



FINAL JEE-MAIN EXAMINATION - SEPTEMBER, 2020

Held On Thursday, 3 September 2020 TIME: 9: 00 AM to 12: 00 PM

- 1. If the surface area of a cube is increasing at a rate of 3.6 cm²/sec, retaining its shape; then the rate of change of its volume (in cm³/sec), when the length of a side of the cube is 10 cm, is:
 - (1) 9

∜Saral

- (2) 18
- (3) 10
- (4) 20

Official Ans. by NTA (1)

Sol.
$$\frac{d}{dt}(6a^2) = 3.6 \Rightarrow 12a \frac{da}{dt} = 3.6$$

$$a\frac{da}{dt} = 0.3$$

$$\frac{dv}{dt} = \frac{d}{dt}(a^3) = 3a\left(a\frac{da}{dt}\right)$$
$$= 3 \times 10 \times 0.3 = 9$$

- 2. If the value of the integral $\int_0^{1/2} \frac{x^2}{(1-x^2)^{3/2}} dx$ is
 - $\frac{k}{6}$, then k is equal to:
 - (1) $2\sqrt{3} \pi$
- (2) $3\sqrt{2} + \pi$
- (3) $3\sqrt{2} \pi$
- (4) $2\sqrt{3} + \pi$

Official Ans. by NTA (1)

Sol.
$$\int_0^{1/2} \frac{((x^2 - 1) + 1)}{(1 - x^2)^{3/2}} dx$$

$$\int_0^{1/2} \frac{\mathrm{d}x}{(1-x^2)^{3/2}} - \int_0^{1/2} \frac{\mathrm{d}x}{\sqrt{1-x^2}}$$

$$\int_0^{1/2} \frac{x^{-3}}{(x^{-2}-1)^{3/2}} dx - (\sin^{-1} x)_0^{1/2}$$

Let
$$x^{-2} - 1 = t^2 \Rightarrow x^{-3} dx = -t dt$$

$$\int_{\infty}^{\sqrt{3}} \frac{-t \, dt}{t^3} - \frac{\pi}{6} = \int_{\sqrt{3}}^{\infty} \frac{dt}{t^2} - \frac{\pi}{6} = \frac{1}{\sqrt{3}} - \frac{\pi}{6} = \frac{k}{6}$$

$$k = 2\sqrt{3} - \pi$$

3. Let R₁ and R₂ be two relations defined as follows:

$$R_1 = \{(a, b) \in R^2 : a^2 + b^2 \in Q\}$$
 and

$$R_2 = \{(a, b) \in R^2 : a^2 + b^2 \notin Q\},\$$

where Q is the set of all rational numbers. Then:

- (1) R_2 is transitive but R_1 is not transitive
- (2) R_1 is transitive but R_2 is not transitive
- (3) R_1 and R_2 are both transitive
- (4) Neither R₁ nor R₂ is transitive

Official Ans. by NTA (4)

Sol. Let $a^2 + b^2 \in Q \& b^2 + c^2 \in Q$

eg.
$$a = 2 + \sqrt{3} \& b = 2 - \sqrt{3}$$

$$a^2 + b^2 = 14 \in Q$$

Let
$$c = (1+2\sqrt{3})$$

$$b^2 + c^2 = 20 \in Q$$

But
$$a^2 + c^2 = (2 + \sqrt{3})^2 + (1 + 2\sqrt{3})^2 \notin Q$$

for R₂ Let
$$a^2 = 1$$
, $b^2 = \sqrt{3}$ & $c^2 = 2$

$$a^2 + b^2 \notin Q \& b^2 + c^2 \notin Q$$

But
$$a^2 + c^2 \in Q$$

- **4.** Let the latus ractum of the parabola $y^2 = 4x$ be the common chord to the circles C_1 and C_2 each of them having radius $2\sqrt{5}$. Then, the distance between the centres of the circles C_1 and C_2 is:
 - (1) 8
- (2) $4\sqrt{5}$
- (3) 12
- $(4) \ 8\sqrt{5}$

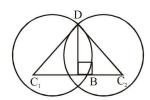
Official Ans. by NTA (1)





