FINAL JEE-MAIN EXAMINATION - SEPTEMBER, 2020

Held On Saturday, 5 September 2020

TIME : 3: 00 PM to 6 : 00 PM

1. If the system of linear equations

$x + y + 3z = 0$

$x + 3y + k^2z = 0$

$3x + y + 3z = 0$

has a non-zero solution (x, y, z) for some $k \in R$, then $x + \left(\frac{y}{z}\right)$ is equal to :

(1) 9 (2) –3

(3) –9 (4) 3

Official Ans. by NTA (2)

Sol. $x + y + 3z = 0$ .....(i)

$x + 3y + k^2z = 0$ .....(ii)

$3x + y + 3z = 0$ .....(iii)

$$\begin{vmatrix} 1 & 1 & 3 \\ 1 & 3 & k^2 \\ 3 & 1 & 3 \end{vmatrix} = 0$$

$\Rightarrow 9 + 3 + 3k^2 - 27 - k^2 - 3 = 0$

$\Rightarrow k^2 = 9$

(i) – (iii) $\Rightarrow -2x = 0 \Rightarrow x = 0$

Now from (i) $\Rightarrow y + 3z = 0$

$\Rightarrow \frac{y}{z} = -3$

$x + \frac{y}{z} = -3$

2. If α and β are the roots of the equation, $7x^2 - 3x - 2 = 0$, then the value of $\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2}$ is equal to :

(1) $\frac{27}{16}$ (2) $\frac{1}{24}$

(3) $\frac{27}{32}$ (4) $\frac{3}{8}$

Official Ans. by NTA (1)

Sol. $7x^2 - 3x - 2 = 0$

$\alpha + \beta = \frac{3}{7}$ $\alpha\beta = \frac{-2}{7}$

$$\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2} = \frac{\alpha+\beta-\alpha\beta(\alpha+\beta)}{1-\alpha^2-\beta^2+\alpha^2\beta^2}$$

$$= \frac{\frac{3}{7}+\frac{2}{7}\left(\frac{3}{7}\right)}{1-(\alpha+\beta)^2+2\alpha\beta+\alpha^2\beta^2} = \frac{27}{16}$$

3. If the sum of the first 20 terms of the series $\log_{(7^{1/2})} x + \log_{(7^{1/3})} x + \log_{(7^{1/4})} x + \ldots$ is 460, then x is equal to :

(1) $7^{46/21}$ (2) $7^{1/2}$

(3) e^2 (4) 7^2

Official Ans. by NTA (4)

Sol. $460 = \log_7 x \cdot (2 + 3 + 4 + \ldots.. + 20 + 21)$

$\Rightarrow 460 = \log_7 x \cdot \left(\frac{21\times 22}{2} - 1\right)$

$\Rightarrow 460 = 230 \cdot \log_7 x$

$\Rightarrow \log_7 x = 2 \Rightarrow x = 49$

4. $$\lim_{x\to 0} \frac{x\left(e^{\left(\sqrt{1+x^2+x^4}-1\right)/x} - 1\right)}{\sqrt{1+x^2+x^4}-1}$$

(1) does not exist. (2) is equal to $\sqrt{e}$.

(3) is equal to 0. (4) is equal to 1.

Official Ans. by NTA (4)

Sol. $$\lim_{x\to 0} \frac{x\left(e^{\left(\sqrt{1+x^2+x^4}-1\right)/x} - 1\right)}{\sqrt{1+x^2+x^4}-1}$$

