset D$  का प्रतिपरिवर्तित है-

- (1)  $C \not\subset D$  या  $A \not\subset B$  या  $B \not\subset C$                       (2)  $C \subset D$  और  $A \not\subset B$  या  $B \not\subset C$   
 (3)  $C \subset D$  या  $A \not\subset B$  और  $B \not\subset C$                       (4)  $C \subset D$  या  $A \not\subset B$  या  $B \not\subset C$

Ans. (4)

Sol. Let  $P = A \subset B, Q = B \subset C, R = C \subset A$   
 Contrapositive of  $(P \wedge Q) \rightarrow R$  is  $\sim R \rightarrow \sim (P \wedge Q)$   
 $R \vee (\sim P \vee \sim Q)$

Sol. माना  $P = A \subset B, Q = B \subset C, R = C \subset A$   
 $(P \wedge Q) \rightarrow R$  का प्रतिपरिवर्तित  $\sim R \rightarrow \sim (P \wedge Q)$   
 $R \vee (\sim P \vee \sim Q)$

5. Let  $y(x)$  is solution of differential equation  $(y^2 - x) \frac{dy}{dx} = 1$  and  $y(0) = 1$ , then find the value of  $x$  where curve cuts the  $x$ -axis

माना  $y(x)$ , अवकल समीकरण  $(y^2 - x) \frac{dy}{dx} = 1$  का हल है तथा  $y(0) = 1$ , तब  $x$  का मान होगा, जहां वक्र  $x$  अक्ष को प्रतिच्छेद करता है-

- (1)  $2 - e$                       (2)  $2 + e$                       (3)  $2$                       (4)  $e$

Ans. (1)

Sol. Sol.  $\frac{dx}{dy} + x = y^2$

I.F. =  $e^{\int 1 \cdot dy} = e^y$

$x \cdot e^y = \int y^2 \cdot e^y \cdot dy$

$= y^2 \cdot e^y - \int 2y \cdot e^y \cdot dy$

$$\Rightarrow y^2 e^y - 2(y \cdot e^y - e^y) + c$$

$$x \cdot e^y = y^2 e^y - 2y e^y + 2e^y + C$$

$$x = y^2 - 2y + 2 + c \cdot e^{-y}$$

$$x = 0, \quad y = 1$$

$$0 = 1 - 2 + 2 + \frac{c}{e}$$

$$c = -e$$

$$y = 0, \quad x = 0 - 0 + 2 + (-e)(e^{-0})$$

$$x = 2 - e$$

6. Let  $\theta_1$  and  $\theta_2$  (where  $\theta_1 < \theta_2$ ) are two solutions of  $2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0, \theta \in [0, 2\pi)$  then  $\int_{\theta_1}^{\theta_2} \cos^2 3\theta d\theta$  is equal to

माना  $\theta_1$  और  $\theta_2$  (जहाँ  $\theta_1 < \theta_2$ ) समीकरण  $2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0, \theta \in [0, 2\pi)$  के दो हल हैं तब  $\int_{\theta_1}^{\theta_2} \cos^2 3\theta d\theta$  का

मान है—

(1)  $\frac{\pi}{3}$

(2)  $\frac{2\pi}{3}$

(3)  $\frac{\pi}{9}$

(4)  $\frac{\pi}{3} + \frac{1}{6}$

Ans. (1)

Sol.  $2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0$

$$\frac{2\cos^2\theta}{\sin^2\theta} - \frac{5}{\sin\theta} + 4 = 0$$

$$2\cos^2\theta - 5\sin\theta + 4\sin^2\theta = 0, \sin\theta \neq 0$$

$$2\sin^2\theta - 5\sin\theta + 2 = 0$$

$$(2\sin\theta - 1)(\sin\theta - 2) = 0$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\therefore \int_{\pi/6}^{5\pi/6} \cos^2 3\theta d\theta = \int_{\pi/6}^{5\pi/6} \frac{1 + \cos 6\theta}{2} d\theta$$

$$= \frac{1}{2} \left[ \theta + \frac{\sin 6\theta}{6} \right]_{\pi/6}^{5\pi/6} = \frac{1}{2} \left[ \frac{5\pi}{6} - \frac{\pi}{6} + \frac{1}{6}(0 - 0) \right] = \frac{1}{2} \cdot \frac{4\pi}{6} = \frac{\pi}{3}$$