Sol. Length of latus rectum = 4

∜Saral



$$DB = 2$$

$$C_1B = \sqrt{(C_1D)^2 - (DB)^2} = 4$$

$$C_1C_2 = 8$$

If $\int \sin^{-1}\left(\sqrt{\frac{x}{1+x}}\right) dx = A(x)\tan^{-1}\left(\sqrt{x}\right) + B(x) + C$,

where C is a constant of integration, then the ordered pair (A(x), B(x)) can be:

(1)
$$\left(x-1,\sqrt{x}\right)$$

(2)
$$\left(x+1,\sqrt{x}\right)$$

(3)
$$(x+1, -\sqrt{x})$$
 (4) $(x-1, -\sqrt{x})$

(4)
$$(x-1, -\sqrt{x})$$

Official Ans. by NTA (3)

Sol. Put $x = \tan^2 \theta \Rightarrow dx = 2 \tan \theta \sec^2 \theta d\theta$

 $\int \theta \cdot (2 \tan \theta \cdot \sec^2 \theta) \, d\theta$

$$\downarrow$$
 \downarrow

II

(By parts)

$$= \theta . \tan^2 \theta - \int \tan^2 \theta \ d\theta$$

$$= \theta \cdot \tan^2 \theta - \int (\sec^2 \theta - 1) d\theta$$

$$= \theta(1 + \tan^2 \theta) - \tan \theta + C$$

$$= \tan^{-1}\left(\sqrt{x}\right)(1+x) - \sqrt{x} + C$$

The probability that a randomly chosen 5-digit 6. number is made from exactly two digits is:

(1)
$$\frac{121}{10^4}$$

(2)
$$\frac{150}{10^4}$$

(3)
$$\frac{135}{10^4}$$

(4)
$$\frac{134}{10^4}$$

Official Ans. by NTA (3)

Sol. First Case: Choose two non-zero digits ⁹C₂

Now, number of 5-digit numbers containing both digits = $2^5 - 2$

Second Case: Choose one non-zero & one zero as digit ⁹C₁.

Number of 5-digit numbers containg one non zero and one zero both = $(2^4 - 1)$

Required prob.

$$=\frac{\left({}^{9}\mathrm{C}_{2}\!\times\!\!\left(2^{5}\!-\!2\right)+{}^{9}\mathrm{C}_{1}\!\times\!\!\left(2^{4}\!-\!1\right)\right)}{9\!\times\!10^{4}}$$

$$=\frac{36\times(32-2)+9\times(16-1)}{9\times10^4}$$

$$=\frac{4\times30+15}{10^4}=\frac{135}{10^4}$$

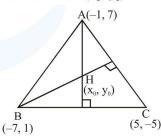
7. If a $\triangle ABC$ has vertices A(-1, 7), B(-7, 1) and C(5, -5), then its orthocentre has coordinates:

$$(2)\left(-\frac{3}{5},\frac{3}{5}\right)$$

$$(4) \left(\frac{3}{5}, -\frac{3}{5}\right)$$

Official Ans. by NTA (3)

Sol. Let orthocentre is $H(x_0, y_0)$



$$m_{AH}.m_{BC} = -1$$

$$\Rightarrow \left(\frac{y_0 - 7}{x_0 + 1}\right) \left(\frac{1 + 5}{-7 - 5}\right) = -1$$

$$\Rightarrow$$
 $2x_0 - y_0 + 9 = 0$ (1)

and
$$m_{BH}.m_{AC} = -1$$





$$\Rightarrow \left(\frac{y_0 - 1}{x_0 + 7}\right) \left(\frac{7 - (-5)}{-1 - 5}\right) = -1$$

$$\Rightarrow$$
 $x_0 - 2y_0 + 9 = 0$ (2)

Solving equation (1) and (2) we get

$$(x_0, y_0) \equiv (-3, 3)$$

If z_1 , z_2 are complex numbers such that $Re(z_1) = |z_1 - 1|, Re(z_2) = |z_2 - 1|$ and

 $arg(z_1 - z_2) = \frac{\pi}{6}$, then $Im(z_1 + z_2)$ is equal to:

(1) $\frac{\sqrt{3}}{2}$

∜Saral

- (3) $\frac{1}{\sqrt{3}}$
- (4) $2\sqrt{3}$

Official Ans. by NTA (4)