$\because \lim_{x\to 0} \frac{\sqrt{1+x^2+x^4}-1}{x}$ $\left(\frac{0}{0}\text{ from}\right)$

$$\lim_{x\to 0} \frac{(1+x^2+x^4)-1}{x(\sqrt{1+x^2+x^4}+1}$$

$$\lim_{x\to 0} \frac{x(1+x^2)}{\left(\sqrt{1+x^2+x^4}+1\right)} = 0$$

So $\lim_{x\to 0} \frac{x\left(e^{\left(\frac{\sqrt{1+x^2+x^4}-1}{x}\right)} - 1\right)}{\sqrt{1+x^2+x^4}-1}$ $\left(\frac{0}{0}\text{ from}\right)$

$$\lim_{x\to 0} \frac{e^{\frac{\sqrt{1+x^2+x^4}-1}{x}} - 1}{\left(\frac{\sqrt{1+x^2+x^4}-1}{x}\right)} = 1$$

5. If the sum of the second, third and fourth terms of a positive term G.P. is 3 and the sum of its sixth, seventh and eighth terms is 243, then the sum of the first 50 terms of this G.P. is :

(1) $\frac{2}{13}(3^{50}-1)$ (2) $\frac{1}{26}(3^{50}-1)$

(3) $\frac{1}{13}(3^{50}-1)$ (4) $\frac{1}{26}(3^{49}-1)$

Official Ans. by NTA (2)

Sol. Let first term = a > 0

Common ratio = r > 0

$ar + ar^2 + ar^3 = 3$ ....(i)

$ar^5 + ar^6 + ar^7 = 243$ ....(ii)

$r^4(ar + ar^2 + ar^3) = 243$

$r^4(3) = 243 \Rightarrow r = 3$ as $r > 0$

from (1)

$3a + 9a + 27a = 3$

$a = \frac{1}{13}$

$S_{50} = \frac{a(r^{50}-1)}{(r-1)} = \frac{1}{26}(3^{50}-1)$

6. The value of $\left(\frac{-1+i\sqrt{3}}{1-i}\right)^{30}$ is :

(1) $2^{15}i$ (2) -2^{15}

(3) $-2^{15}i$ (4) 6^5

Official Ans. by NTA (3)

Sol. $\left(\frac{-1+i\sqrt{3}}{1-i}\right)^{30} = \left(\frac{2\omega}{1-i}\right)^{30}$

$= \frac{2^{30}\cdot\omega^{30}}{\left((1-i)^2\right)^{30}}$

$= \frac{2^{30}\cdot 1}{(1+i^2-2i)^{15}}$

$= \frac{2^{30}}{-2^{15}\cdot i^{15}}$

$= -2^{15}i$

7. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x = \frac{1}{2}$ is :

(1) $\frac{\sqrt{3}}{12}$ (2) $\frac{\sqrt{3}}{10}$

(3) $\frac{2\sqrt{3}}{5}$ (4) $\frac{2\sqrt{3}}{3}$

Official Ans. by NTA (2)

Sol. Let $f = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

$f = \tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)$

$f = \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right) = \frac{\theta}{2}$

$f = \frac{\tan^{-1}x}{2} \Rightarrow \frac{df}{dx} = \frac{1}{2(1+x^2)}$ ....(i)

Let $g = \tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$

Put $x = \sin\theta \Rightarrow \theta = \sin^{-1}x$

$g = \tan^{-1}\left(\frac{2\sin\theta\cos\theta}{1-2\sin^2\theta}\right)$

$g = \tan^{-1}(\tan 2\theta) = 2\theta$

$g = 2\sin^{-1}x$

$\frac{dg}{dx} = \frac{2}{\sqrt{1-x^2}}$ ....(ii)

$\frac{df}{dg} = \frac{1}{2(1+x^2)}\frac{\sqrt{1-x^2}}{2}$

at $x = \frac{1}{2}$ $\left(\frac{df}{dg}\right)_{x=\frac{1}{2}} = \frac{\sqrt{3}}{10}$

8. The area (in sq. units) of the region $A = \{(x, y) : (x - 1)[x] \le y \le 2\sqrt{x}, 0 \le x \le 2\}$, where [t] denotes the greatest integer function, is :

