FINAL JEE-MAIN EXAMINATION - JANUARY, 2020
Held On Tuesday, 7 January 2020
TIME : 9 : 30 AM to 12 : 30 PM

1. Let y = f(x) is a solution of differential equation $e^y\left(\frac{dy}{dx}-1\right)=e^x$ and f(0) = 0 then f(1) is equal to :

यदि y = f(x) अवकल समीकरण $e^y\left(\frac{dy}{dx}-1\right)=e^x$ का हल है तथा f(0) = 0 तब f(1) का मान है–

(1) $\ell n2$ (2) $2+\ell n2$ (3) $1+\ell n2$ (4) $3+\ell n2$

Ans. **(3)**

Sol. $e^y = t$

$e^y\frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dt}{dx} - t = e^x$

$IF = e^{\int -1.dx} = e^{-x}$

$t(e^{-x}) = \int e^x.e^{-x}dx$

$e^{y-x} = x + c$

Put x = 0, y = 0 रखने पर then c = 1

$e^{y-x} = x + 1$

$y = x + \ell n(x+1)$

at x = 1 रखने पर, $y = 1 + \ell n(2)$

2. If α is a roots of equation $x^2+x+1=0$ and $A=\frac{1}{\sqrt{3}}\begin{bmatrix}1&1&1\\1&\alpha&\alpha^2\\1&\alpha^2&\alpha\end{bmatrix}$ then A^{31} equal to :

यदि α समीकरण $x^2+x+1=0$ का एक मूल है तथा $A=\frac{1}{\sqrt{3}}\begin{bmatrix}1&1&1\\1&\alpha&\alpha^2\\1&\alpha^2&\alpha\end{bmatrix}$ तब A^{31} बराबर है–

(1) A (2) A^2 (3) A^3 (4) A^4

Ans. (3)

Sol. $A^2=\frac{1}{3}\begin{bmatrix}1&1&1\\1&\omega&\omega^2\\1&\omega^2&\omega\end{bmatrix}\begin{bmatrix}1&1&1\\1&\omega&\omega^2\\1&\omega^2&\omega\end{bmatrix}=\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}$

$\Rightarrow A^4 = I$

$\Rightarrow A^{30} = A^{28}\times A^3 = A^3$

3. The six digit numbers that can be formed using digits 1, 3, 5, 7, 9 such that each digit is used at least once.

अंको 1, 3, 5, 7, 9 का प्रयोग कर बनाई जा सकने वाली छः अंको की संख्याओं की संख्या कि प्रत्येक अंक कम से कम एक बार प्रयुक्त हो, होगी–

Ans. **1800**

Sol. 1, 3, 5, 7, 9

For digit to repeat we have $^5C_1$ choice

And six digits can be arrange in $\frac{\lfloor 6}{\lfloor 2}$ ways.

Hence total such numbers = $\frac{5\lfloor 6}{\lfloor 2}$

Sol. 1, 3, 5, 7, 9

पुनरावृत्ति वाले अंक के लिए $^5C_1$ विकल्प है।

और छः अंकों की व्यवस्था के तरीके $\frac{\lfloor 6}{\lfloor 2}$

अतः कुल संख्याऐं = $\frac{5\lfloor 6}{\lfloor 2}$

4. The area that is enclosed in the circle $x^2 + y^2 = 2$ which is not common area enclosed by $y = x$ & $y^2 = x$ is

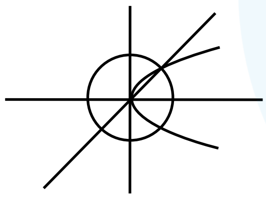
वृत्त $x^2 + y^2 = 2$ के अन्तर्गत परिबद्ध वह क्षेत्रफल जो $y = x$ & $y^2 = x$ द्वारा परिबद्ध क्षेत्रफल के साथ उभयनिष्ठ न हो–

(1) $\frac{1}{12}(24\pi - 1)$ (2) $\frac{1}{6}(12\pi - 1)$ (3) $\frac{1}{12}(6\pi - 1)$ (4) $\frac{1}{12}(12\pi - 1)$