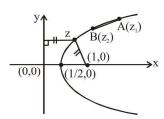
Sol.
$$Re(z) = |z - 1|$$

$$\Rightarrow$$
 $x = \sqrt{(x-1)^2 + (y-0)^2}$ $(x > 0)$

$$\Rightarrow \quad y^2 = 2x - 1 = 4 \cdot \frac{1}{2} \left(x - \frac{1}{2} \right)$$

 \Rightarrow a parabola with focus (1, 0) & directrix as imaginary axis.

$$\therefore$$
 Vertex = $\left(\frac{1}{2}, 0\right)$



 $A(z_1)$ & $B(z_2)$ are two points on it such that

slope of AB =
$$\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

$$(arg (z_1-z_2) = \frac{\pi}{6})$$

for
$$y^2 = 4ax$$

Let
$$A(at_1^2, 2at_1) \& B(at_2^2, 2at_2)$$

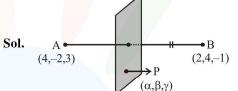
$$m_{AB} = \frac{2}{t_1 + t_2} = \frac{4a}{y_1 + y_2} = \frac{1}{\sqrt{3}}$$

Here
$$a = \frac{1}{2}$$

$$\Rightarrow$$
 $y_1 + y_2 = 4a\sqrt{3} = 2\sqrt{3}$

- The plane which bisects the line joining the points (4, -2, 3) and (2, 4, -1) at right angles also passes through the point:
 - (1) (4, 0, -1)
- (2) (4, 0, 1)
- (3) (0, 1, -1)
- (4) (0, -1, 1)

Official Ans. by NTA (1)



$$PA = PB$$

$$\Rightarrow$$
 PA² = PB²

$$\Rightarrow \quad (\alpha-4)^2+(\beta+2)^2+(\gamma-3)^2$$

$$= (\alpha - 2)^2 + (\beta - 4)^2 + (\gamma + 1)^2$$

$$\Rightarrow$$
 $-4\alpha + 12\beta - 8\gamma = -8$

$$\Rightarrow$$
 2x - 6y + 4z = 4

- $\lim_{x \to a} \frac{(a+2x)^{\frac{1}{3}} (3x)^{\frac{1}{3}}}{(3a+x)^{\frac{1}{3}} (4x)^{\frac{1}{3}}} (a \neq 0) \text{ is equal to :}$
 - (1) $\left(\frac{2}{3}\right) \left(\frac{2}{9}\right)^{\frac{1}{3}}$ (2) $\left(\frac{2}{3}\right)^{\frac{4}{3}}$

 - (3) $\left(\frac{2}{9}\right)^{\frac{4}{3}}$ (4) $\left(\frac{2}{9}\right)\left(\frac{2}{3}\right)^{\frac{1}{3}}$

Official Ans. by NTA (1)





Sol. Required limit

∜Saral

$$L = \lim_{h \to 0} \frac{(a + 2(a + h))^{1/3} - (3(a + h))^{1/3}}{(3a + a + h)^{1/3} - (4(a + h))^{1/3}}$$

$$= \lim_{h \to 0} \frac{(3a)^{1/3} \left(1 + \frac{2h}{3a}\right)^{1/3} - (3a)^{1/3} \left(1 + \frac{h}{a}\right)^{1/3}}{(4a)^{1/3} \left(1 + \frac{h}{4a}\right)^{1/3} - (4a)^{1/3} \left(1 + \frac{h}{a}\right)^{1/3}}$$

$$= \lim_{h \to 0} \left(\frac{3^{1/3}}{4^{1/3}} \right) \left[\frac{\left(1 + \frac{2h}{9a} \right) - \left(1 + \frac{h}{3a} \right)}{\left(1 + \frac{h}{12a} \right) - \left(1 + \frac{h}{3a} \right)} \right]$$

$$= \left(\frac{3}{4}\right)^{1/3} \frac{\left(\frac{2}{9} - \frac{1}{3}\right)}{\left(\frac{1}{12} - \frac{1}{3}\right)} = \left(\frac{3}{4}\right)^{1/3} \left(\frac{8 - 12}{3 - 12}\right)$$

$$= \left(\frac{3}{4}\right)^{1/3} \left(\frac{-4}{-9}\right) = \frac{4^{1-\frac{1}{3}}}{3^{2-\frac{1}{3}}} = \frac{4^{2/3}}{3^{5/3}}$$

$$= \frac{(8 \times 2)^{1/3}}{(27 \times 9)^{1/3}} = \frac{2}{3} \left(\frac{2}{9}\right)^{1/3}$$