7. Let  $3 + 4 + 8 + 9 + 13 + 14 + 18 + \dots \dots \dots 40$  terms = S. If  $S = (102)m$  then  $m =$

माना  $3 + 4 + 8 + 9 + 13 + 14 + 18 + \dots \dots \dots 40$  पद तक = S. यदि  $S = (102)m$  तब  $m =$

(1) 20

(2) 25

(3) 10

(4) 5

Ans. (1)



Sol.  $S = \underbrace{3+4} + \underbrace{8+9} + \underbrace{13+14} + \underbrace{18+19} + \dots + 40$  terms पद तक

$S = 7 + 17 + 27 + 37 + 47 + \dots + 20$  terms पद तक

$$S_{40} = \frac{20}{2} [2 \times 7 + (19)10] = 10[14 + 190] = 10[2040] = (102) (20)$$

$$\Rightarrow m = 20$$

8. If  $({}^{36}C_{r+1}) \times (k^2 - 3) = {}^{35}C_r - 6$ , then number of ordered pairs (r, k) are – (where  $k \in I$ ).

यदि  $({}^{36}C_{r+1}) \times (k^2 - 3) = {}^{35}C_r - 6$ , तो क्रमित युग्मों (r, k) की संख्या है – (जहाँ  $k \in I$ ).

- (1) 6                                      (2) 2                                      (3) 3                                      (4) 4

Ans. (4)

Sol.  $\frac{36}{r+1} \times \frac{35}{C_r} (k^2 - 3) = \frac{35}{C_r}$

$$k^2 - 3 = \frac{r+1}{6} \Rightarrow k^2 = 3 + \frac{r+1}{6}$$

r can be 5, 35

for  $r = 5, k = \pm 2$

$r = 35, k = \pm 3$

Hence number of order pair = 4

Hindi.  $\frac{36}{r+1} \times \frac{35}{C_r} (k^2 - 3) = \frac{35}{C_r}$

$$k^2 - 3 = \frac{r+1}{6} \Rightarrow k^2 = 3 + \frac{r+1}{6}$$

r 5, 35 हो सकता है।

$r = 5$ , के लिए  $k = \pm 2$

$r = 35$ , के लिए  $k = \pm 3$

अतः क्रमित युग्मों की संख्या = 4

9. Let  $4\alpha \int_{-1}^2 e^{-\alpha|x|} dx = 5$  then  $\alpha =$

माना कि  $4\alpha \int_{-1}^2 e^{-\alpha|x|} dx = 5$  तो  $\alpha =$

- (1)  $\ln 2$                                       (2)  $\ln \sqrt{2}$                                       (3)  $\ln \frac{3}{4}$                                       (4)  $\ln \frac{4}{3}$

Ans. (1)

Sol.  $4\alpha \left\{ \int_{-1}^0 e^{\alpha x} dx + \int_0^2 e^{-\alpha x} dx \right\} = 5$

$$\Rightarrow 4\alpha \left\{ \left( \frac{e^{\alpha x}}{\alpha} \right)_{-1}^0 + \left( \frac{e^{-\alpha x}}{-\alpha} \right)_0^2 \right\} = 5$$

$$\Rightarrow 4\alpha \left\{ \left( \frac{1 - e^{-\alpha}}{\alpha} \right) - \left( \frac{e^{-2\alpha} - 1}{\alpha} \right) \right\} = 5 \quad \Rightarrow 4(2 - e^{-\alpha} - e^{-2\alpha}) = 5 \quad \text{Put } e^{-\alpha} = t$$

$$\Rightarrow 4t^2 + 4t - 3 = 0 \quad \Rightarrow (2t + 3)(2t - 1) = 0$$

$$\Rightarrow e^{-\alpha} = \frac{1}{2} \quad \Rightarrow \alpha = \ln 2$$

10. Let  $f(x)$  is a five degree polynomial which has critical points  $x = \pm 1$  and  $\lim_{x \rightarrow 0} \left( 2 + \frac{f(x)}{x^3} \right) = 4$  then which one is incorrect.