Ans. **(2)**

Sol. Total area – enclosed area

कुल क्षेत्रफल – उभयनिष्ठ क्षेत्रफल



$$2\pi - \int_0^1 \sqrt{x} - x\,dx$$

$$2\pi - \left(\frac{2x^{3/2}}{3} - \frac{x^2}{2}\right)_0^1$$

$$2\pi - \left(\frac{2}{3} - \frac{1}{2}\right) \Rightarrow 2\pi - \left(\frac{1}{6}\right) \Rightarrow \frac{12\pi - 1}{6}$$

5. If sum of all the coefficient of even powers in $(1-x+x^2-x^3 \ldots\ldots x^{2n})(1+x+x^2+x^3 \ldots\ldots+x^{2n})$ is 61 then n is equal to

$(1-x+x^2-x^3 \ldots\ldots x^{2n})(1+x+x^2+x^3 \ldots\ldots+x^{2n})$ में समघातों के सभी गुणांको का योगफल 61 है। तब n का मान बराबर है–

(1) 30 (2) 32 (3) 28 (4) 36

Ans. **(1)**

Sol. Let माना $(1-x+x^2 \ldots\ldots)(1+x+x^2 \ldots\ldots) = a_0 + a_1 x + a_2 x^2 + \ldots\ldots$

put $x = 1$ रखने पर

$1(2n+1) = a_0 + a_1 + a_2 + \ldots\ldots a_{2n}$ ....(i)

put $x = -1$ रखने पर

$(2n+1)\times 1 = a_0 - a_1 + a_2 + \ldots\ldots a_{2n}$ ....(ii)

Form (i) + (ii) से

$4n + 2 = 2(a_0 + a_2 + \ldots.)$

$= 2 \times 61$

$\Rightarrow 2n+1 = 61 \Rightarrow n = 30$

6. If variance of first n natural numbers is 10 and variance of first m even natural numbers is 16 then the value of m + n is

प्रथम n प्राकृत संख्याओं का प्रसरण 10 है तथा प्रथम m सम प्राकृत संख्याओं का प्रसरण 16 है तो m + n का मान है–

Ans. **18**

Sol. Var (1, 2, ...,n) = 10 $\Rightarrow \dfrac{1^2+2^2+\ldots\ldots+n^2}{n} - \left(\dfrac{1+2+\ldots\ldots+n}{n}\right)^2 = 10$

$\Rightarrow \dfrac{(n+1)(2n+1)}{6} - \left(\dfrac{n+1}{2}\right)^2 = 10$

$\Rightarrow n^2 - 1 = 120 \qquad \Rightarrow n = 11$

Var (2, 4, 6,............,2m) = 16 $\Rightarrow$ var (1, 2,........,m) = 4

$\Rightarrow m^2 - 1 = 48 \quad \Rightarrow \quad m = 7 \Rightarrow m + n = 18$

7. Evaluate $\lim\limits_{x\to 2} \dfrac{3^x + 3^{x-1} - 12}{3^{\frac{-x}{2}} - 3^{1-x}}$ ज्ञात कीजिए।

Ans. **72**

Sol. Put $3^{\frac{x}{2}} = t$ रखने पर

$\Rightarrow \lim\limits_{t\to 3} \dfrac{\dfrac{4t^2}{3} - 12}{-\dfrac{3}{t^2} + \dfrac{1}{t}} = \lim\limits_{t\to 3} \dfrac{4(t^2-9)t^2}{3(-3+t)} = \lim\limits_{t\to 3} \dfrac{4t^2(3+t)}{3} = \dfrac{4\times 9\times 6}{3} = 72$

8. If f(x) is continuous and differentiable in $x \in [-7, 0]$ and $f'(x) \le 2\ \forall x \in [-7, 0]$, also $f(-7) = -3$ then range of $f(-1) + f(0)$

यदि f(x), $x \in [-7, 0]$ में सतत् तथा अवकलनीय है तथा $f'(x) \le 2\ \forall x \in [-7, 0]$ और $f(-7) = -3$ तो $f(-1) + f(0)$ का परिसर है–

(1) [–5, –7] (2) $(-\infty, 6]$ (3) $(-\infty, 20]$ (4) [-5, 3]

Ans. **(3)**

Sol. Lets use LMVT for $x \in [-7, -1]$ में लांग्राज मध्यमान प्रमेय से

$$\frac{f(-1)-f(-7)}{(-1+7)} \le 2$$

$$\frac{f(-1)+3}{6} \le 2 \Rightarrow f(-1) \le 9$$

Also use LMVT for $x \in [-7, 0]$ में लांग्राज मध्यमान प्रमेय से

$$\frac{f(0)-f(-7)}{(0+7)} \le 2$$

$$\frac{f(0)+3}{7} \le 2 \Rightarrow f(0) \le 11 \qquad \therefore \qquad f(0) + f(-1) \le 20$$