Let A be a 3×3 matrix such that

$$adj A = \begin{vmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{vmatrix} and$$

If $|A| = \lambda$ and $|(B^{-1})^T| = \mu$, then the ordered pair, $(|\lambda|, \mu)$ is equal to :

$$(1)$$
 $\left(9,\frac{1}{9}\right)$

(1)
$$\left(9, \frac{1}{9}\right)$$
 (2) $\left(9, \frac{1}{81}\right)$

$$(3)\left(3,\frac{1}{81}\right)$$

Official Ans. by NTA (3)

Sol.
$$C = adj A = \begin{vmatrix} +2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{vmatrix}$$

$$|C| = |adj A| = +2(0 + 4) + 1.(1 - 2) + 1.(2, 4)$$

= +8 - 1 + 2
 $|adj A| = |A|^2 = 9 = 9$
 $\lambda = |A| = \pm 3$

$$|\lambda| = 3$$

$$B = adj C$$

$$|B| = |adj C| = |C|^2 = 81$$

$$|(\mathbf{B}^{-1})^{\mathrm{T}}| = |\mathbf{B}|^{-1} = \frac{1}{81}$$

$$(|\lambda|, \mu) = \left(3, \frac{1}{81}\right)$$

12. Suppose f(x) is a polynomial of degree four, having critical points at -1, 0, 1. If $T = \{x \in R | f(x) = f(0)\}, \text{ then the sum of squares}$ of all the elements of T is:

Official Ans. by NTA (3)

Sol.
$$f'(x) = x(x+1)(x-1) = x^3 - x$$

$$\int df(x) = \int x^3 - x \, dx$$

$$f(x) = \frac{x^4}{4} - \frac{x^2}{2} + C$$

$$f(x) = f(0)$$

$$\frac{x^4}{4} - \frac{x^2}{2} = 0$$

$$x^2 (x^2 - 2) = 0$$

$$x = 0, 0, \sqrt{2}, -\sqrt{2}$$

$$x_1^2 + x_2^2 + x_3^2 = 0 + 2 + 2 = 4$$





Let a, b, $c \in R$ be such that $a^2 + b^2 + c^2 = 1$.

If a cos
$$\theta = b \cos \left(\theta + \frac{2\pi}{3}\right) = \cos \left(\theta + \frac{4\pi}{3}\right)$$
,

where $\theta = \frac{\pi}{9}$, then the angle between the

vectors $a\hat{i} + b\hat{j} + c\hat{k}$ and $b\hat{i} + c\hat{j} + a\hat{k}$ is:

 $(1) \frac{\pi}{2}$

∜Saral

- (2) 0
- (3) $\frac{\pi}{0}$

Official Ans. by NTA (1)

Sol. $\cos \phi = \frac{\overline{p}.\overline{q}}{|\overline{p}||\overline{q}|} = \frac{ab + bc + ca}{a^2 + b^2 + c^2} = \frac{\sum ab}{1}$

$$=abc\bigg(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\bigg)$$

$$= \frac{abc}{\lambda} \left(\cos \theta + \frac{\cos \left(\theta + \frac{2\pi}{3} \right) + \cos \left(\theta + \frac{4\pi}{3} \right) \right)$$

$$= \frac{abc}{\lambda} \left(\cos + 2\cos(\theta + \pi)\cos\frac{\pi}{3} \right)$$

$$=\frac{abc}{\lambda}(\cos\theta-\cos\theta)=0$$

$$\phi = \frac{\pi}{2}$$

14. If the sum of the series

 $20+19\frac{3}{5}+19\frac{1}{5}+18\frac{4}{5}+...$ upto nth term is 488

and the nth term is negative, then:

- (1) n^{th} term is $-4\frac{2}{5}$ (2) n = 41
- (3) n^{th} term is -4
- (4) n = 60

Official Ans. by NTA (3)

