FINAL JEE-MAIN EXAMINATION - JANUARY, 2020
Held On Thursday, 9 January 2020
TIME : 9 : 30 AM to 12 : 30 PM

1. Kinetic energy of the particle is E and it's De–Broglie wavelength is λ. On increasing it's KE by ΔE, it's new De–Broglie wavelength becomes $\frac{\lambda}{2}$. Then ΔE is

(1) 3E (2) E (3) 2E (4) 4E

Ans. **(1)**

Sol. $\lambda = \frac{h}{\sqrt{2(KE)m}} \Rightarrow \lambda \propto \frac{1}{\sqrt{KE}}$

$\frac{\lambda}{\lambda/2} = \sqrt{\frac{KE_f}{KE_i}}$

$4KE_i = KE_f$

$\Rightarrow \Delta E = 4KE_i - KE_i = 3KE = 3E$

2. The dimensional formula of $\sqrt{\frac{hc^5}{G}}$ is

(1) $[ML^2T^{-3}]$ (2) $[ML^2T^{-2}]$ (3) $[ML^{-2}T^2]$ (4) $[MLT^{-2}]$

Ans. **(2)**

Sol. $[ML^2T^{-2}]$

$[hc] = [ML^3T^{-2}]$

$[c] = [LT^{-1}]$

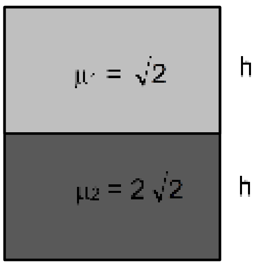
$[G] = [M^{-1}L^3T^{-2}]$

3. Two immiscible liquids of refractive index $\sqrt{2}$ and $2\sqrt{2}$ are filled with equal height h in a vessel. Then apparent depth of bottom surface of the container given that outside medium is air :

(1) $\frac{3\sqrt{2}h}{4}$ (2) $\frac{3h}{4}$ (3) $\frac{3h}{2}$ (4) $\frac{3h}{4\sqrt{2}}$

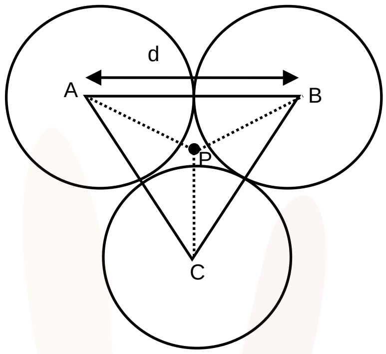
Ans. **(1)**

Sol.



$d = \frac{h}{\sqrt{2}} + \frac{h}{2\sqrt{2}}$ $\Rightarrow d = \frac{h}{\sqrt{2}} \times \frac{3}{2} = \frac{3\sqrt{2}h}{4}$

4. Three identical solid spheres each having mass 'm' and diameter 'd' are touching each other as shown in figure. Calculate ratio of moment of inertia about an axis (perpendicular to plane of paper) passing through point P and B as shown in figure. Given P is centroid of triangle ABC.



(1) 13 / 23 (2) 11/19 (3) 7/9 (4) 13/11

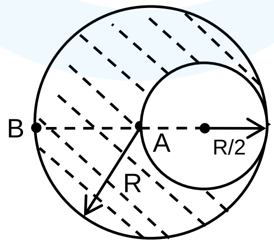
Ans. **(1)**

Sol. M.I about P = $3\left[\frac{2}{5}M\left(\frac{d}{2}\right)^2 + M\left(\frac{d}{\sqrt{3}}\right)^2\right] = \frac{13}{10}Md^2$

M.I about B = $2\left[\frac{2}{5}M\left(\frac{d}{2}\right)^2 + M(d)^2\right] + \frac{2}{5}M\left(\frac{d}{2}\right)^2 = \frac{23}{10}Md^2$

Now ratio = $\frac{13}{23}$

5. A solid sphere having radius R and Uniform charge density ρ has a cavity of radius R/2 as shown in figure. Find the ratio of magnitude of electric field at point A and B i.e. $\left|\frac{E_A}{E_B}\right|$.