9. If $y = mx + 4$ is common tangent to parabolas $y^2 = 4x$ and $x^2 = 2by$. Then value of b is

यदि $y = mx + 4$ परवलय $y^2 = 4x$ तथा $x^2 = 2by$ की उभयनिष्ठ स्पर्श रेखा है तो b का मान है–

(1) –64 (2) –32 (3) –128 (4) 16

Ans. **(3)**

Sol. $y = mx + 4$ ......(i)

$y^2 = 4x$ tangent की स्पर्श रेखा $y = mx + \frac{a}{m} \Rightarrow y = mx + \frac{1}{m}$ ....(ii)

from (i) and (ii) से

$$4 = \frac{1}{m} \Rightarrow m = \frac{1}{4}$$

So line $y = \frac{1}{4}x + 4$ is also tangent to parabola $x^2 = 2by$, so solve

अतः रेखा $y = \frac{1}{4}x + 4$ परवलय $x^2 = 2by$ की स्पर्श रेखा है अतः

$$x^2 = 2b\left(\frac{x+16}{4}\right)$$

$\Rightarrow 2x^2 - bx - 16b = 0 \quad \Rightarrow D = 0 \Rightarrow b^2 - 4 \times 2 \times (-16b) = 0$

$\Rightarrow b^2 + 32 \times 4b = 0$

$b = -128$, $b = 0$ (not possible असंभव)

10. If α and β are the roots of equation $(k+1)\tan^2 x - \sqrt{2}\lambda \tan x = 1 - k$ and $\tan^2(\alpha+\beta) = 50$. Find value of λ.

यदि α तथा β समीकरण $(k+1)\tan^2 x - \sqrt{2}\lambda \tan x = 1 - k$ के मूल है तथा $\tan^2(\alpha+\beta) = 50$ तो λ का मान है–

(1) 10 (2) 5 (3) 7 (4) 12

Ans. **(1)**

$(k+1)\tan^2 x - \sqrt{2}\lambda \tan x + (k-1) = 0$

$\tan\alpha + \tan\beta = \frac{\sqrt{2}\lambda}{k+1}$

$\tan\alpha \tan\beta = \frac{k-1}{k+1}$

$\tan(\alpha+\beta) = (k-1) = 0 \frac{\frac{\sqrt{2}\lambda}{k+1}}{1-\frac{k-1}{k+1}} = \frac{\sqrt{2}\lambda}{2} = \frac{\lambda}{\sqrt{2}}$

$\tan^2(\alpha+\beta) = \frac{\lambda^2}{2} = 50$

$\lambda = 10$

11. Find image of point (2, 1, 6) in the plane containing points (2, 1, 0), (6, 3, 3) and (5, 2, 2)

बिन्दुओं (2, 1, 0), (6, 3, 3) तथा (5, 2, 2) को समाहित करने वाले समतल में बिन्दु (2, 1, 6) का प्रतिबिम्ब है–

(1) (6, 5, –2) (2) (6, –5, 2) (3) (2, –3, 4) (4) (2, –5, 6)

Ans. **(1)**

Sol. Plane is समतल $x + y - 2z = 3$ $\Rightarrow \frac{x-2}{1} = \frac{y-1}{1} = \frac{z-6}{-2} = \frac{-2(2+1-12-3)}{6} \Rightarrow (x, y, z) = (6, 5, -2)$

12. Let माना $(x)^k + (y)^k = (a)^k$ where जहां $a, k > 0$ and तथा $\frac{dy}{dx} + \left(\frac{y}{x}\right)^{\frac{1}{3}} = 0$, then find तब k बराबर है–

(1) $\frac{1}{3}$ (2) $\frac{2}{3}$ (3) $\frac{4}{3}$ (1) 2

Ans. **(2)**

Sol. $k.x^{k-1} + k.y^{k-1}\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\left(\frac{x}{y}\right)^{k-1}$