(1) $\frac{8}{3}\sqrt{2} - \frac{1}{2}$ (2) $\frac{8}{3}\sqrt{2} - 1$

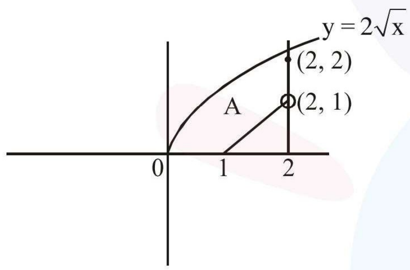
(3) $\frac{4}{3}\sqrt{2} - \frac{1}{2}$ (4) $\frac{4}{3}\sqrt{2} + 1$

Official Ans. by NTA (1)

Sol. $(x - 1)[x] \le y \le 2\sqrt{x}$, $\boxed{0 \le x \le 2}$

Draw $y = 2\sqrt{x} \Rightarrow y^2 = 4x$ $\boxed{x \ge 0}$

$$y = (x-1)[x] = \begin{cases} 0 & , 0 \le x < 1 \\ x - 1 & , 1 \le x < 2 \\ 2 & , x = 2 \end{cases}$$



$$A = \int_0^2 2\sqrt{x}\,dx - \frac{1}{2}1\cdot1$$

$$A = 2\cdot\left[\frac{x^{3/2}}{(3/2)}\right]_0^2 - \frac{1}{2} = \frac{8\sqrt{2}}{3} - \frac{1}{2}$$

9. If the length of the chord of the circle, $x^2 + y^2 = r^2$ $(r > 0)$ along the line, $y - 2x = 3$ is r, then r^2 is equal to :

(1) $\frac{9}{5}$ (2) $\frac{12}{5}$

(3) 12 (4) $\frac{24}{5}$

Official Ans. by NTA (2)

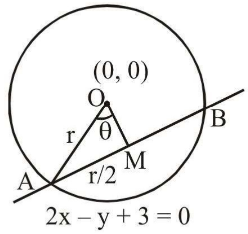
Sol. Let chord

AB = r

$\because$ ΔAOM is right angled triangle

$\therefore$ $OM = \frac{r\sqrt{3}}{2}$ = perpendicular distance of line AB from (0,0)

$$\frac{r\sqrt{3}}{2} = \left|\frac{3}{\sqrt{5}}\right|$$



$$r^2 = \frac{12}{5}$$

10. If x = 1 is a critical point of the function $f(x) = (3x^2 + ax - 2 - a)e^x$, then :

(1) x = 1 is a local minima and $x = -\frac{2}{3}$ is a local maxima of f.

(2) x = 1 is a local maxima and $x = -\frac{2}{3}$ is a local minima of f.

(3) x = 1 and $x = -\frac{2}{3}$ are local minima of f.

(4) x = 1 and $x = -\frac{2}{3}$ are local maxima of f.

Official Ans. by NTA (1)

Sol. $f(x) = (3x^2 + ax - 2 - a)e^x$

$f'(x) = (3x^2 + ax - 2 - a)e^x + e^x(6x + a)$

$= e^x(3x^2 + x(6 + a) - 2)$

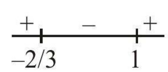
$f'(x) = 0$ at $x = 1$

$\Rightarrow 3 + (6 + a) - 2 = 0$

$a = -7$

$f'(x) = e^x(3x^2 - x - 2)$

$= e^x(x - 1)(3x + 2)$



x = 1 is point of local minima

$x = \frac{-2}{3}$ is point of local maxima

11. If the mean and the standard deviation of the data 3, 5, 7, a, b are 5 and 2 respectively, then a and b are the roots of the equation :

(1) $2x^2 - 20x + 19 = 0$
(2) $x^2 - 10x + 19 = 0$
(3) $x^2 - 10x + 18 = 0$
(4) $x^2 - 20x + 18 = 0$

Official Ans. by NTA (2)

Sol. Mean = 5

$$\frac{3+5+7+a+b}{5} = 5$$

$a + b = 10$ ....(i)

$$\text{S.d.} = 2 \Rightarrow \sqrt{\frac{\sum_{i=1}^{5}(x_i - \bar{x})^2}{5}} = 2$$