- (1)  $f(x)$  has minima at  $x = 1$  & maxima at  $x = -1$
- (2)  $f(1) - 4f(-1) = 4$
- (3)  $f(x)$  is maxima at  $x = 1$  and minima at  $x = -1$
- (4)  $f(x)$  is odd

माना कि  $f(x)$  एक 5 घात का बहुपद है जिसके क्रांतिक बिन्दु  $x = \pm 1$  है तथा  $\lim_{x \rightarrow 0} \left( 2 + \frac{f(x)}{x^3} \right) = 4$  तो निम्न में से कौनसा गलत है।

- (1)  $f(x)$  का  $x = 1$  पर निम्ननिष्ठ तथा  $x = -1$  पर उच्चिष्ठ है।
- (2)  $f(1) - 4f(-1) = 4$
- (3)  $f(x)$  का  $x = 1$  उच्चिष्ठ तथा  $x = -1$  पर निम्ननिष्ठ है।
- (4)  $f(x)$  विषम घात है।

**Ans.**

**Sol.**

$$f(x) = ax^5 + bx^4 + cx^3$$

$$\lim_{x \rightarrow 0} \left( 2 + \frac{ax^5 + bx^4 + cx^3}{x^3} \right) = 4 \Rightarrow 2 + c = 4 \Rightarrow c = 2$$

$$f'(x) = 5ax^4 + 4bx^3 + 6x^2$$

$$= x^2 (5ax^2 + 4bx + 6)$$

$$f'(1) = 0 \quad \Rightarrow \quad 5a + 4b + 6 = 0$$

$$f'(-1) = 0 \quad \Rightarrow \quad 5a - 4b + 6 = 0$$

$$b = 0$$

$$a = -\frac{6}{5}$$

$$f(x) = -\frac{6}{5}x^5 + 2x^3$$

$$f'(x) = -6x^4 + 6x^2$$

$$= 6x^2 (-x^2 + 1)$$

$$= -6x^2 (x+1)(x-1)$$

$$\frac{-1}{1-} \quad + \quad \frac{1-}{1}$$

Minimal at  $x = -1$

Maxima at  $x = 1$

$x = -1$  पर निम्ननिष्ठ

$x = 1$  पर उच्चिष्ठ



11. If  $\vec{a}, \vec{b}, \vec{c}$  are unit vectors such that  $\vec{a} + \vec{b} + \vec{c} = \vec{0}$  and  $\lambda = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$  and  $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$  then  $(\lambda, \vec{d}) =$

यदि  $\vec{a}, \vec{b}, \vec{c}$  इकाई सदिश इस प्रकार है कि  $\vec{a} + \vec{b} + \vec{c} = \vec{0}$  तथा  $\lambda = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$  और  $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$  तब  $(\lambda, \vec{d}) =$

- (1)  $\left(\frac{3}{2}, 3\vec{a} \times \vec{b}\right)$       (2)  $\left(\frac{3}{2}, 3\vec{a} \times \vec{c}\right)$       (3)  $\left(-\frac{3}{2}, 3\vec{a} \times \vec{c}\right)$       (4)  $\left(-\frac{3}{2}, 3\vec{a} \times \vec{b}\right)$

Ans. (4)

Sol.  $|\vec{a} + \vec{b} + \vec{c}|^2 = 0$

$$3 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = \frac{-3}{2}$$

$$\Rightarrow \lambda = \frac{-3}{2}$$

$$\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times (-\vec{a} - \vec{b}) + (-\vec{a} - \vec{b}) \times \vec{a}$$

$$= \vec{a} \times \vec{b} + \vec{a} \times \vec{b} + \vec{a} \times \vec{b}$$

$$\vec{d} = 3(\vec{a} \times \vec{b})$$

12. Coefficient of  $x^7$  in  $(1+x)^{10} + x(1+x)^9 + x^2(1+x)^8 + \dots + x^{10}$  is-

$(1+x)^{10} + x(1+x)^9 + x^2(1+x)^8 + \dots + x^{10}$  में  $x^7$  का गुणांक है-

- (1) 330      (2) 210      (3) 420      (4) 260

Ans. (1)