$\frac{dy}{dx} + \left(\frac{x}{y}\right)^{k-1} = 0$

$k - 1 = -\frac{1}{3}$

$-\frac{1}{3} = \frac{2}{3}$

13. If $g(x) = x^2 + x - 1$ and $g(f(x)) = 4x^2 - 10x + 5$, then find $f\left(\frac{5}{4}\right)$.

यदि $g(x) = x^2 + x - 1$ और $g(f(x)) = 4x^2 - 10x + 5$, तब $f\left(\frac{5}{4}\right)$ का मान है–

(1) $\frac{1}{2}$ (2) $-\frac{1}{2}$ (3) $-\frac{1}{3}$ (4) $\frac{1}{3}$

Ans. **(2)**

Sol. $g(f(x)) = f^2(x) + f(x) - 1$

$$g\left(f\left(\frac{5}{4}\right)\right) = 4\left(\frac{5}{4}\right)^2 - 10.\ \frac{5}{4} + 5 = -\frac{5}{4}$$

$$g\left(f\left(\frac{5}{4}\right)\right) = f^2\left(\frac{5}{4}\right) + f\left(\frac{5}{4}\right) - 1$$

$$-\frac{5}{4} = f^2\left(\frac{5}{4}\right) + f\left(\frac{5}{4}\right) - 1$$

$$f^2\left(\frac{5}{4}\right) + f\left(\frac{5}{4}\right) + \frac{1}{4} = 0$$

$$\left(f\left(\frac{5}{4}\right) + \frac{1}{2}\right)^2 = 0$$

$$f\left(\frac{5}{4}\right) = -\frac{1}{2}$$

14. If $z = x + iy$ and real part $\left(\frac{z-1}{2z+i}\right) = 1$ then locus of z is

(1) Straight line with slope 2

(2) Straight line with slope $-\frac{1}{2}$

(3) circle with diameter $\frac{\sqrt{5}}{2}$

(4) circle with diameter $\frac{1}{2}$

यदि $z = x + iy$ तथा $\text{Re}\left(\frac{z-1}{2z+i}\right) = 1$ तब z का बिन्दुपथ है

(1) प्रवणता 2 की सरल रेखा

(2) प्रवणता $-\frac{1}{2}$ की सरल रेखा

(3) वृत्त जिसका व्यास $\frac{\sqrt{5}}{2}$ है

(4) वृत्त जिसका व्यास $\frac{1}{2}$ है

Ans. **(3)**

Sol. $z = x + iy$

$$\left(\frac{z-1}{2z+i}\right) = \frac{(x-1)+iy}{2(x+iy)+i} = \frac{(x-1)+iy}{2x+(2y+1)i} \times \frac{2x-(2y+1)i}{2x-(2y+1)i}$$

$$\text{Re}\left(\frac{z+1}{2z+i}\right) = \frac{2x(x-1)+y(2y+1)}{(2x)^2+(2y+1)^2} = 1$$

$\Rightarrow 2x^2 + 2y^2 - 2x + y = 4x^2 + 4y^2 + 4y + 1$ $\qquad \Rightarrow 2x^2 + 2y^2 + 2x + 3y + 1 = 0$

$\Rightarrow x^2 + y^2 + x + \frac{3}{2}y + \frac{1}{2} = 0$ Circle with centre (वृत्त जिसका केन्द्र) $\left(-\frac{1}{2}, -\frac{3}{4}\right)$

$r = \sqrt{\frac{1}{4} + \frac{9}{16} - \frac{1}{2}} = \sqrt{\frac{4+9-8}{16}} = \frac{\sqrt{5}}{4}$

15. If distance between the foci of an ellipse is 6 and distance between its directrices is 12, then length of its latus rectum is

यदि एक दीर्घवृत्त के नाभियों के मध्य की दूरी 6 है तथा इसकी नियताओं के मध्य की दूरी 12 है तब इसके नाभिलम्ब की लम्बाई है–

(1) 4 (2) $3\sqrt{2}$ (3) 9 (4) $2\sqrt{2}$

Ans. **(2)**

Sol. $2ae = 6$ and और $\frac{2a}{e} = 12$

$\Rightarrow ae = 3$ and और $\frac{a}{e} = 6$

$\Rightarrow a^2 = 18$

$\Rightarrow b^2 = a^2 - a^2e^2 = 18 - 9 = 9$ $\qquad \Rightarrow$ L.R. $= \frac{2b^2}{a} = \frac{2\times 9}{3\sqrt{2}} = 3\sqrt{2}$