(1) $\frac{18}{19}$ (2) $\frac{11}{17}$ (3) $\frac{9}{17}$ (4) $\frac{9}{19}$

Ans. **(3)**

Sol. For a solid sphere

$E = \frac{\rho r}{3\varepsilon_0}$

$E_A = \frac{-\rho R}{2(3\varepsilon_0)}$

$|E_A| = \frac{\rho R}{6\varepsilon_0}$

Electric field at point B = $E_B = E_{1A} + E_{2A}$

E_{1A} = Electric Field Due to solid sphere of radius R = $\frac{\rho R}{3\varepsilon_0}$

E_{2A} = Electric Field Due to solid sphere of radius R/2 (which having charge density $-\rho$)

$= -\frac{KQ' \times 4}{9R^2} = -\frac{\rho R}{54\varepsilon_0}$

$E_B = E_{1A} + E_{2A} = \frac{\rho R}{3\varepsilon_0} - \frac{\rho R}{54\varepsilon_0} = \frac{17\rho R}{54\varepsilon_0}$

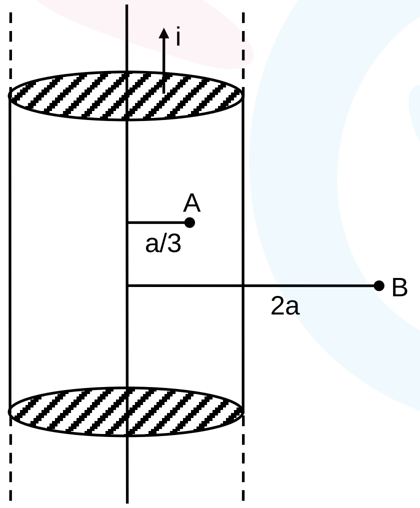
$\frac{|E_A|}{|E_B|} = \frac{9}{17}$

6. Consider an infinitely long current carrying cylindrical straight wire having radius 'a'. Then the ratio of magnetic field at distance $\frac{a}{3}$ and 2a from axis of wire is.

(1) $\frac{3}{5}$ (2) $\frac{2}{3}$ (3) $\frac{1}{2}$ (4) $\frac{4}{3}$

Ans. (2)

Sol.

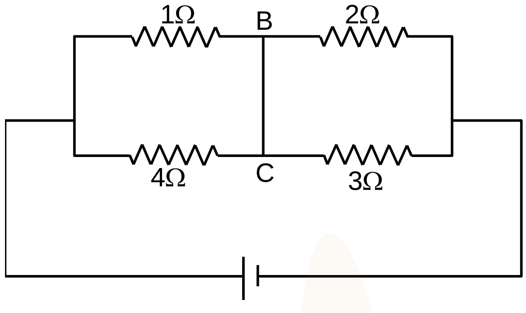


$B_A = \frac{\mu_0 i r}{2\pi a^2} = \frac{\mu_0 i \frac{a}{3}}{2\pi a^2} = \frac{\mu_0 i}{\pi a^2}\frac{a}{6} = \frac{\mu_0 i}{6\pi a}$

$B_B = \frac{\mu_0 i}{2\pi(2a)}$

$\frac{B_A}{B_B} = \frac{4}{6} = \frac{2}{3}$

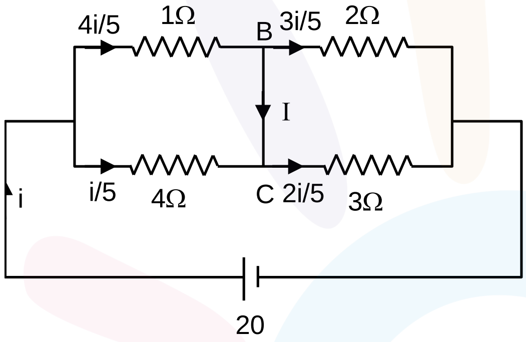
7. Find current in the wire BC.