Sol.

$$(1+x)^{10} \left[ 1 - \left(\frac{x}{1+x}\right)^{11} \right]$$

$$\frac{(1+x)^{10} \left[ (1+x)^{11} - x^{11} \right]}{(1+x)^{11} \times \frac{1}{(1+x)}}$$

$$= (1+x)^{11} - x^{11}$$

Coefficient of  $x^7$  is  ${}^{11}C_7 = {}^{11}C_4 = 330$

$x^7$  का गुणांक  ${}^{11}C_7 = {}^{11}C_4 = 330$

13. Let  $\alpha$  and  $\beta$  are the roots of  $x^2 - x - 1 = 0$  such that  $P_k = \alpha^k + \beta^k, k \geq 1$  then which one is incorrect?

माना कि  $\alpha$  तथा  $\beta$  समीकरण  $x^2 - x - 1 = 0$  के मूल इस प्रकार है कि  $P_k = \alpha^k + \beta^k, k \geq 1$  तो निम्न में से कौनसा गलत है-

- (1)  $P_5 = P_2 \times P_3$       (2)  $P_1 + P_2 + P_3 + P_4 + P_5 = 26$   
 (3)  $P_3 = P_5 - P_4$       (4)  $P_4 = 11$

Ans. (1)

**Sol.**  $\alpha^5 = 5\alpha + 3$   
 $\beta^5 = 5\beta + 3$

$$P_5 = 5(\alpha + \beta) + 6$$

$$= 5(1) + 6$$

$$P_5 = 11 \text{ and तथा } P_5 = \alpha^2 + \beta^2 = \alpha + 1 + \beta + 1$$

$$P_2 = 3 \text{ and तथा } P_3 = \alpha^3 + \beta^3 = 2\alpha + 1 + 2\beta + 1 = 2(1) + 2 = 4$$

$$P_2 \times P_3 = 12 \text{ and तथा } P_5 = 11 \Rightarrow P_5 \neq P_2 \times P_3$$

**14.** Let  $f(x) = x^3 - 4x^2 + 8x + 11$ , if LMVT is applicable on  $f(x)$  in  $[0, 1]$ , value of  $c$  is :  
 माना कि  $f(x) = x^3 - 4x^2 + 8x + 11$ ,  $f(x)$  के लिए LMVT अन्तराल  $[0, 1]$  के लिए लागू होती तो  $c$  है।

(1)  $\frac{4 - \sqrt{7}}{3}$                       (2)  $\frac{4 - \sqrt{5}}{3}$                       (3)  $\frac{4 + \sqrt{7}}{3}$                       (4)  $\frac{4 + \sqrt{5}}{3}$

**Ans.** (1)

**Sol.**  $f(x)$  is a polynomial function

$\therefore$  it is continuous and differentiable in  $[0, 1]$

Here  $f(0) = 11$ ,  $f(1) = 1 - 4 + 8 + 11 = 16$

$$f'(x) = 3x^2 - 8x + 8$$

$$\therefore f'(c) = \frac{f(1) - f(0)}{1 - 0} = \frac{16 - 11}{1} = 3c^2 - 8c + 8$$

$$\Rightarrow 3c^2 - 8c + 3 = 0$$

$$C = \frac{8 \pm 2\sqrt{7}}{6} = \frac{4 \pm \sqrt{7}}{3}$$

$$\therefore c = \frac{4 - \sqrt{7}}{3} \in (0, 1)$$

**Hindi.**  $f(x)$  एक बहुपदीय फलन है।

$\therefore$  यह अन्तराल  $[0, 1]$  में सतत तथा अन्तराल  $(0, 1)$  में अवकलनीय है।