16. If $y = \sqrt{\frac{2(\tan\alpha + \cot\alpha)}{1+\tan^2\alpha} + \frac{1}{\sin^2\alpha}}$ when $\alpha \in \left(\frac{3\pi}{4}, \pi\right)$ then find $\frac{dy}{d\alpha}$ at $\alpha = \frac{5\pi}{6}$

यदि $y = \sqrt{\frac{2(\tan\alpha + \cot\alpha)}{1+\tan^2\alpha} + \frac{1}{\sin^2\alpha}}$ जब $\alpha \in \left(\frac{3\pi}{4}, \pi\right)$ तब $\alpha = \frac{5\pi}{6}$ पर $\frac{dy}{d\alpha}$ का मान होगा–

(1) 4 (2) 2 (3) 3 (4) –4

Ans. **(1)**

Sol. $y = \sqrt{\frac{2\cos^2\alpha}{\sin\alpha\cos\alpha} + \frac{1}{\sin^2\alpha}} = \sqrt{2\cot\alpha + \operatorname{cosec}^2\alpha} = |1 + \cot\alpha| = -1 - \cot\alpha$

$\frac{dy}{d\alpha} = \operatorname{cosec}^2\alpha \Rightarrow \left(\frac{dy}{d\alpha}\right) \text{at } \alpha = \frac{5\pi}{6} \text{ will be} = 4$

17. If A(1, 1), B(6, 5), $C\left(\frac{3}{2},2\right)$ are vertices of ΔABC. A point P is such that area of ΔPAB, ΔPAC, ΔPBC are equal, also $Q\left(\frac{-7}{6},\frac{-1}{3}\right)$, then length of PQ is

यदि A(1, 1), B(6, 5), $C\left(\frac{3}{2},2\right)$ त्रिभुज ΔABC के शीर्ष है एक बिन्दु P इस प्रकार है कि ΔPAB, ΔPAC, ΔPBC के क्षेत्रफल बराबर है तथा $Q\left(\frac{-7}{6},\frac{-1}{3}\right)$ तब PQ की लम्बाई है–

(1) 2 (2) 3 (3) 4 (4) 5

Ans. **(4)**

Sol. P will be centroid of ΔABC

P त्रिभुज ΔABC का केन्द्रक होगा

$$P\left(\frac{17}{6},\frac{8}{3}\right) \quad \Rightarrow \quad PQ=\sqrt{\left(\frac{24}{6}\right)^2+\left(\frac{9}{3}\right)}=5$$

18. $(p \to q) \wedge (q \to \sim p)$ is equivalent to

$(p \to q) \wedge (q \to \sim p)$ के तुल्य है–

(1) ~p (2) p (3) $p \wedge q$ (4) $p \vee q$

Ans. **(1)**

Sol.

p	q	$p \to q$	~p	$q \to \sim p$	$(p \to q) \wedge (p \to \sim q)$
T	T	T	F	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	T	T

Clearly $(p \to q) \wedge (q \to \sim p)$ is equivalent to ~p

स्पष्टतया $(p \to q) \wedge (q \to \sim p)$ के तुल्य ~p है

19. Find greatest value of k for which $49^k + 1$ is factor of $1 + 49 + 49^2 \ldots..(49)^{125}$

k का अधिकतम मान होगा जबकि $1 + 49 + 49^2 \ldots..(49)^{125}$ का एक गुणनखण्ड $49^k + 1$ है–

(1) 63 (2) 65 (3) 2 (4) 5

Ans. **(1)**

Sol. $$\frac{(49)^{126}-1}{48}=\frac{((49)^{63}+1)(49^{63}-1)}{48}$$

20. If f(x) = |2 – |x – 3|| is non differentiable in x ∈ S.Then value of $\sum_{x\in S}(f(f(x))$ is

यदि f(x) = |2 – |x – 3|| सभी x ∈ S में अवकलनीय नहीं है तब $\sum_{x\in S}(f(f(x))$ का मान है–

Ans. 3

Sol. $\because$ f(x) is non differentiable at x = 1, 3, 5

$\Sigma f(f(x)) = f(f(1) + f(f(3)) + f(f(5))$

$= 1 + 1 + 1$

$= 3$

Sol. $\because$ f(x), x = 1, 3, 5 पर अवकलनीय नहीं है

$\Sigma f(f(x)) = f(f(1) + f(f(3)) + f(f(5))$

$= 1 + 1 + 1$

$= 3$

21. If system of equations

$2x + 2ay + az = 0$

$2x + 3by + bz = 0$

$2x + 4cy + cz = 0$ have non-trivial solution

then

(1) a + b + c = 0

(2) a,b,c are in A.P.