$(3-5)^2 + (5-5)^2 + (7-5)^2 + (a-5)^2 + (b-5)^2 = 20$

$\Rightarrow 4 + 0 + 4 + (a-5)^2 + (b-5)^2 = 20$

$a^2 + b^2 - 10(a+b) + 50 = 12$

$(a+b)^2 - 2ab - 100 + 50 = 12$

$ab = 19$ ....(ii)

Equation is $x^2 - 10x + 19 = 0$

12. If $a + x = b + y = c + z + 1$, where a, b, c, x, y, z are non-zero distinct real numbers, then

$$\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix}$$ is equal to :

(1) 0 (2) $y(a-b)$
(3) $y(b-a)$ (4) $y(a-c)$

Official Ans. by NTA (2)

Sol. $a + x = b + y = c + z + 1$

$$\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix} \quad C_3 \to C_3 - C_1$$

$$\begin{vmatrix} x & a+y & a \\ y & b+y & b \\ z & c+y & c \end{vmatrix} \quad C_2 \to C_2 - C_3$$

$$\begin{vmatrix} x & y & a \\ y & y & b \\ z & y & c \end{vmatrix} \quad R_3 \to R_3 - R_1,\ R_2 \to R_2 - R_1$$

$$\begin{vmatrix} x & y & a \\ y-x & 0 & b-a \\ z-x & 0 & c-a \end{vmatrix}$$

$= (-y)[(y-x)(c-a) - (b-a)(z-x)]$

$= (-y)[(a-b)(c-a) + (a-b)(a-c-1)]$

$= (-y)[(a-b)(c-a) + (a-b)(a-c) + b - a)$

$= -y(b-a) = y(a-b)$

13. If $\int \frac{\cos\theta}{5 + 7\sin\theta - 2\cos^2\theta} d\theta = A \log_e |B(\theta)| + C$, where C is a constant of integration, then $\frac{B(\theta)}{A}$ can be :

(1) $\frac{2\sin\theta + 1}{5(\sin\theta + 3)}$ (2) $\frac{2\sin\theta + 1}{\sin\theta + 3}$

(3) $\frac{5(\sin\theta + 3)}{2\sin\theta + 1}$ (4) $\frac{5(2\sin\theta + 1)}{\sin\theta + 3}$

Official Ans. by NTA (4)

Sol. $$\int \frac{\cos\theta \, d\theta}{5 + 7\sin\theta - 2\cos^2\theta}$$

$$\int \frac{\cos\theta \, d\theta}{3 + 7\sin\theta + 2\sin^2\theta} \qquad \boxed{\begin{array}{l} \sin\theta = t \\ \cos\theta d\theta = dt \end{array}}$$

$$\int \frac{dt}{2t^2 + 7t + 3} = \int \frac{dt}{(2t+1)(t+3)}$$

$$= \frac{1}{5}\int\left(\frac{2}{2t+1} - \frac{1}{t+3}\right)dt$$

$$= \frac{1}{5} \ln\left|\frac{2t+1}{t+3}\right| + C$$

$$= \frac{1}{5} \ln\left|\frac{2\sin\theta + 1}{\sin\theta + 3}\right| + C$$

$$A = \frac{1}{5} \text{ and } B(\theta) = \frac{2\sin\theta + 1}{\sin\theta + 3}$$

14. If the line y = mx + c is a common tangent to the hyperbola $\frac{x^2}{100}-\frac{y^2}{64}=1$ and the circle $x^2 + y^2 = 36$, then which one of the following is true?