यहां  $f(0) = 11$ ,  $f(1) = 1 - 4 + 8 + 11 = 16$

$$f'(x) = 3x^2 - 8x + 8$$

$$\therefore f'(c) = \frac{f(1) - f(0)}{1 - 0} = \frac{16 - 11}{1} = 3c^2 - 8c + 8$$

$$\Rightarrow 3c^2 - 8c + 3 = 0$$

$$C = \frac{8 \pm 2\sqrt{7}}{6} = \frac{4 \pm \sqrt{7}}{3}$$

$$\therefore c = \frac{4 - \sqrt{7}}{3} \in (0, 1)$$

15. The area bounded by  $4x^2 \leq y \leq 8x + 12$  is -

$4x^2 \leq y \leq 8x + 12$  द्वारा घिरा हुआ क्षेत्रफल है-

(1)  $\frac{127}{3}$

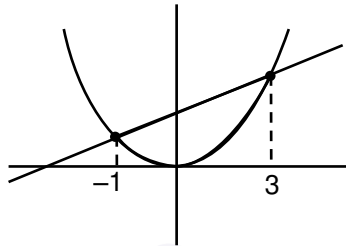
(2)  $\frac{128}{3}$

(3)  $\frac{124}{3}$

(4)  $\frac{125}{3}$

Ans. (2)

Sol.



$$\begin{aligned} 4x^2 &= y \\ y &= 8x + 12 \\ 4x^2 &= 8x + 12 \\ x^2 - x - 3 &= 0 \\ x^2 - 2x - 3 &= 0 \\ x^2 - 3x + x - 3 &= 0 \\ (x + 1)(x - 3) &= 0 \\ x &= -1 \end{aligned}$$

$$A = \int_{-1}^3 (8x + 12 - 4x^2) dx$$

$$\begin{aligned} A &= \left[ \frac{8x^2}{2} + 12x - \frac{4x^3}{3} \right]_{-1}^3 = (4(9) + 36 - 36) - \left( 4 - 12 + \frac{4}{3} \right) = 36 + 8 - \frac{4}{3} \\ &= 44 - \frac{4}{3} = \frac{132 - 4}{3} = \frac{128}{3} \end{aligned}$$

16. There are 5 machines. Probability of a machine being faulted is  $\frac{1}{4}$ . Probability of atmost two machines

is faulted, is  $\left(\frac{3}{4}\right)^3 k$  then value of k is

5 मशीन इस प्रकार है कि किसी एक मशीन के खराब होने की प्रायिकता  $\frac{1}{4}$  है, यदि अधिकतम दो मशीनों के खराब होने

की प्रायिकता  $\left(\frac{3}{4}\right)^3 k$  है, तो k का मान होगा।

(1)  $\frac{17}{2}$

(2) 4

(3)  $\frac{17}{8}$

(4)  $\frac{17}{4}$

Ans. (3)



**Sol.** Required probability = when no. machine has fault + when only one machine has fault + when only two machines have fault.

अभीष्ट प्रायिकता = जब कोई भी मशीन खराब नहीं है + जब केवल एक मशीन खराब है + जब केवल दो मशीन खराब है

$$\begin{aligned}
 &= {}^5C_0 \left(\frac{3}{4}\right)^5 + {}^5C_1 \left(\frac{1}{4}\right) \left(\frac{3}{4}\right)^4 + {}^5C_2 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^3 \\
 &= \frac{243}{1024} + \frac{405}{1024} + \frac{270}{1024} = \frac{918}{1024} = \frac{459}{512} = \frac{27 \times 17}{64 \times 8} \\
 &= \left(\frac{3}{4}\right)^3 \times k = \left(\frac{3}{4}\right)^3 \times \frac{17}{8} \\
 \therefore k &= \frac{17}{8}
 \end{aligned}$$

**17.**  $3x + 4y = 12\sqrt{2}$  is the tangent to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$  then the distance between foci of ellipse is-

यदि  $3x + 4y = 12\sqrt{2}$ , दीर्घवृत्त  $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$  की स्पर्श रेखा है तो दीर्घवृत्त की नाभियों के मध्य दूरी होगी-

