FINAL JEE-MAIN EXAMINATION - JANUARY, 2020

Held On Thursday, 9 January 2020

TIME : 2 : 30 PM to 05 : 30 PM

1. If $f(x) = \begin{cases} x & 0 < x < \frac{1}{2} \\ \frac{1}{2} & x = \frac{1}{2} \\ 1-x & \frac{1}{2} < x < 1 \end{cases}$

$g(x) = \left(x-\frac{1}{2}\right)^2$ then find the area bounded by f(x) and g(x) from $x = \frac{1}{2}$ to $x = \frac{\sqrt{3}}{2}$.

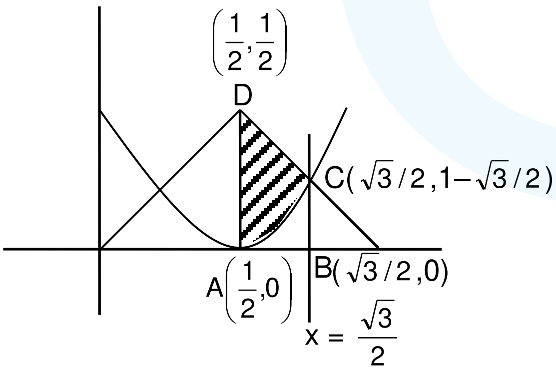
यदि $f(x) = \begin{cases} x & 0 < x < \frac{1}{2} \\ \frac{1}{2} & x = \frac{1}{2} \\ 1-x & \frac{1}{2} < x < 1 \end{cases}$

$g(x) = \left(x-\frac{1}{2}\right)^2$ तब f(x) तथा g(x) के द्वारा $x = \frac{1}{2}$ से $x = \frac{\sqrt{3}}{2}$ तक परिबद्व क्षेत्रफल ज्ञात कीजिये। .

(1) $\frac{\sqrt{3}}{4} - \frac{1}{3}$ (2) $\frac{\sqrt{3}}{4} + \frac{1}{3}$ (3) $2\sqrt{3}$ (4) $3\sqrt{3}$

Ans. **(1)**

Sol.



Required area = Area of trapezium ABCD – $\int_{1/2}^{\sqrt{3}/2}\left(x-\frac{1}{2}\right)^2 dx$

अभीष्ट क्षेत्रफल = समलम्ब चतुर्भज ABCD का क्षेत्रफल – $\int_{1/2}^{\sqrt{3}/2}\left(x-\frac{1}{2}\right)^2 dx$

$$= \frac{1}{2}\left(\frac{\sqrt{3}-1}{2}\right)\left(\frac{1}{2}+1-\frac{\sqrt{3}}{2}\right)-\frac{1}{3}\left(\left(x-\frac{1}{2}\right)^3\right)_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}$$

$$= \frac{\sqrt{3}}{4}-\frac{1}{3}$$

2. z is a complex number such that |Re(z)| + |Im (z)| = 4 then |z| can't be

z एक समिश्र संख्या इस प्रकार है कि |Re(z)| + |Im (z)| = 4 तब |z| नही हो सकता

(1) $\sqrt{7}$ (2) $\sqrt{10}$ (3) $\sqrt{\frac{17}{2}}$ (4) $\sqrt{8}$

Ans. **(1)**

Sol. z = x + iy

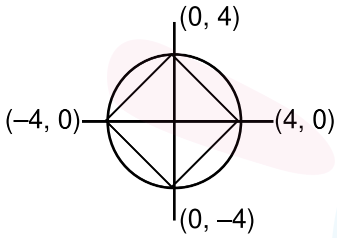
|x| + |y| = 4

Minimum value of |z| = $2\sqrt{2}$

Maximum value of |z| = 4

$|z| \in \left[\sqrt{8}, \sqrt{16}\right]$

So |z| can't be $\sqrt{7}$



Sol. z = x + iy

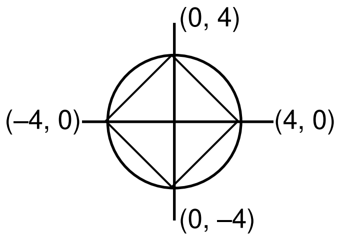
|x| + |y| = 4

|z| का न्यूनतम मान = $2\sqrt{2}$

|z| का अधिकतम मान = 4

$|z| \in \left[\sqrt{8}, \sqrt{16}\right]$

अतः |z|= $\sqrt{7}$ नही हो सकता



3. If यदि $f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$ and तथा $a - 2b + c = 1$ then तब

(1) $f(50) = 1$ (2) $f(-50) = -1$

(3) $f(50) = 501$ (4) $f(50) = -501$

Ans. **(1)**

Sol. Apply $R_1 = R_1 + R_3 - 2R_2$ प्रयोग करने पर

$$\Rightarrow f(x) = \begin{vmatrix} 1 & 0 & 0 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix} \quad \Rightarrow f(x) = 1 \quad \Rightarrow f(50) = 1$$