(3) $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in A.P.

(4) a, b, c in G.P.

यदि समीकरणों का निकाय

$2x + 2ay + az = 0$

$2x + 3by + bz = 0$

$2x + 4cy + cz = 0$ के अशून्य हल है तब

(1) a + b + c = 0

(2) a,b,c समान्तर श्रेणी में है

(3) $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ समान्तर श्रेणी में है

(4) a, b, c गुणोत्तर श्रेणी में है

Sol. For non-trivial solution

अशून्य हल के लिए

$$\begin{vmatrix} 2 & 2a & a \\ 2 & 3b & b \\ 2 & 4c & c \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$$

$(3bc - 4bc) - (2ac - 4ac) + (2ab - 3ab) = 0$

$-bc + 2ac - ab = 0$

$ab + bc = 2ac$

a, b, c in H.P.

$\Rightarrow \quad \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ in A.P.

22. If sum of 5 consecutive terms of 'an A.P is 25 & product of these terms is 2520. If one of the terms is – 1/2 then the value of greatest term is

यदि समान्तर श्रेणी के 5 क्रमागत पदों का योगफल 25 है तथा इन पदों का गुणनफल 2520 है यदि उनमे से एक पद – 1/2 है तब सबसे बडे पद का मान है–

(1) $\frac{21}{2}$ (2) 16 (3) 5 (4) 7

Ans. **(2)**

Sol. Let terms be (माना कि पद) a – 2d, a –d, a, a + d , a + 2d .

sum योगफल= 25 $\Rightarrow$ 5a = 25 $\Rightarrow$ a = 5

Product गुणनफल= 2520

$(5-2d)\ (5-d)\ 5(5+d)\ (5+2d) = 2520$

$\Rightarrow (25-4d^2)\ (25-d^2) = 504$

$\Rightarrow 625 - 100d^2 - 25d^2 + 4d^4 = 504$

$\Rightarrow 4d^4 - 125d^2 + 625 - 504 = 0$

$\Rightarrow 4d^4 - 125d^2 + 121 = 0$

$\Rightarrow 4d^4 - 121d^2 - 4d^2 + 121 = 0$

$\Rightarrow (d^2 - 1)\ (4d^2 - 121) = 0$

$\Rightarrow d = \pm 1, \quad d = \pm\frac{11}{2}$

d = ±1, does not give $\frac{-1}{2}$ as a term

d = ±1, जो $\frac{-1}{2}$ नहीं देता है

$\therefore d = \frac{11}{2}$

$\therefore$ Largest term अधिकतम पद = 5 + 2d = 5 + 11 = 16

23. Let $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$

$\vec{a}$ lies in plane of $\vec{b}$ & $\vec{c}$

$\vec{b} = \hat{i} + \hat{j}$ & $\vec{c} = \hat{i} - \hat{j} + 4\hat{k}$

of $\vec{a}$ bisectors angle between $\vec{b}$ & $\vec{c}$, then

माना $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$

तथा $\vec{a}$ सदिश $\vec{b}$ और $\vec{c}$ के समतल में है

जहां $\vec{b} = \hat{i} + \hat{j}$ और $\vec{c} = \hat{i} - \hat{j} + 4\hat{k}$ है $\vec{b}$ और $\vec{c}$ के मध्य कोण अर्द्धक $\vec{a}$ है तब

(1) $\vec{a}\cdot\hat{k} + 2 = 0$ (2) $\vec{a}\cdot\hat{k} + 4 = 0$ (3) $\vec{a}\cdot\hat{k} - 2 = 0$ (4) $\vec{a}\cdot\hat{k} + 5 = 0$

Ans. **(1)**

Sol. angle bisector can be कोण अर्द्धक हो सकता है $\vec{a} = \vec{a} = \lambda(\hat{b} + \hat{c})$ or या $\vec{a} = \mu(\hat{b} - \hat{c})$

$$\vec{a} = \lambda\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}} + \frac{\hat{i}+\hat{j}+4\hat{k}}{3\sqrt{2}}\right) = \frac{\lambda}{3\sqrt{2}}\left[3\hat{i}+3\hat{j}+\hat{i}-\hat{j}+4\hat{k}\right] = \frac{\lambda}{3\sqrt{2}}\left[4\hat{i}+2\hat{j}+4\hat{k}\right]$$