- (1)  $2\sqrt{5}$                       (2)  $2\sqrt{3}$                       (3)  $2\sqrt{7}$                       (4) 4

**Ans.** (3)

**Sol.**  $3x + 4y = 12\sqrt{2}$

$$\Rightarrow 4y = -3x + 12\sqrt{2}$$

$$\Rightarrow y = -\frac{3}{4}x + 3\sqrt{2}$$

condition of tangency स्पर्शता का प्रतिबन्ध  $c^2 = a^2m^2 + b^2$

$$18 = a^2 \cdot \frac{9}{16} + 9$$

$$a^2 \cdot \frac{9}{16} = 9$$

$$a^2 = 16$$

$$1a = 4$$

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$$

$$\therefore ae = \frac{\sqrt{7}}{4} \cdot 4 = \sqrt{7}$$

$\therefore$  focus are नाभियां  $(\pm\sqrt{7}, 0)$

$\therefore$  distance between foci नाभियों के मध्य दूरी =  $2\sqrt{7}$

18. If  $z = \left(\frac{3 + i\sin\theta}{4 - i\cos\theta}\right)$  is purely real and  $\theta \in \left(\frac{\pi}{2}, \pi\right)$  then  $\arg(\sin\theta + i\cos\theta)$  is -

यदि  $z = \left(\frac{3 + i\sin\theta}{4 - i\cos\theta}\right)$  पूर्णतया वास्तविक है तथा  $\theta \in \left(\frac{\pi}{2}, \pi\right)$  तो  $\arg(\sin\theta + i\cos\theta)$  का मान होगा-

- (1)  $-\tan^{-1}\frac{3}{4}$                       (2)  $\pi - \tan^{-1}\frac{3}{4}$                       (3)  $\pi - \tan^{-1}\frac{4}{3}$                       (4)  $\tan^{-1}\frac{4}{3}$

Ans. (3)

Sol.  $z = \frac{(3 + i\sin\theta)}{(4 - i\cos\theta)} \times \frac{(4 + i\cos\theta)}{(4 + i\cos\theta)}$

as चूंकि z is purely real पूर्णतः वास्तविक है  $\Rightarrow 3\cos\theta + 4\sin\theta = 0 \Rightarrow \tan\theta = -\frac{3}{4}$

$\arg(\sin\theta + i\cos\theta) = \pi + \tan^{-1}\left(\frac{\cos\theta}{\sin\theta}\right) = \pi + \tan^{-1}\left(-\frac{4}{3}\right) = \pi - \tan^{-1}\left(-\frac{4}{3}\right)$

19.  $a_1, a_2, a_3, \dots, a_9$  are in GP where  $a_1 < 0$ ,

$a_1 + a_2 = 4, a_3 + a_4 = 16$ , if  $\sum_{i=1}^9 a_i = 4\lambda$ , then  $\lambda$  is equal to

$a_1, a_2, a_3, \dots, a_9$  गुणोत्तर श्रेणी में है जहां  $a_1 < 0$ ,

$a_1 + a_2 = 4, a_3 + a_4 = 16$ , if  $\sum_{i=1}^9 a_i = 4\lambda$ , तो  $\lambda$  का मान होगा-

- (1) -513                      (2)  $-\frac{511}{3}$                       (3) -171                      (4) 171

Ans. (3)

Sol.  $a_1 + a_2 = 4 \Rightarrow a_1 + a_1r = 4 \dots\dots(i)$

$a_3 + a_4 = 16 \Rightarrow a_1r^2 + a_1r^3 = 16 \dots\dots(ii)$

$\frac{1}{r^2} + \frac{1}{4} \Rightarrow r^2 = 4$

$r = \pm 2$

$r = 2, a_1(1+2) = 4 \Rightarrow a_1 = \frac{4}{3}$

$r = -2, a_1(1-2) = 4 \Rightarrow a_1 = -4$

$\sum_{i=1}^9 a_i = \frac{a_1(r^9 - 1)}{r - 1} = \frac{(-4)((-2)^9 - 1)}{-2 - 1} = \frac{4}{3}(-513) = 4\lambda$