4. Let a_n is a positive term of a GP and $\sum_{n=1}^{100} a_{2n+1} = 200$, $\sum_{n=1}^{100} a_{2n} = 100$ find $\sum_{n=1}^{200} a_n$

माना a_n गुणोत्तर श्रेढ़ी का धनात्मक पद है तथा $\sum_{n=1}^{100} a_{2n+1} = 200$, $\sum_{n=1}^{100} a_{2n} = 100$, $\sum_{n=1}^{200} a_n$ का मान है–

(1) 300 (2) 150 (3) 175 (4) 225

Ans. **(2)**

Sol. Let GP is a, ar, ar^2 ....

$$\sum_{n=1}^{100} a_{2n+1} = a_3 + a_5 + \ldots.. a_{201} = 200 \Rightarrow \frac{ar^2(r^{200}-1)}{r^2-1} = 200 \quad \ldots.(1)$$

$$\sum_{n=1}^{100} a_{2n} = a_2 + a_4 + \ldots\ldots. a_{200} = 100 = \frac{ar(r^{200}-1)}{r^2-1} = 100 \quad \ldots.(2)$$

Form (1) and (2) r = 2

add both

$\Rightarrow a_2 + a_3 + \ldots\ldots\ldots. a_{200} + a_{201} = 300 \quad \Rightarrow \quad r(a_1 + \ldots\ldots\ldots.. a_{200}) = 300$

$$\sum_{n=1}^{200} a_n = \frac{300}{r} = 150$$

Sol. माना a, ar, ar^2 .... गुणोत्तर श्रेढ़ी में है

$$\sum_{n=1}^{100} a_{2n+1} = a_3 + a_5 + \ldots.. a_{201} = 200 \Rightarrow \frac{ar^2(r^{200}-1)}{r^2-1} = 200 \quad \ldots.(1)$$

$$\sum_{n=1}^{100} a_{2n} = a_2 + a_4 + \ldots\ldots. a_{200} = 100 = \frac{ar(r^{200}-1)}{r^2-1} = 100 \quad \ldots.(2)$$

समीकरण (1) तथा (2) से r = 2

दोनों का योग करने पर

$\Rightarrow a_2 + a_3 + \ldots\ldots\ldots. a_{200} + a_{201} = 300 \quad \Rightarrow \quad r(a_1 + \ldots\ldots\ldots.. a_{200}) = 300$

$$\sum_{n=1}^{200} a_n = \frac{300}{r} = 150$$

5. If यदि $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$, y(1) = 1 and तथा y(x) = e then तब x = ?

(1) $\frac{\sqrt{3}}{2}e$ (2) $\sqrt{3}\,e$ (3) $\sqrt{2}\,e$ (4) $\frac{e}{\sqrt{2}}$

Ans. (2)

Sol. Put y = vx रखने पर

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{vx^2}{x^2+v^2x^2}$$

$$\Rightarrow \quad \frac{1+v^2}{v^3}dv = -\frac{1}{x}dx$$

$$\Rightarrow \quad \int\left(\frac{1}{v^3}+\frac{1}{v}\right)dv = \int\frac{-1}{x}dx$$

$$\Rightarrow \quad \frac{-1}{2}\frac{1}{v^2} + \ell nv = -\ell nx + c$$

$$\Rightarrow \quad -\frac{x^2}{2y^2} = -\ell ny + c$$

When जब x = 1, y = 1 then तब

$$-\frac{1}{2} = c$$

$\Rightarrow \quad x^2 = y^2(1 + 2\ell ny)$

$\Rightarrow \quad x^2 = e^2(3)$

$\Rightarrow \quad x = \pm\sqrt{3}\,e$

So इसलिये $x = \sqrt{3}e$

6. Let probability distribution is

माना प्रायिकता वितरण इस प्रकार है

x_i :	1	2	3	4	5
P_i :	k^2	2k	k	2k	$5k^2$

then value of p(x > 2) is

तब p(x > 2) का मान है

(1) $\frac{7}{12}$ (2) $\frac{1}{36}$ (3) $\frac{1}{6}$ (4) $\frac{23}{36}$

Ans. **(4)**

Sol. $\sum p_i = 1 \Rightarrow 6k^2 + 5k = 1$

$6k^2 + 5k - 1 = 0$

$6k^2 + 6k - k - 1 = 0$

$(6k - 1)(k + 1) = 0 \Rightarrow k = -1$ (rejected अमान्य) ; $k = \frac{1}{6}$

$P(x > 2) = k + 2k + 5k^2$

$= \frac{1}{6} + \frac{2}{6} + \frac{5}{36} = \frac{6+12+5}{36} = \frac{23}{36}$

7. $\int \frac{d\theta}{\cos^2\theta(\sec 2\theta + \tan 2\theta)} = \lambda\tan\theta + 2\log f(x) + c$ then ordered pair $(\lambda, f(x))$ is