(1) 1.6A (2) 2A (3) 2.4A (4) 3A

Ans. **(2)**

Sol.



$R_{eff} = \frac{4}{5} + \frac{6}{5} = 2\Omega$

$i = \frac{20}{2}$

$= 10A$

$I = \frac{4i}{5} - \frac{3i}{5} = + \frac{i}{5} = 2A$

8. Two electromagnetic waves are moving in free space whose electric field vectors are given by $\vec{E}_1 = E_0\hat{j}\cos(kx - \omega t)$ & $\vec{E}_2 = E_0\hat{k}\cos(ky - \omega t)$. A charge q is moving with velocity $\vec{v} = 0.8\,c\,\hat{j}$. Find the net Lorentz force on this charge at t = 0 and when it is at origin.

(1) $qE_0(0.4\hat{i} + 0.2\hat{j} + 0.2\hat{k})$ (2) $qE_0(0.8\hat{i} + \hat{j} + 0.2\hat{k})$

(3) $qE_0(0.6\hat{i} + \hat{j} + 0.2\hat{k})$ (4) $qE_0(0.8\hat{i} + \hat{j} + \hat{k})$

Ans. **(2)**

Sol. Magnetic field vectors associated with this electromagnetic wave are given by

$\vec{B}_1 = \frac{E_0}{c}\hat{k}\cos(kx - \omega t)$ & $\vec{B}_2 = \frac{E_0}{c}\hat{i}\cos(ky - \omega t)$

$\vec{F} = q\vec{E} + q(\vec{V} \times \vec{B})$

$= q(\vec{E}_1 + \vec{E}_2) + q(\vec{V} \times (\vec{B}_1 + \vec{B}_2))$

by putting the value of $\vec{E}_1$, $\vec{E}_2$, $\vec{B}_1$ & $\vec{B}_2$

The net Lorentz force on the charged particle is

$$\vec{F} = qE_0\left[0.8\cos(kx - \omega t)\hat{i} + \cos(kx - \omega t)\hat{j} + 0.2\cos(ky - \omega t)\hat{k}\right]$$

at t = 0 and at x = y = 0

$$\vec{F} = qE_0[0.8\hat{i} + \hat{j} + 0.2\hat{k}]$$

9. Two ideal di-atomic gases A and B. A is rigid, B has an extra degree of freedom due to vibration. Mass of A is m and mass of B is $\frac{m}{4}$. The ratio of molar specific heat of A to B at constant volume is :

(1) $\frac{7}{9}$ (2) $\frac{5}{9}$ (3) $\frac{5}{11}$ (4) $\frac{5}{7}$

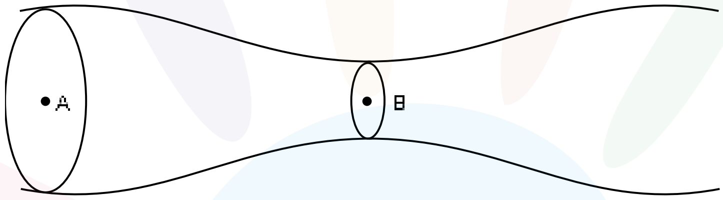
Ans. **(4)**

Sol. Molar heat capacity of A at constant volume = $\frac{5R}{2}$

Molar heat capacity of B at constant volume = $\frac{7R}{2}$

Dividing both, $\frac{(C_V)_A}{(C_V)_B} = \frac{5}{7}$

10. An ideal liquid (water) flowing through a tube of non-uniform cross section area at A and B are 40 cm^2 and 20 cm^2 respectively. If pressure difference between A & B is 700 N/m^2 then volume flow rate is :



(1) 2732 cm^3 /s (2) 2142 cm^3/s (3) 1832 cm^3/s (4) 3218 cm^3/s

Ans. **(1)**

Sol. using equation of continuity

$40\,V_A = 20\,V_B$

$\Rightarrow \quad 2V_A = V_B$

Using Bernoullies equation

$P_A + \frac{1}{2}\rho V_A^2 = P_B + \frac{1}{2}\rho V_B^2$

$\Rightarrow \quad P_A - P_B = \frac{1}{2}\rho(V_B^2 - V_A^2)$

$\Rightarrow \quad \Delta P = \frac{1}{2}1000\left(V_B^2 - \frac{V_B^2}{4}\right)$