$\lambda = -171$

20. If यदि  $y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$  and और  $y\left(\frac{1}{2}\right) = -\frac{1}{4}$ . Then तो  $\frac{dy}{dx}$  at  $x = \frac{1}{2}$  is होगा -

- (1)  $\frac{2}{\sqrt{5}}$                       (2)  $-\frac{\sqrt{5}}{4}$                       (3)  $-\frac{\sqrt{5}}{2}$                       (4)  $\frac{\sqrt{5}}{2}$

Ans. (3)

**Sol.**  $x = \frac{1}{2}, y = \frac{-1}{4} \Rightarrow xy = \frac{-1}{8}$

$$y \cdot \frac{1 \cdot (-2x)}{2\sqrt{1-x^2}} + y' \cdot \sqrt{1-x^2} = - \left\{ 1 \cdot \sqrt{1-y^2} + \frac{x \cdot (-2y)}{2\sqrt{1-y^2}} y' \right\}$$

$$-\frac{xy}{\sqrt{1-x^2}} + y' \sqrt{1-x^2} = -\sqrt{1-y^2} + \frac{xy \cdot y'}{\sqrt{1-y^2}}$$

$$y' \left( \sqrt{1-x^2} - \frac{xy}{\sqrt{1-y^2}} \right) = \frac{xy}{\sqrt{1-x^2}} - \sqrt{1-y^2}$$

$$y' \left( \frac{\sqrt{3}}{2} + \frac{1}{8 \cdot \frac{\sqrt{15}}{4}} \right) = \frac{-1}{8 \cdot \frac{\sqrt{3}}{2}} - \frac{\sqrt{15}}{4}$$

$$y' \left( \frac{\sqrt{45} + 1}{2\sqrt{15}} \right) = - \frac{(1 + \sqrt{45})}{4\sqrt{3}}$$

$$y' = - \frac{\sqrt{5}}{2}$$

**21.** Let  $X = \{x : 1 \leq x \leq 50, x \in \mathbb{N}\}$

$A = \{x : x \text{ is multiple of } 2\}$

$B = \{x : x \text{ is multiple of } 7\}$

Then find number of elements in the smallest subset of X which contain elements of both A and B

माना  $X = \{x : 1 \leq x \leq 50, x \in \mathbb{N}\}$

$A = \{x : x, 2 \text{ का गुणज है}\}$

$B = \{x : x, 7 \text{ का गुणज है}\}$

तो X के सबसे छोटे उपसमुच्चय, जो A तथा B दोनों के अवयवों को समाहित करता है, में अवयवों की संख्या है—

**Ans.** 29

**Sol.**  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$   
 $= 25 + 7 - 3$   
 $= 29$

**22.** If  $Q\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$  is foot of perpendicular drawn from  $P(1, 0, 3)$  on a line L and if line L is passing through

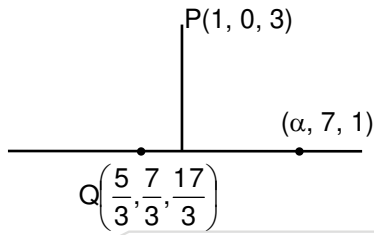
$(\alpha, 7, 1)$ , then value of  $\alpha$  is

यदि  $P(1, 0, 3)$  से किसी रेखा L पर लम्ब का पाद  $Q\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$  है तथा रेखा L  $(\alpha, 7, 1)$  से गुजरती है तो  $\alpha$  का मान