$\int \frac{d\theta}{\cos^2\theta(\sec 2\theta + \tan 2\theta)} = \lambda\tan\theta + 2\log f(x) + c$ तब क्रमित युग्म $(\lambda, f(x))$ है–

(1) $(1, 1 + \tan\theta)$ (2) $(1, 1 - \tan\theta)$ (3) $(-1, 1 + \tan\theta)$ (4) $(-1, 1 - \tan\theta)$

Ans. (3)

Sol. $\int \frac{\sec^2\theta}{\frac{1+\tan^2\theta}{1-\tan^2\theta} + \frac{2\tan\theta}{1-\tan^2\theta}} d\theta$

$= \int \frac{\sec^2\theta(1-\tan^2\theta)}{(1+\tan\theta)^2} d\theta$

$= \int \frac{\sec^2\theta(1-\tan\theta)}{1+\tan\theta} d\theta$

$\tan\theta = t \Rightarrow \sec^2\theta\, d\theta = dt$

$= \int \left(\frac{1-t}{1+t}\right) dt = \int \left(-1 + \frac{2}{1+t}\right) dt$

$= -t + 2\log(1+t) + C$

$= -\tan\theta + 2\log(1 + \tan\theta) + C$

$\Rightarrow \quad \lambda = -1$ and $f(x) = 1 + \tan\theta$

8. If $p \to (p \wedge \sim q)$ is false. Truth value of p & q will be

यदि $p \to (p \wedge \sim q)$ असत्य है, तब p & q का सत्यतता मान होगा

(1) TT (2) TF (3) F T (4) F F

Ans. (1)

Sol.

p	q	~q	p∧~q	p→(p∧~q)
T	T	F	F	F
T	F	T	T	T
F	T	F	F	T
F	F	T	F	T

9. If $\lim_{x\to 0} x\left[\frac{4}{x}\right]$ = A then the value of x at which f(x) = [x^2] sinπx is discontinuous

(where [.] denotes greatest integer function)

यदि $\lim_{x\to 0} x\left[\frac{4}{x}\right]$ = A तब x का मान जिसके लिये f(x) = [x^2] sinπx असतत् हो

(जहाँ [.]महत्तम पूर्णांक फलन को प्रदर्शित करता है)

(1) $\sqrt{A+1}$ (2) $\sqrt{A+21}$ (3) $\sqrt{A}$ (4) $\sqrt{A+5}$

Ans. (1)

Sol. $\lim_{x\to 0} x\left(\frac{4}{x}-\left\{\frac{4}{x}\right\}\right) = A \Rightarrow \lim_{x\to 0} 4 - x\left\{\frac{4}{x}\right\} = A \Rightarrow 4 - 0 = A$

check when जाँचने पर

(A) x = $\sqrt{A+1}$ $\Rightarrow$ x = $\sqrt{5}$ $\Rightarrow$ discontinuous असतत्

(B) x = $\sqrt{A+21}$ $\Rightarrow$ x = 5 $\Rightarrow$ continuous सतत्

(C) x = $\sqrt{A}$ $\Rightarrow$ x = 2 $\Rightarrow$ continuous सतत्

(D) x = $\sqrt{A+5}$ $\Rightarrow$ x = 3 $\Rightarrow$ continuous सतत्

10. Let one end of focal chord of parabola $y^2 = 8x$ is $\left(\frac{1}{2},-2\right)$, then equation of tangent at other end of this focal chord is

माना परवलय $y^2 = 8x$ की नाभिय जीवा के एक सिरे के निर्देशांक $\left(\frac{1}{2},-2\right)$ है, तो दूसरे सिरे पर खींची गई स्पर्श रेखा का समीकरण ज्ञात कीजिये।

(1) x + 2y + 8 = 0 (2) x + 2y = 8 (3) x – 2y = 8 (4) x – 2y + 8 = 0

Ans. **(4)**

Sol. Let माना $\left(\frac{1}{2},-2\right)$ is $(2t^2, 4t) \Rightarrow t = \frac{-1}{2}$