Compare with से तुलना करने पर $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$

$\frac{2\lambda}{3\sqrt{2}} = 2 \Rightarrow \lambda = 3\sqrt{2}$

$\vec{a} = 4\hat{i} + 2\hat{j} + 4\hat{k}$

Not in option so now consider माना कि $\vec{a} = \mu\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}} - \frac{\hat{i}-\hat{j}+4\hat{k}}{3\sqrt{2}}\right)$

$\vec{a} = \frac{\mu}{3\sqrt{2}}\left(3\hat{i}+3\hat{j}-\hat{i}+\hat{j}-4\hat{k}\right)$

$= \frac{\mu}{3\sqrt{2}}\left(2\hat{i}+4\hat{j}-4\hat{k}\right)$

Compare withतुलना करने पर $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$

$\frac{4\mu}{3\sqrt{2}} = 2 \Rightarrow \mu = \frac{3\sqrt{2}}{2}$

$\vec{a} = \hat{i} + 2\hat{j} - 2\hat{k}$

$\vec{a}.\hat{k} + 2 = 0$

$-2 + 2 = 0$

24. Given f(a + b + 1 – x) = f(x) $\forall$ x $\in$ R then the value of $\frac{1}{(a+b)}\int_a^b x[f(x)+f(x+1)]dx$ is equal to

दिया गया है f(a + b + 1 – x) = f(x) $\forall$ x $\in$ R तब $\frac{1}{(a+b)}\int_a^b x[f(x)+f(x+1)]dx$ का मान बराबर है–

(1) $\int_{a+1}^{b+1} f(x)dx$ (2) $\int_{a+1}^{b+1} f(x+1)dx$ (3) $\int_{a-1}^{b-1} f(x)dx$ (4*) $\int_{a-1}^{b-1} f(x+1)dx$

Ans. **(4)**

Sol. $I = \frac{1}{(a+b)}\int_a^b x[f(x)+f(x+1)]dx$ ......(1)

$x \to a + b - x$

$I = \frac{1}{(a+b)}\int_a^b (a+b-x)[f(a+b-x)+f(a+b+1-x)]dx$

$I = \frac{1}{(a+b)}\int_a^b (a+b-x)[f(x+1)+f(x)]dx$ ....(2)

[$\because$ put x $\to$ x + 1 in given equation]

[$\because$दी गई समीकरण में x $\to$ x + 1 रखने पर]

(1) + (2)

$2I = \int_a^b [f(x+1)+f(x)]dx$

$$2I = \int_a^b f(x+1)\,dx + \int_a^b f(x)\,dx$$

$$\int_a^b f(a+b+1-x)\,dx + \int_a^b f(x)\,dx$$

$$2I = 2\int_a^b f(x)\,dx$$

25. An unbiased coin is thrown 5 times. Let X be a random variable and k be the value assigned to X for k = 3, 4, 5 times Head occurs consecutively and otherwise the value of X is assigned –1. What is value of expectation.

एक निष्पक्षपाती सिक्के को 5 बार उछाला जाता है माना X यादृच्छिक चर है तथा X से अर्जित मान k है जहां k = 3, 4, 5 बार लगातार शीर्ष आता है अन्यथा X को –1 मान दिया जाता है तब इसकी आकाक्षां है–

(1) $\frac{1}{8}$ (2) $-\frac{1}{8}$ (3) $\frac{3}{8}$ (4) $-\frac{3}{8}$

Ans. **(1)**

Sol.

k	0	1	2	3	4	5
P(k)	$\frac{1}{32}$	$\frac{12}{32}$	$\frac{11}{32}$	$\frac{5}{32}$	$\frac{2}{32}$	$\frac{1}{32}$

k = no. of times head occur consecutively

k = शीर्ष के क्रमागत आने की संख्या

Now expectation अब आकांक्षा

$$= \sum xP(k) = (-1)\times\frac{1}{32} + (-1)\times\frac{12}{32} + (-1)\times\frac{11}{32} + 3\times\frac{5}{32} + 4\times\frac{2}{32} + 5\times\frac{1}{32} = \frac{1}{8}$$