होगा—

**Ans.** 4

**Sol.** Since PQ is perpendicular to L, therefore  
चूँकि PQ रेखा L के लम्बवत् है इसलिए



$$\left(1 - \frac{5}{3}\right)\left(\alpha - \frac{5}{3}\right) + \left(\frac{-7}{3}\right)\left(7 - \frac{7}{3}\right) + \left(3 - \frac{17}{3}\right)\left(1 - \frac{17}{3}\right) = 0$$

$$\Rightarrow \frac{-2\alpha}{3} + \frac{10}{9} - \frac{98}{9} + \frac{112}{9} = 0$$

$$\Rightarrow \frac{2\alpha}{3} = \frac{24}{9} \Rightarrow \alpha = 4$$

**23.** If  $f(x)$  is defined in  $x \in \left(-\frac{1}{3}, \frac{1}{3}\right)$

$$f(x) = \begin{cases} \left(\frac{1}{x}\right) \log_e \left(\frac{1+3x}{1-2x}\right) & x \neq 0 \\ k & x = 0 \end{cases} \quad \text{Find } k \text{ such that } f(x) \text{ is continuous}$$

यदि  $f(x)$ ,  $x \in \left(-\frac{1}{3}, \frac{1}{3}\right)$  में परिभाषित है

$$f(x) = \begin{cases} \left(\frac{1}{x}\right) \log_e \left(\frac{1+3x}{1-2x}\right) & x \neq 0 \\ k & x = 0 \end{cases} \quad \text{यदि } f(x) \text{ सतत् है तो } k \text{ का मान होगा—}$$

**Ans.** 5

**Sol.**  $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \left( \frac{1}{x} \ln \left( \frac{1+3x}{1-2x} \right) \right) = \lim_{x \rightarrow 0} \left( \frac{\ln(1+3x)}{x} - \frac{\ln(1-2x)}{x} \right)$

$$= \lim_{x \rightarrow 0} \left( \frac{3 \ln(1+3x)}{3x} - \frac{2 \ln(1-2x)}{-2x} \right) = 3 + 2 = 5$$

$\therefore f(x)$  will be continuous सतत् होगा if यदि  $f(0) = \lim_{x \rightarrow 0} f(x)$

**24.** If system of equation यदि समीकरणों का निकाय

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$3x + 2y + \lambda z = \mu$$

has more than two solutions. Find  $(\mu - \lambda^2)$

के दो से अधिक हल है तब  $(\mu - \lambda^2)$  का मान होगा—

**Ans.** 13

**Sol.**  $x + y + z = 6$  ..... (1)

$x + 2y + 3z = 10$  ..... (2)

$3x + 2y + \lambda z = \mu$  ..... (3)

from (1) and तथा (2) से

if  $z = 0 \Rightarrow x + y = 6$  and और  $x + 2y = 10$

$\Rightarrow y = 4, x = 2$

(2, 4, 0)

if  $y = 0 \Rightarrow x + z = 6$  and और  $x + 3z = 10$

$\Rightarrow z = 2$  and और  $x = 4$

(4, 0, 2)

so इसलिए  $3x + 2y + \lambda z = \mu$

must pass through गुजरती है (2, 4, 0) and तथा (4, 0, 2)

so इसलिए  $6 + 8 = \mu \Rightarrow \mu = 14$

and तथा  $12 + 2\lambda = \mu$

$12 + 2\lambda = 14 \Rightarrow \lambda = 1$

so इसलिए  $\mu - \lambda^2 = 14 - 1$

$= 13$

**25.** If mean and variance of 2, 3, 16, 20, 13, 7, x, y are 10 and 25 respectively then find xy  
यदि चरों 2, 3, 16, 20, 13, 7, x, y का माध्य और प्रसरण क्रमशः 10 और 25 है तब xy का मान है—

**Ans.** 124

**Sol.** mean माध्य  $= \bar{x} = \frac{2+3+16+20+13+7+x+y}{8} = 10 \Rightarrow x+y = 19 \dots\dots(i)$

variance प्रसरण  $\sigma^2 = \frac{\sum(x_i)^2}{8} - (\bar{x})^2 = 25$

$\frac{4+9+256+400+169+49+x^2+y^2}{8} - 100 = 25$

$\Rightarrow x^2 + y^2 = 113 \dots\dots(ii)$

$(x+y)^2 = (19)^2 \Rightarrow x^2 + y^2 + 2xy = 361 \Rightarrow xy = 124$

(exact data is not retrieved so ans. can vary)