Parameter of other end of focal chord is 2

$\Rightarrow$ point is (8, 8)

$\Rightarrow$ equation of tangent is 8y – 4(x+8) = 0

$\Rightarrow$ 2y – x = 8

नाभिय जीवा के दूसरे सिरे का प्राचल 2

$\Rightarrow$ अतः अन्य सिरे के निर्देशांक (8, 8) है

$\Rightarrow$ अतः स्पर्श रेखा का समीकरण 8y – 4(x+8) = 0

$\Rightarrow$ 2y – x = 8

11. Let x + 6y = 8 is tangent to standard ellipse where minor axis is $\frac{4}{\sqrt{3}}$, then eccentricity of ellipse is

माना x + 6y = 8 मानक दीर्घवृत्त की स्पर्श रेखा का समीकरण है जिसकी लघु अक्ष की लंबाई $\frac{4}{\sqrt{3}}$ है, तो दीर्घवृत्त की उत्केन्द्रता है

(1) $\sqrt{\frac{5}{6}}$ (2) $\sqrt{\frac{11}{12}}$ (3) $\frac{1}{3}\sqrt{\frac{11}{3}}$ (4) $\frac{1}{4}\sqrt{\frac{11}{12}}$

Ans. (2)

Sol. $2b = \frac{4}{\sqrt{3}} \Rightarrow b = \frac{2}{\sqrt{3}}$

Equation of tangent $\equiv y = mx \pm \sqrt{a^2m^2 + b^2}$

स्पर्श रेखा का समीकरण $\equiv y = mx \pm \sqrt{a^2m^2 + b^2}$

comparing with $\equiv y = \frac{-x}{6} + \frac{4}{3}$

$\equiv y = \frac{-x}{6} + \frac{4}{3}$ से तुलना करने पर

$m = \frac{-1}{6}$ and तथा $a^2m^2 + b^2 = \frac{16}{9}$

$\Rightarrow \frac{a^2}{36} + \frac{4}{3} = \frac{16}{9}$

$\Rightarrow \frac{a^2}{36} = \frac{16}{9} - \frac{4}{3} = \frac{4}{9}$

$\Rightarrow a^2 = 16$

$e = \sqrt{1 - \frac{b^2}{a^2}}$

$e = \sqrt{1 - \frac{4}{3 \times 16}} = \sqrt{\frac{11}{12}}$

12. if f(x) and g(x) are continuous functions, fog is identity function, $g'(b) = 5$ and $g(b) = a$ then $f'(a)$ is
यदि f(x) तथा g(x) सतत् फलन है, fog तत्समक फलन है, $g'(b) = 5$ तथा $g(b) = a$ तब $f'(a)$ है–

(1) $\frac{2}{5}$ (2) $\frac{1}{5}$ (3) $\frac{3}{5}$ (4) 5

Ans. **(2)**

Sol. $f(g(x)) = x$

$\Rightarrow f'(g(x)).\, g'(x) = 1$

Put x = b

$\Rightarrow f'(g(b))\, g'(b) = 1$

$\Rightarrow f'(a) \times 5 = 1$

$\Rightarrow f'(a) = \frac{1}{5}$

13. If यदि $7x + 6y - 2z = 0$

$3x + 4y + 2z = 0$

$x - 2y - 6z = 0$ then which option is correct

$x - 2y - 6z = 0$ तब निम्न में से कौनसा विकल्प सत्य है–

(1) no. solution
(2) only trivial solution
(3) Infinite non trivial solution for $x = 2z$
(4) Infinite non trivial solution for $y = 2z$

(1) कोई हल नही
(2) केवल तुच्छ हल
(3) $x = 2z$ के लिए अनन्त अतुच्छ हल
(4) $y = 2z$ के लिये अनन्त अतुच्छ हल

Ans. **(3)**

Sol. $\begin{vmatrix} 7 & 6 & -2 \\ 3 & 4 & 2 \\ 1 & -2 & -6 \end{vmatrix}$

$= 7(-20) - 6(-20) - 2(-10)$

$= -140 + 120 + 20 = 0$

so infinite non-trivial solution exist

अतः अनन्त अतुच्छ हल का अस्तित्व होगा

now equation (1) + 3 equation (3)

अब समीकरण (1) + 3 समीकरण (3)