$\Rightarrow \quad \Delta P = 500 \times \frac{3V_B^2}{4}$

$\Rightarrow \quad V_B = \sqrt{\frac{(\Delta P)\times 4}{1500}} = \sqrt{\frac{(700)\times 4}{1500}}$ m/s

Volume flow rate = 20 × 100 × V_B = 2732 cm^3/s

11. A screw gauge advances by 3mm in 6 rotations. There are 50 divisions on circular scale. Find least count of screw gauge ?

(1) 0.002 cm (2) 0.001 cm (3) 0.01 cm (4) 0.02 cm

Ans. **(2)**

Sol. Pitch = $\frac{3}{6}$ = 0.5 mm

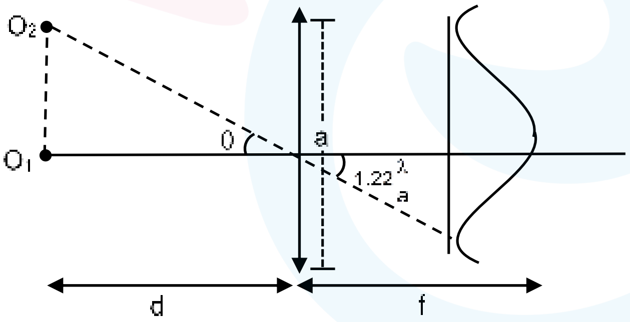
L.C. = $\frac{0.5\text{ mm}}{50} = \frac{1}{100}$ mm = 0.01 mm = 0.001 cm

12. A telescope of aperture diameter 5m is used to observe the moon from the earth. Distance between the moon and earth is 4×10^5 km. Determine the minimum distance between two points on the moon's surface which can be resolved using this telescope. (Wave length of light is 5893 Å.

(1) 60 m (2) 20 m (3) 600 m (4) 200 m

Ans. **(1)**

Sol.



$\theta = 1.22\frac{\lambda}{a}$

distance = $O_1O_2 = d\theta$

$= 1.22\frac{\lambda}{a}d$

distance = $O_1O_2 = \frac{1.22 \times 5893 \times 10^{-10} \times 4 \times 10^8}{5} \approx 57.5$ m

∴ answer from options = 60m

(minimum distance)

13. A particle of mass m is revolving around a planet in a circular orbit of radius R. At the instant the particle has velocity $\vec{V}$, another particle of mass $\frac{m}{2}$ moving at velocity $\frac{\vec{V}}{2}$ collides perfectly in-elastically with the first particle. The new path of the combined body will take is

(1) Circular
(2) Elliptical
(3) Straight line
(4) Fall directly below on the ground

Ans. **(2)**

Sol. Conserving momentum:

$$\frac{m}{2}\frac{v}{2} + mv = \left(m + \frac{m}{2}\right)v_f$$

$$v_f = \frac{5mV}{4 \times \frac{3m}{2}} = \frac{5V}{6}$$

$v_f < v_{orb}$ (= v) thus the combined mass will go on to an elliptical path

14. Two particles of same mass 'm' moving with velocities $\vec{v}_1 = v\hat{i}$ and $\vec{v}_2 = \frac{v}{2}\hat{i} + \frac{v}{2}\hat{j}$ collide in-elastically. Find the loss in kinetic energy.