(1) 5m = 4 (2) $4c^2 = 369$

(3) $c^2 = 369$ (4) 8m + 5 = 0

Official Ans. by NTA (2)

Sol. y = mx + c is tangent to

$\frac{x^2}{100}-\frac{y^2}{64}=1$ and $x^2 + y^2 = 36$

$c^2 = 100\,m^2 - 64 \mid c^2 = 36\,(1 + m^2)$

$\Rightarrow 100\,m^2 - 64 = 36 + 36m^2$

$m^2 = \frac{100}{64} \Rightarrow m = \pm\frac{10}{8}$

$c^2 = 36\left(1+\frac{100}{64}\right) = \frac{36\times164}{64}$

$4c^2 = 369$

15. There are 3 sections in a question paper and each section contains 5 questions. A candidate has to answer a total of 5 questions, choosing at least one question from each section. Then the number of ways, in which the candidate can choose the questions, is :

(1) 1500 (2) 2255

(3) 3000 (4) 2250

Official Ans. by NTA (4)

Sol.

A	B	C
5	5	5
1	2	2
2	1	2
2	2	1
1	1	3
1	3	1
3	1	1

Total number of selection

$= ({}^5C_1\ {}^5C_2\ {}^5C_2)\cdot 3 + ({}^5C_1\ {}^5C_1\ {}^5C_3)\cdot 3$

$= 5 \cdot 10 \cdot 10 \cdot 3 + 5 \cdot 5 \cdot 10 \cdot 3$

$= 2250$

16. If for some $\alpha \in R$, the lines

$L_1 : \frac{x+1}{2}=\frac{y-2}{-1}=\frac{z-1}{1}$ and

$L_2 : \frac{x+2}{\alpha}=\frac{y+1}{5-\alpha}=\frac{z+1}{1}$ are coplanar, then the line L_2 passes through the point :

(1) (–2, 10, 2) (2) (10, 2, 2)

(3) (10, –2, –2) (4) (2, –10, –2)

Official Ans. by NTA (4)

Sol. $L_1 \equiv \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$

$L_2 \equiv \frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1}$

Point A(–1, 2, 1) B(–2, –1, –1)

$\because L_1$ and L_2 are coplanar

$\Rightarrow \begin{vmatrix} 2 & -1 & 1 \\ \alpha & 5-\alpha & 1 \\ 1 & 3 & 2 \end{vmatrix} = 0$

$\alpha = -4$

$L_2 \equiv \frac{x+2}{-4} = \frac{y+1}{9} = \frac{z+1}{1}$

Check options (2, –10, –2) lies on L_2

17. Let y = y(x) be the solution of the differential equation $\cos x\frac{dy}{dx} + 2y\sin x = \sin 2x$, $x \in \left(0, \frac{\pi}{2}\right)$. If $y(\pi/3) = 0$, then $y(\pi/4)$ is equal to :

(1) $\sqrt{2}-2$ (2) $\frac{1}{\sqrt{2}} - 1$

(3) $2 - \sqrt{2}$ (4) $2 + \sqrt{2}$

Official Ans. by NTA (1)

Sol. $\cos x \dfrac{dy}{dx} + 2y \sin x = \sin 2x$

$\dfrac{dy}{dx} + \dfrac{2\sin x}{\cos x} y = 2 \sin x$

I.F. $= e^{\int 2\frac{\sin x}{\cos x}dx}$

$= e^{2\ln \sec x} = \sec^2 x$

$y \cdot \sec^2 x = \int 2\sin x \cdot \sec^2 x dx$

$y \sec^2 x = 2 \int \tan x \sec x dx$

$y \sec^2 x = 2 \sec x + c$

At $x = \dfrac{\pi}{3}$, $y = 0$

$\Rightarrow 0 = 2 \sec \dfrac{\pi}{3} + C \Rightarrow C = -4$

$\boxed{y\sec^2 x = 2\sec x - 4}$

Put $x = \dfrac{\pi}{4}$

$y \cdot 2 = 2\sqrt{2} - 4$

$y = \sqrt{2} - 2$

18. Which of the following points lies on the tangent to the curve $x^4e^y + 2\sqrt{y+1} = 3$ at the point (1, 0) ?

(1) (2, 2) (2) (–2, 6)