$10x - 20z = 0$

$x = 2z$

14. Let $x = 2\sin\theta - \sin 2\theta$ and

$y = 2\cos\theta - \cos 2\theta$

find $\frac{d^2y}{dx^2}$ at $\theta = \pi$

माना $x = 2\sin\theta - \sin 2\theta$ तथा

$y = 2\cos\theta - \cos 2\theta$

$\theta = \pi$ पर $\frac{d^2y}{dx^2}$ का मान ज्ञात कीजिये।

(1) $\frac{3}{8}$
(2) $\frac{3}{2}$
(3) $\frac{5}{8}$
(4) $\frac{7}{8}$

Sol. $\frac{dx}{d\theta} = 2\cos\theta - 2\cos 2\theta$

$\frac{dy}{d\theta} = -2\sin\theta + 2\sin 2\theta$

$\therefore \frac{dy}{dx} = \frac{\sin 2\theta - \sin\theta}{\cos\theta - \cos 2\theta}$

$= \frac{2\sin\frac{\theta}{2}.\cos\frac{3\theta}{2}}{2\sin\frac{\theta}{2}\sin\frac{3\theta}{2}} = \cot\frac{3\theta}{2}$

$$\frac{d^2y}{dx^2} = \frac{-3}{2}\text{cosec}^2\frac{3\theta}{2}.\frac{d\theta}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{-\frac{3}{2}\text{cosec}^2\frac{3\theta}{2}}{2(\cos\theta - \cos 2\theta)}$$

$$\frac{d^2y}{dx^2}_{(\theta=\pi)} = -\frac{3}{4(-1-1)} = \frac{3}{8}$$

15. $f(x) : [0, 5] \to R$, $F(x) = \int_0^x x^2\, g(x)$, $f(1) = 3$

$g(x) = \int_1^x f(t)\,dt$ then correct choice is

(1) F(x) has local minimum at x = 1
(2) F(x) has local maximum at x = 1
(3) F(x) has point of inflection at x = 1
(4) F(x) has no critical point

$f(x) : [0, 5] \to R$, $F(x) = \int_0^x x^2\, g(x)$, $f(1) = 3$

$g(x) = \int_1^x f(t)\,dt$ तब सही विकल्प है–

(1) F(x), x = 1 पर स्थानिय निम्निष्ठ है
(2) F(x), x = 1 पर स्थानीय उच्चिष्ठ है
(3) F(x), x = 1 पर नति परिवर्तन बिंदु रखता है
(4) F(x) का कोई क्रांतिक बिंदु नही है

Ans. **(1)**

Sol. $F'(x) = x^2 g(x)$

$\Rightarrow F'(1) = 1.g(1) = 0$ .... (1) $(\because g(1) = 0)$

Now अब $F''(x) = 2xg(x) + x^2g'(x)$

$\Rightarrow F''(x) = 2xg(x) + x^2f(x)$ $(\because g'(x) = f(x))$

$\Rightarrow F''(1) = 0 + 1 \times 3$

$\Rightarrow F''(1) = 3$ .... (2)

From (1) and (2) F(x) has local minimum at x = 1

(1) तथा (2) से F(x), x = 1 पर स्थानिय निम्निष्ठ है

16. Let both root of equation $ax^2 - 2bx + 5 = 0$ are α and root of equation $x^2 - 2bx - 10 = 0$ are α and β. Find the value of $\alpha^2 + \beta^2$

माना समीकरण $ax^2 - 2bx + 5 = 0$ के दोनों मूल α हैं एवं समीकरण $x^2 - 2bx - 10 = 0$ के मूल α तथा β हैं, तब $\alpha^2 + \beta^2$ का मान ज्ञात करो।

(1) 20 (2) 25 (3) 15 (4) 30

Ans. **(2)**

Sol. $2\alpha = \frac{2b}{a} \Rightarrow \alpha = \frac{b}{a}$ and $\alpha^2 = \frac{5}{a}$ $\Rightarrow$ $\frac{b^2}{a^2} = \frac{5}{a}$

$\Rightarrow b^2 = 5a$ ..... (i) $(a \neq 0)$

$\alpha + \beta = 2b$ ....(ii)

and तथा $\alpha\beta = -10$ ....(iii)