(1) $\frac{mv^2}{8}$
(2) $\frac{5mv^2}{8}$
(3) $\frac{mv^2}{4}$
(4) $\frac{3mv^2}{8}$

Ans. **(1)**

Sol. Conserving momentum

$$mv\hat{i} + m\left(\frac{v}{2}\hat{i} + \frac{v}{2}\hat{j}\right) = 2m(v_1\hat{i} + v_2\hat{j})$$

on solving

$$v_1 = \frac{3v}{4} \text{ and } v_2 = \frac{v}{4}$$

Change in K.E.

$$\left[\frac{1}{2}mv^2 + \frac{1}{2}m\left(\frac{v}{2}\sqrt{2}\right)^2\right] - \left[\frac{1}{2}(2m)\left(\frac{9v^2}{16} + \frac{v^2}{16}\right)\right]$$

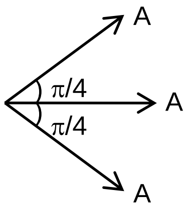
$$= \frac{3mv^2}{4} - \frac{5mv^2}{8} = \frac{mv^2}{8}$$

15. Three waves of same intensity (I_0) having initial phases $0, \frac{\pi}{4}, -\frac{\pi}{4}$ rad respectively interfere at a point. Find the resultant Intensity

(1) I_0
(2) 0
(3) $5.8\ I_0$
(4) $0.2\ I_0$

Ans. **(3)**

Sol.

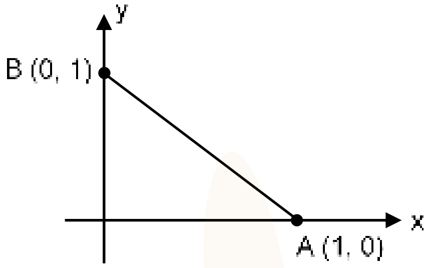


$$A_{res} = (\sqrt{2} + 1)A$$

$$I_{res} = (\sqrt{2} + 1)^2 I_0$$

$$= (3 + 2\sqrt{2})I_0 = 5.8\ I_{0s}$$

16. Particle moves from point A to point B along the line shown in figure under the action of force. $\vec{F} = -x\hat{i} + y\hat{j}$. Determine the work done on the particle by $\vec{F}$ in moving the particle from point A to point B.



(A) 1J (2) $\frac{1}{2}$ J (3) 2J (4) 3J

Ans. **(1)**

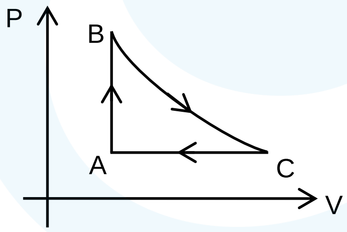
Sol. $W = \int \vec{F}.d\vec{s}$

$= (-x\hat{i} + y\hat{j}).(dx\hat{i} + dy\hat{j})$

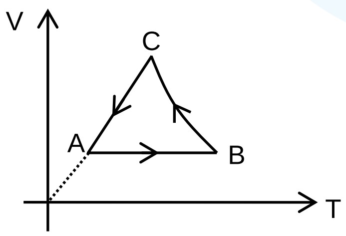
$= \int_1^0 -xdx + \int_0^1 ydy$

$= -\frac{x^2}{2}\Big|_1^0 + \frac{y^2}{2}\Big|_0^1 = \left(0 + \frac{1}{2}\right) + \left(\frac{1}{2}\right) = 1J$

17. For the given P-V graph for an ideal gas, chose the correct V- T graph. Process BC is adiabatic.



(1)



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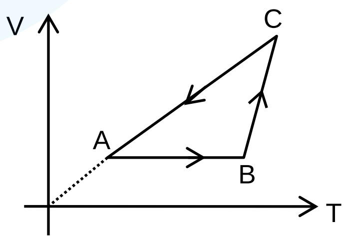

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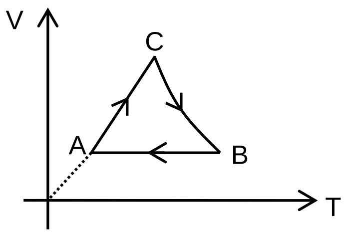

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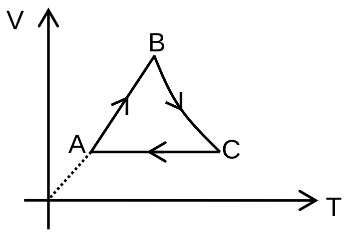