Sol. $S = \frac{100}{5} + \frac{98}{5} + \frac{96}{5} + \frac{94}{5} + \dots n$

$$S_n = \frac{n}{2} \left(2 \times \frac{100}{5} + (n-1) \left(-\frac{2}{5} \right) \right) = 188$$

$$n(100 - n + 1) = 488 \times 5$$

$$n^2 - 101n + 488 \times 5 = 0$$

$$n = 61, 40$$

$$T_n = a + (n-1)d = \frac{100}{5} - \frac{2}{5} \times 60$$

$$=20-24=-4$$

Let x_i ($1 \le i \le 10$) be ten observations of a random

variable X. If
$$\sum_{i=1}^{10} (x_i - p) = 3$$
 and $\sum_{i=1}^{10} (x_i - p)^2 = 9$

where $0 \neq p \in R$, then the standard deviation of these observations is:

- (1) $\sqrt{\frac{3}{5}}$
- (2) $\frac{7}{10}$

Official Ans. by NTA (3)

Sol. Variance = $\frac{\sum (x_i - p)^2}{n} - \left(\frac{\sum (x_i - p)}{n}\right)^2$

$$=\frac{9}{10}-\left(\frac{3}{10}\right)^2=\frac{81}{100}$$

S.D. =
$$\frac{9}{10}$$

- **16.** If $x^3dy + xy dx = x^2 dy + 2y dx$; y(2) = e and x > 1, then y(4) is equal to :
 - (1) $\frac{3}{2} + \sqrt{e}$ (2) $\frac{3}{2}\sqrt{e}$
 - (3) $\frac{1}{2} + \sqrt{e}$ (4) $\frac{\sqrt{e}}{2}$

Official Ans. by NTA (2)





Sol.
$$x^3 dy + xy dx = x^2 dy + 2y dx$$

∜Saral

$$\Rightarrow dy(x^3 - x^2) = dx (2y - xy)$$

$$\Rightarrow \qquad -\int \frac{1}{y} dy = \int \frac{x-2}{x^2(x-1)} dx$$

$$\Rightarrow \qquad -\ell ny = \int \left(\frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x-1)}\right) dx$$

Where A = 1, B = +2, C = -1

$$\Rightarrow \qquad -\ell ny = \ell n \, x - \frac{2}{x} - \ell n \, (x - 1) + \lambda$$

$$\Rightarrow$$
 y(2) = e

$$\Rightarrow$$
 $-1 = \ell n \ 2 - 1 - 0 + \lambda$

$$\lambda = - \ln 2$$

$$\Rightarrow \qquad \ell n \, y = - \ \ell n x + \frac{2}{x} + \ell n (x - 1) + \ell n 2$$

Now put x = 4 in equation

$$\Rightarrow \qquad \ell n y = -\ell n 4 + \frac{1}{2} + \ell n 3 + \ell n 2$$

$$\Rightarrow \qquad \ell n y = \ell n \left(\frac{3}{2}\right) + \frac{1}{2} \ell n e$$

$$\Rightarrow y = \frac{3}{2}\sqrt{e}$$

17. Let e_1 and e_2 be the eccentricities of the ellipse,

$$\frac{x^2}{25} + \frac{y^2}{h^2} = 1(b < 5)$$
 and the hyperbola,

$$\frac{x^2}{16} - \frac{y^2}{h^2} = 1$$
 respectively satisfying $e_1 e_2 = 1$. If

 α and β are the distances between the foci of the ellipse and the foci of the hyperbola respectively, then the ordered pair (α, β) is equal to:

(3)
$$\left(\frac{20}{3}, 12\right)$$
 (4) $\left(\frac{24}{5}, 10\right)$

$$(4) \left(\frac{24}{5}, 10\right)$$

Official Ans. by NTA (1)

Sol. For ellipse
$$\frac{x^2}{25} + \frac{y^2}{h^2} = 1$$
 (b < 5)

Let e₁ is eccentricity of ellipse

$$b^2 = 25 (1 - e_1^2) \dots (1)$$

Again for hyperbola

$$\frac{x^2}{16} - \frac{y^2}{b^2} = 1$$

Let e₂ is eccentricity of hyperbola.