$\alpha = \frac{b}{a}$ is also root of $x^2 - 2bx - 10 = 0$

$\alpha = \frac{b}{a}$, $x^2 - 2bx - 10 = 0$ का मूल है

$\Rightarrow b^2 - 2ab^2 - 10a^2 = 0$

by (i) से $\Rightarrow 5a - 10a^2 - 10a^2 = 0$

$\Rightarrow 20a^2 = 5a$

$\Rightarrow a = \frac{1}{4}$ and तथा $b^2 = \frac{5}{4}$

$\alpha^2 = 20$ and तथा $\beta^2 = 5$

Now अब $\alpha^2 + \beta^2$

$= 5 + 20$

$= 25$

17. Let $A = \{x : |x| < 2\}$ and $B = \{x : |x - 2| \geq 3\}$ then

माना $A = \{x : |x| < 2\}$ तथा $B = \{x : |x - 2| \geq 3\}$ तब

(1) $A \cap B = [-2, -1]$ (2) $A \cup B = R - (2, 5)$

(3) $A - B = [-1, 2)$ (4) $B - A = R - (-2, 5)$

Ans. **(4)**

Sol. $A = \{ x : x \in (-2, 2) \}$

$B = \{ x : x \in (-\infty, -1] \cup [5, \infty) \}$

$A \cap B = \{ x : x \in (-2, -1] \}$

$A \cup B = \{ x : x \in (-\infty, 2) \cup [5, \infty) \}$

$A - B = \{ x : x \in (-1, 2) \}$

$B - A = \{ x : x \in (-\infty, -2] \cup [5, \infty) \}$

18. Let $x = \sum_{n=0}^{\infty} (-1)^n (\tan\theta)^{2n}$ and $y = \sum_{n=0}^{\infty} (\cos\theta)^{2n}$ where $\theta \in (0, \pi/4)$, then

माना $x = \sum_{n=0}^{\infty} (-1)^n (\tan\theta)^{2n}$ तथा $y = \sum_{n=0}^{\infty} (\cos\theta)^{2n}$ जहाँ $\theta \in (0, \pi/4)$, तब

(1) $x (y + 1) = 1$ (2) $y (1 - x) = 1$ (3) $y (x - 1) = 1$ (4) $y (1 + x) = 1$

Ans. **(2)**

Sol. $y = 1 + \cos^2\theta + \cos^4\theta + ...$

$\Rightarrow y = \frac{1}{1 - \cos^2\theta} \Rightarrow \frac{1}{y} = \sin^2\theta$

$x = \frac{1}{1 - (-\tan^2\theta)} = \frac{1}{\sec^2\theta}$

$\Rightarrow \cos^2\theta = x$

$\Rightarrow \frac{1}{y} + x = 1$

19. Let the distance between plane passing through lines $\frac{x+1}{2}=\frac{y-3}{2}=\frac{z+1}{8}=8$ and $\frac{x+3}{2}=\frac{y+2}{1}=\frac{z-1}{\lambda}$ and plane $23x-10y-2z+48=0$ is $\frac{k}{\sqrt{633}}$ then k is equal to

माना सरल रेखाओं $\frac{x+1}{2}=\frac{y-3}{2}=\frac{z+1}{8}=8$ तथा $\frac{x+3}{2}=\frac{y+2}{1}=\frac{z-1}{\lambda}$ से गुजरने वाले समतल तथा समतल $23x-10y-2z+48=0$ के मध्य दूरी $\frac{k}{\sqrt{633}}$ है तब k का मान है–

(1) 1 (2) 2 (3) 3 (4) 4

Ans. **(3)**

Sol. Lines must be intersecting

$\Rightarrow (2s-1, 3s+3, 8s-1) = (2t-3, t-2, \lambda t+1)$

$2s-1 = 2t-3,\ 3s+3 = t-2,\ 8s-1 = \lambda t+1 \quad \Rightarrow t=-1, s=-2, \lambda=18$

distance of plane contains given lines from given plane is same as distance between point (–3, –2,1) from given plane.

Required distance equal to $\frac{|-69+20-2+48|}{\sqrt{529+100+4}}=\frac{3}{\sqrt{633}}=\frac{k}{\sqrt{633}} \Rightarrow k=3$

Sol. रेखाऐं प्रतिच्छेदी होगी

$\Rightarrow (2s-1, 3s+3, 8s-1) = (2t-3, t-2, \lambda t+1)$

$2s-1 = 2t-3,\ 3s+3 = t-2,\ 8s-1 = \lambda t+1 \quad \Rightarrow t=-1, s=-2, \lambda=18$

रेखाओं का समाहित करने वाले समतल की दिये हुये समतल से दूरी = बिंदु (–3, –2,1) से दिये गये समतल की दूरी

अभीष्ट दूरी = $\frac{|-69+20-2+48|}{\sqrt{529+100+4}}=\frac{3}{\sqrt{633}}=\frac{k}{\sqrt{633}} \Rightarrow k=3$

SECTION – 2

- This section contains **FOUR (04)** questions. The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto one digit.
- If the numerical value has more than two decimal places **truncate/round-off** the value upto **TWO** decimal places.
  - Full Marks : **+4** If ONLY the correct option is chosen.
  - Zero Marks : **0** In all other cases