(3) (–2, 4) (4) (2, 6)

Official Ans. by NTA (2)

Sol. $x^4e^y + 2\sqrt{y+1} = 3$

d.w.r. to x

$x^4 e^y y' + e^y 4x^3 + \dfrac{2y'}{2\sqrt{y+1}} = 0$

at P(1, 0)

$y'_P + 4 + y'_P = 0$

$\Rightarrow y'_P = -2$

Tangent at P(1, 0) is

$y - 0 = -2 (x - 1)$

$2x + y = 2$

(–2, 6) lies on it

19. The statement $(p \to (q \to p)) \to (p \to (p \vee q))$ is:

(1) a contradiction

(2) equivalent to $(p \wedge q) \vee (\sim q)$

(3) a tautology

(4) equivalent to $(p \vee q) \wedge (\sim p)$

Official Ans. by NTA (3)

Sol.

p	q	$q \to p$	$p \to (q \to p)$	$p \vee q$	$p \to p \vee q$	$(p \to (q \to p)) \to (p \to (p \vee q))$
T	T	T	T	T	T	T
T	F	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	T	F	T	T

20. If $L = \sin^2\left(\dfrac{\pi}{16}\right) - \sin^2\left(\dfrac{\pi}{8}\right)$ and

$M = \cos^2\left(\dfrac{\pi}{16}\right) - \sin^2\left(\dfrac{\pi}{8}\right)$, then :

(1) $M = \dfrac{1}{2\sqrt{2}} + \dfrac{1}{2}\cos\dfrac{\pi}{8}$

(2) $L = \dfrac{1}{4\sqrt{2}} - \dfrac{1}{4}\cos\dfrac{\pi}{8}$

(3) $M = \dfrac{1}{4\sqrt{2}} + \dfrac{1}{4}\cos\dfrac{\pi}{8}$

(4) $L = -\dfrac{1}{2\sqrt{2}} + \dfrac{1}{2}\cos\dfrac{\pi}{8}$

Official Ans. by NTA (1)

Sol. $L = \sin^2\left(\dfrac{\pi}{16}\right) - \sin^2\left(\dfrac{\pi}{8}\right)$

$\left(\because \sin^2\theta = \dfrac{1-\cos 2\theta}{2}\right)$

$\Rightarrow L = \left(\dfrac{1-\cos(\pi/8)}{2}\right) - \left(\dfrac{1-\cos(\pi/4)}{2}\right)$

$$L = \frac{1}{2}\left[\cos\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{8}\right)\right]$$

$$L = \frac{1}{2\sqrt{2}} - \frac{1}{2}\cos\left(\frac{\pi}{8}\right)$$

$$M = \cos^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$$

$$M = \frac{1+\cos(\pi/8)}{2} - \frac{1-\cos(\pi/4)}{2}$$

$$M = \frac{1}{2}\cos\left(\frac{\pi}{8}\right) + \frac{1}{2\sqrt{2}}$$

21. In a bombing attack, there is 50% chance that a bomb will hit the target. At least two independent hits are required to destroy the target completely. Then the minimum number of bombs, that must be dropped to ensure that there is at least 99% chance of completely destroying the target, is _______.

Official Ans. by NTA (11.00)

Sol. $P(H) = \frac{1}{2}$

$P(\bar{H}) = \frac{1}{2}$

Let total 'n' bomb are required to destroy the target

$$1 - {}^nC_n\left(\frac{1}{2}\right)^n - {}^nC_1\left(\frac{1}{2}\right)^n \geq \frac{99}{100}$$

$$1 - \frac{1}{2^n} - \frac{n}{2^n} \geq \frac{99}{100}$$

$$\frac{1}{100} \geq \frac{n+1}{2^n}$$

Now check for value of n

$\boxed{n = 11}$

22. Let A = {a, b, c} and B = {1, 2, 3, 4}. Then the number of elements in the set C = {f : A → B| 2 ∈ f(A) and f is not one-one} is _______.