$$b^2 = 16(e_2^2 - 1) \dots (2)$$

by (1) & (2)

$$25(1 - e_1^2) = 16(e_2^2 - 1)$$

Now $e_1.e_2 = 1$ (given)

$$\therefore \quad 25(1 - e_1^2) = 16 \left(\frac{1 - e_1^2}{e_1^2} \right)$$

or
$$e_1 = \frac{4}{5}$$
 : $e_2 = \frac{5}{4}$

Now distance between foci is 2ae

$$\therefore \text{ distance for ellipse} = 2 \times 5 \times \frac{4}{5} = 8 = \alpha$$

distance for hyperbola = $2 \times 4 \times \frac{5}{4} = 10 = \beta$

$$(\alpha, \beta) \equiv (8, 10)$$

18. The set of all real values of λ for which the quadratic equations,

> $(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$ always have exactly one root in the interval (0, 1) is:

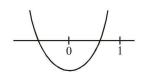
$$(1)(-3,-1)$$

Official Ans. by NTA (2)

Sol. If exactly one root in (0, 1) then







$$\Rightarrow$$
 f(0).f(1) < 0

∜Saral

$$\Rightarrow$$
 2($\lambda^2 - 4\lambda + 3$) < 0

$$\Rightarrow$$
 1 < λ < 3

Now for
$$\lambda = 1, 2x^2 - 4x + 2 = 0$$

$$(x-1)^2 = 0, x = 1, 1$$

So both roots doesn't lie between (0, 1)

$$\lambda \neq 1$$

Again for
$$\lambda = 3$$

$$10x^2 - 12x + 2 = 0$$

$$\Rightarrow$$
 $x = 1, \frac{1}{5}$

so if one root is 1 then second root lie between (0, 1) so $\lambda = 3$ is correct.

$$\lambda \in (1, 3].$$

19. If the term independent of x in the expansion of

$$\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$$
 is k, then 18 k is equal to :

- (1)9
- (2) 11
- (3) 5
- (4) 7

Official Ans. by NTA (4)

Sol.
$$T_{r+1} = {}^{9}C_{r} \left(\frac{3}{2}x^{2}\right)^{9-r} \left(-\frac{1}{3x}\right)^{r}$$

$$T_{r+1} = {}^{9}C_{r} \left(\frac{3}{2}\right)^{9-r} \left(-\frac{1}{3}\right)^{r} x^{18-3r}$$

For independent of x

$$18 - 3r = 0, r = 6$$

$$T_7 = {}^{9}C_6 \left(\frac{3}{2}\right)^3 \left(-\frac{1}{3}\right)^6 = \frac{21}{54} = k$$

$$\therefore 18k = \frac{21}{54} \times 18 = 7$$

- **20.** Let p, q, r be three statements such that the truth value of $(p \land q) \rightarrow (\neg q \lor r)$ is F. Then the truth values of p, q, r are respectively:
 - (1) T, F, T
- (2) F, T, F
- (3) T, T, F
- (4) T, T, T

Official Ans. by NTA (3)

Sol.
$$(p \land q) \rightarrow (\neg q \lor r) = false$$

when
$$(p \land q) = T$$

and
$$(\sim q \vee r) = F$$

So
$$(p \land q) = T$$
 is possible when $p = q = true$

$$\therefore$$
 ~q = False (q = true)

So $(\neg q \lor r)$ = False is possible if r is false

$$\therefore$$
 p = T, q = T, r = F

21. If m arithmetic means (A.Ms) and three geometric means (G.Ms) are inserted between 3 and 243 such that 4th A.M. is equal to 2nd G.M., then m is equal to ____.

Official Ans. by NTA (39)

$$d = \frac{243 - 3}{m + 1} = \frac{240}{m + 1}$$

Now 3, G₁, G₂, G₃, 243

$$r = \left(\frac{243}{3}\right)^{\frac{1}{3+1}} = 3$$

$$A_4 = G_2$$

$$\Rightarrow$$
 a + 4d = ar²

$$3+4\left(\frac{240}{m+1}\right)=3(3)^2$$

$$m = 39$$





22. If the tangent of the curve, $y = e^x$ at a point (c, e^c) and the normal to the parabola, $y^2 = 4x$ at the point (1, 2) intersect at the same point on the x-axis, then the value of c is _____.