खंड 2

- इस खंड में **चार (04)** प्रश्न है। प्रत्येक प्रश्न का उत्तर संख्यात्मक मान **(NUMERICAL VALUE)** हैं, जो द्वि–अंकीय पूर्णांक तथा दशमलव एकल–अंकन में है।
- यदि संख्यात्मक मान में दो से अधिक दशमलव स्थान है , तो संख्यात्मक मान को दशमलव के दो स्थानों तक **ट्रंकेट/राउंड ऑफ (truncate/round-off)** करें।
- अंकन योजना :
  - पूर्ण अंक : **+4** यदि सिर्फ सही विकल्प ही चुना गया है।
  - शून्य अंक : **0** अन्य सभी परिस्थितियों में।

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20. If ${}^{25}C_0 + 5\ {}^{25}C_1 + 9\ {}^{25}C_2 \ldots\ldots\ldots 101\ {}^{25}C_{25} = 2^{25}k$ find k = ?

यदि ${}^{25}C_0 + 5\ {}^{25}C_1 + 9\ {}^{25}C_2 \ldots\ldots\ldots 101\ {}^{25}C_{25} = 2^{25}k$ तब k = ?

Ans. **(51)**

Sol. $\sum_{r=0}^{25}(4r+1)^{25}C_r = 4\sum_{r=0}^{25} r.^{25}C_r + \sum_{r=0}^{25} {}^{25}C_r$

$= 4\sum_{r=1}^{25} r\times\frac{25}{r}\,{}^{24}C_{r-1} + 2^{25} = 100\sum_{r=1}^{25} {}^{24}C_{r-1} + 2^{25}$

$= 100.2^{24} + 2^{25} = 2^{25}(50+1) = 51.2^{25}$

So इसलिये k = 51

21. Let circles $(x-0)^2 + (y-4)^2 = k$ and $(x-3)^2 + (y-0)^2 = 1^2$ touches each other than find the maximum value of 'k'

माना वृत्त $(x-0)^2 + (y-4)^2 = k$ तथा $(x-3)^2 + (y-0)^2 = 1^2$ एक दूसरे को स्पर्श करते है तब 'k' का अधिकतम मान ज्ञात कीजिये।

Ans. **36.00**

Sol. Two circles touches each other if $C_1C_2 = |r_1 \pm r_2|$

Distance between $C_2(3, 0)$ and $C_1(0, 4)$ is either $\sqrt{k}+1$ or $\left|\sqrt{k}-1\right|$ $(C_1C_2 = 5)$

$\Rightarrow \sqrt{k}+1 = 5$ or $\left|\sqrt{k}-1\right| = 5$ $\Rightarrow k = 16$ or $k = 36$ $\Rightarrow$ maximum value of k is 36

Sol. दो वृत्त एक दूसरे को स्पर्श करते हैं यदि $C_1C_2 = |r_1 \pm r_2|$

$C_2(3, 0)$ तथा $C_1(0, 4)$ के मध्य दूरी $\sqrt{k}+1$ या $\left|\sqrt{k}-1\right|$ है $(C_1C_2 = 5)$

$\Rightarrow \sqrt{k}+1 = 5$ or $\left|\sqrt{k}-1\right| = 5$ $\Rightarrow k = 16$ or $k = 36$ $\Rightarrow$ k का अधिकतम मान 36 है

22. Let $|\vec{a}| = \sqrt{3},\ |\vec{b}| = 5,\ \vec{b}.\vec{c} = 10$ angle between $\vec{b}\ \&\ \vec{c}$ equal to $\frac{\pi}{3}$

If $\vec{a}$ is perpendicular to $\vec{b}\times\vec{c}$ then find the value of $\left|\vec{a}\times(\vec{b}\times\vec{c})\right|$

माना $|\vec{a}| = \sqrt{3},\ |\vec{b}| = 5,\ \vec{b}.\vec{c} = 10,\ \vec{b}\ \&\ \vec{c}$ के बीच का कोण $\frac{\pi}{3}$ के बराबर है।

यदि $\vec{a}$, $\vec{b}\times\vec{c}$ के लम्बवत् है तब $\left|\vec{a}\times(\vec{b}\times\vec{c})\right|$ का मान ज्ञात कीजिये।

Ans. **30**

Sol. $\vec{b}.\vec{c} = 10 \Rightarrow |\vec{b}|\ |\vec{c}|\ \cos\left(\frac{\pi}{3}\right) = 10 \Rightarrow$ $5.\ |\vec{c}|\ .\ \frac{1}{2} = 10 \Rightarrow$ $|\vec{c}| = 4$