Ans. **(1)**

Sol. For process A – B ; Volume is constant ;
$PV = nRT$; as P increases; T increases
For process B – C ;
PV^{γ} = Constant ;
$\Rightarrow$ $TV^{\gamma-1}$ = Constant
For process C – A ; pressure is constant
$V = kT$

18. Given $\vec{p} = -\hat{i} - 3\hat{j} + 2\hat{k}$ and $\vec{r} = \hat{i} + 3\hat{j} + 5\hat{k}$. Find vector parallel to electric field at position $\vec{r}$
[Note that $\vec{p}.\vec{r} = 0$]

(1) $\hat{i} + 3\hat{j} - 2\hat{k}$ (2) $3\hat{i} + \hat{j} + 2\hat{k}$ (3) $-3\hat{i} - \hat{j} - 2\hat{k}$ (4) $-\hat{i} + 3\hat{j} + 2\hat{k}$

Ans. **(1)**

Sol. **Since** $\vec{p}.\vec{r} = 0$

$\vec{E}$ must be antiparallel to $\vec{p}$

So, $\vec{E} = -\lambda(\vec{p})$

where λ is a arbitrary positive constant

Now $\vec{A} = a\hat{i} + b\hat{j} + c\hat{k}$

$\vec{A} \,||\, \vec{E}$

$$\frac{a}{\lambda} = \frac{b}{3\lambda} = \frac{c}{-2\lambda} = k$$

So $\vec{A} = \lambda k(\hat{i} + 3\hat{j} - 2\hat{k})$

19. **Coming Soon**

20. **Coming Soon**

Numerical Value Type (संख्यात्मक प्रकार)
This section contains **5 Numerical value type questions.**
इस खण्ड में **5** संख्यात्मक प्रकार के प्रश्न हैं।

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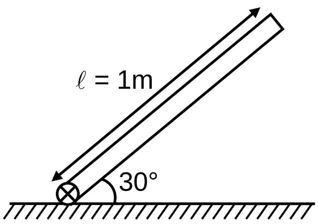
21. A rod of length 1 m is released from rest as shown in the figure below.



If ω of rod is $\sqrt{n}$ at the moment it hits the ground, then find n.

Ans. **15**

Sol. $$mg\frac{\ell}{2}\sin 30^\circ = \frac{1}{2}\frac{m\ell^2}{3}\omega^2$$

Solving

$\omega^2 = 15$

$\omega = \sqrt{15}$

22. If reversible voltage of 200 V is applied across an inductor, current in it reduces from 0.25A to 0A in 0.025ms. Find inductance of inductor (in mH).

Ans. **20**

Sol. $200 = \frac{L(0.25)}{0.025} \times 10^3$

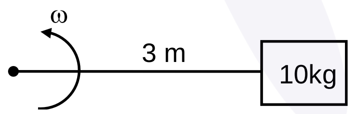
$\therefore \quad L = 200 \times 10^{-4}$ H

$= 20$ mH

23. A wire of length ℓ = 3m and area of cross section 10^{-2}cm^2 and breaking stress 4.8×10^{-7}N/m^2 is attached with block of mass 10kg. Find the maximum possible value of angular velocity with which block can be moved in circle with string fixed at one end.

Ans. 4 rad/s

Sol.



$\frac{T}{A} = \sigma$ ....(1)

$T = m\omega^2\ell$ ....(2)

Solving

$\omega = 4$ rad/s

24. Position of a particle as a function of time is given as $x^2 = at^2 + 2bt + c$, where a, b, c are constants. Acceleration of particle varies with x^{-n} then value of n is.

Ans. **n = 3**

Sol. $x^2 = at^2 + 2bt + c$

$2xv = 2at + 2b$

$xv = at + b$

$v^2 + ax = a$

$$ax = a - \left(\frac{at+b}{x}\right)^2$$

$$a = \frac{a(at^2+2bt+c)-(at+b)^2}{x^3}$$

$$a = \frac{ac-b^2}{x^3}$$

$a \propto x^{-3}$