Official Ans. by NTA (4)

Sol.
$$y = e^x \Rightarrow \frac{dy}{dx} = e^x$$

∜Saral

$$m = \left(\frac{dy}{dx}\right)_{(c,e^c)} = e^c$$

 \Rightarrow Tangent at (c, e^c)

$$y - e^c = e^c (x - c)$$

it intersect x-axis

Put
$$y = 0 \Rightarrow x = c - 1$$
(1)

Now
$$y^2 = 4x \Rightarrow \frac{dy}{dx} = \frac{2}{y} \Rightarrow \left(\frac{dy}{dx}\right)_{(1,2)} = 1$$

 \Rightarrow Slope of normal = -1

Equation of normal y - 2 = -1(x - 1)

$$x + y = 3$$
 it intersect x-axis

Put
$$y = 0 \Rightarrow x = 3$$
(2

Points are same

$$\Rightarrow$$
 $x = c - 1 = 3$

$$\rightarrow$$
 $c = 4$

23. Let a plane P contain two lines

$$\vec{r} = \hat{i} + \lambda (\hat{i} + \hat{j}), \lambda \in R$$
 and

$$\vec{r} = -\hat{j} + \mu(\hat{j} - \hat{k}), \mu \in R$$

If $Q(\alpha, \beta, \gamma)$ is the foot of the perpendicular drawn from the point M(1, 0, 1) to P, then $3(\alpha + \beta + \gamma)$ equals _____.

Official Ans. by NTA (5)

Sol. Dr's normal to plane

$$= \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = -\hat{i} + \hat{j} + \hat{k}$$

Equation of plane

$$-1(x-1) + 1(y-0) + 1(z-0) = 0$$

$$x - y - z - 1 = 0 \qquad \dots \dots (1)$$

Now
$$\frac{\alpha-1}{1} = \frac{\beta-0}{-1} = \frac{\gamma-1}{-1} = -\frac{(1-0-1-1)}{3}$$

$$\frac{\alpha - 1}{1} = \frac{\beta}{-1} = \frac{\gamma - 1}{-1} = \frac{1}{3}$$

$$\alpha = \frac{4}{3}, \beta = -\frac{1}{3}, \gamma = \frac{2}{3}$$

$$3(\alpha + \beta + \gamma) = 3\left(\frac{4}{3} - \frac{1}{3} + \frac{2}{3}\right) = 5$$

24. Let S be the set of all integer solutions, (x, y, z), of the system of equations

$$x - 2y + 5z = 0$$

$$-2x + 4y + z = 0$$

$$-7x + 14y + 9z = 0$$

such that $15 \le x^2 + y^2 + z^2 \le 150$. Then, the number of elements in the set S is equal to

Official Ans. by NTA (8)





Sol.
$$\Delta = \begin{vmatrix} 1 & -2 & 5 \\ -2 & 4 & 1 \\ -7 & 14 & 9 \end{vmatrix} = 0$$

∜Saral

Let
$$x = k$$

 \Rightarrow Put in (1) & (2)
 $k - 2y + 5z = 0$
 $-2k + 4y + z = 0$
 $z = 0, y = \frac{k}{2}$

- ∴ x, y, z are integer
- \Rightarrow k is even integer

Now x = k, $y = \frac{k}{2}$, z = 0 put in condition

$$15 \le k^2 + \left(\frac{k}{2}\right)^2 + 0 \le 150$$

$$12 \le k^2 \le 120$$

- \Rightarrow k = ±4, ±6, ±8, ±10
- \Rightarrow Number of element in S = 8.

25. The total number of 3-digit numbers, whose sum of digits is 10, is _____.

Official Ans. by NTA (54)

Sol. Let three digit number is xyz

$$x + y + z = 10$$
; $x \ge 1, y \ge 0 \ z \ge 0 \dots$ (1)

Let
$$T = x - 1 \Rightarrow x = T + 1$$
 where $T \ge 0$

Put in (1)

$$T + y + z = 9$$
; $0 \le T \le 8, 0 \le y, z \le 9$

No. of non negative integral solution

$$= {}^{9+3-1}C_{3-1} - 1$$
 (when T = 9)

$$= 55 - 1 = 54$$