Also तथा, $\vec{a}.(\vec{b}\times\vec{c}) = 0$

$$|\vec{a}\times(\vec{b}\times\vec{c})| = |\vec{a}||\vec{b}\times\vec{c}|\sin\left(\frac{\pi}{2}\right)$$

$$\sqrt{3}\times|\vec{b}||\vec{c}|\sin\frac{\pi}{3}\times 1 = \sqrt{3}\times 5\times 4\times\frac{\sqrt{3}}{2} = 30$$

23. Number of common terms in both sequence 3, 7, 11, ………….407 and 2, 9, 16, ……..905 is

श्रेणियों 3, 7, 11, ………….407 तथा 2, 9, 16, ……..905 में उभयनिष्ठ पदों की संख्या है–

Ans. (14)

Sol. First common term = 23
common difference = 7 × 4 = 28
Last term ≤ 407
$\Rightarrow$ 23 + (n –1) × 28 ≤ 407
$\Rightarrow$ (n –1) × 28 ≤ 384
$\Rightarrow$ n ≤ 13.71 + 1
n ≤ 14.71
So n = 14

Sol. प्रथम उभयनिष्ठ पद = 23
सार्व अंतर = 7 × 4 = 28
अंतिम पद ≤ 407
$\Rightarrow$ 23 + (n –1) × 28 ≤ 407
$\Rightarrow$ (n –1) × 28 ≤ 384
$\Rightarrow$ n ≤ 13.71 + 1
n ≤ 14.71
इसलिये n = 14

24. If minimum value of term free from x for $\left(\frac{x}{\sin\theta}+\frac{1}{x\cos\theta}\right)^{16}$ is L_1 in $\theta \in \left[\frac{\pi}{8},\frac{\pi}{4}\right]$ and L_2 in $\theta \in \left[\frac{\pi}{16},\frac{\pi}{8}\right]$ find $\frac{L_2}{L_1}$

यदि $\left(\frac{x}{\sin\theta}+\frac{1}{x\cos\theta}\right)^{16}$ में x रहित पद का न्यूनतम मान $\theta \in \left[\frac{\pi}{8},\frac{\pi}{4}\right]$ के लिए L_1 है तथा $\theta \in \left[\frac{\pi}{16},\frac{\pi}{8}\right]$ के लिये L_2 है तब $\frac{L_2}{L_1}$ का मान ज्ञात कीजिये।

Ans. 16

Sol. $T_{r+1} = {}^{16}C_r \left(\frac{x}{\sin\theta}\right)^{16-r}\left(\frac{1}{x\cos\theta}\right)^{r}$

for r = 8 term is free from 'x'

$T_9 = {}^{16}C_8 \frac{1}{\sin^8\theta\cos^8\theta}$

$T_9 = {}^{16}C_8 \frac{2^8}{(\sin 2\theta)^8}$

in $\theta \in \left[\frac{\pi}{8},\frac{\pi}{4}\right]$, $L_1 = {}^{16}C_8\, 2^8$ $\because$ {Min value of L_1 at $\theta = \pi/4$}

in $\theta \in \left[\frac{\pi}{16},\frac{\pi}{8}\right]$, $L_2 = {}^{16}C_8 \frac{2^8}{\left(\frac{1}{\sqrt{2}}\right)^8} = {}^{16}C_8 . 2^8.2^4$ {$\because$ min value of L_2 at $\theta = \pi/8$]

$\frac{L_2}{L_1} = \frac{{}^{16}C_8.2^8 2^4}{{}^{16}C_8.2^8} = 16$

Sol. $T_{r+1} = {}^{16}C_r \left(\frac{x}{\sin\theta}\right)^{16-r} \left(\frac{1}{x\cos\theta}\right)^r$

r = 8 से 'x' रहित पद

$T_9 = {}^{16}C_8 \frac{1}{\sin^8\theta\cos^8\theta}$

$T_9 = {}^{16}C_8 \frac{1}{\sin^8\theta\cos^8\theta}$

$T_9 = {}^{16}C_8 \frac{2^8}{(\sin 2\theta)^8}$

$\theta \in \left[\frac{\pi}{8}, \frac{\pi}{4}\right]$ में, $L_1 = {}^{16}C_8\, 2^8$ $\because$ { L_1 का न्यूनतम मान $\theta = \pi/4$ पर}

$\theta \in \left[\frac{\pi}{16}, \frac{\pi}{8}\right]$ में, $L_2 = {}^{16}C_8 \frac{2^8}{\left(\frac{1}{\sqrt{2}}\right)^8} = {}^{16}C_8 \,.\, 2^8.2^4$ { $\because$ L_2 का न्यूनतम मान $\theta = \pi/8$]

$\frac{L_2}{L_1} = \frac{{}^{16}C_8.2^8 2^4}{{}^{16}C_8.2^8} = 16$