Official Ans. by NTA (19.00)

Sol. C = {f : A → B| 2 ∈ f(A) and f is not one-one}

Case-I : If f(x) = 2 ∀ x ∈ A then number of function = 1

Case-II : If f(x) = 2 for exactly two elements then total number of many-one function = ${}^3C_2\ {}^3C_1 = 9$

Case-III : If f(x) = 2 for exactly one element then total number of many-one functions = ${}^3C_1\ {}^3C_1 = 9$

Total = 19

23. The coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^6$ in powers of x, is ______.

Official Ans. by NTA (120.00)

Sol. $(1 + x + x^2 + x^3)^6 = \left((1+x)(1+x^2)\right)^6$

$= (1 + x)^6 (1 + x^2)^6$

$$= \sum_{r=0}^{6} {}^6C_r x^r \sum_{t=0}^{6} {}^6C_t x^{2t}$$

$$= \sum_{r=0}^{6}\sum_{t=0}^{6} {}^6C_r\,{}^6C_t\, x^{r+2t}$$

For coefficient of $x^4 \Rightarrow r + 2t = 4$

r	t
0	2
2	1
4	0

Coefficient of x^4

$= {}^6C_0\,{}^6C_2 + {}^6C_2\,{}^6C_1 + {}^6C_4\,{}^6C_0$

$= 120$

24. If the lines x + y = a and x – y = b touch the curve $y = x^2 - 3x + 2$ at the points where the curve intersects the x-axis, then $\frac{a}{b}$ is equal to________.

Official Ans. by NTA (0.50)

Sol. $y = x^2 - 3x + 2$

At x-axis $y = 0 = x^2 - 3x + 2$

x = 1, 2

$$\frac{dy}{dx} = 2x - 3$$

A(1, 0) B(2, 0)

$\left(\frac{dy}{dx}\right)_{x=1} = -1$ and $\left(\frac{dy}{dx}\right)_{x=2} = 1$

$x + y = a \Rightarrow \frac{dy}{dx} = -1$ So A(1, 0) lies on it

$\Rightarrow 1 + 0 = a \Rightarrow \boxed{a = 1}$

$x - y = b \Rightarrow \frac{dy}{dx} = 1$ So B(2, 0) lies on it

$2 - 0 = b \Rightarrow \boxed{b = 2}$

$\frac{a}{b} = 0.50$

25. Let the vectors $\vec{a}, \vec{b}, \vec{c}$ be such that $|\vec{a}| = 2, |\vec{b}| = 4$ and $|\vec{c}| = 4$. If the projection of $\vec{b}$ on $\vec{a}$ is equal to the projection of $\vec{c}$ on $\vec{a}$ and $\vec{b}$ is perpendicular to $\vec{c}$, then the value of $|\vec{a} + \vec{b} - \vec{c}|$ is __________.

Official Ans. by NTA (6.00)

Sol. Projection of $\vec{b}$ on $\vec{a}$ = projection of $\vec{c}$ on $\vec{a}$

$\Rightarrow \frac{\vec{b} \cdot \vec{a}}{|\vec{a}|} = \frac{\vec{c} \cdot \vec{a}}{|\vec{a}|} \Rightarrow \vec{b} \cdot \vec{a} = \vec{c} \cdot \vec{a}$

$\because \vec{b}$ is perpendicular to $\vec{c} \Rightarrow \vec{b} \cdot \vec{c} = 0$

Let $|\vec{a} + \vec{b} - \vec{c}| = k$

Square both sides

$k^2 = \vec{a}^2 + \vec{b}^2 + \vec{c}^2 + 2\vec{a} \cdot \vec{b} - 2\vec{a} \cdot \vec{c} - 2\vec{b} \cdot \vec{c}$

$k^2 = \vec{a}^2 + \vec{b}^2 + \vec{c}^2 = 36$

$k = 6 = |\vec{a} + \vec{b} - \vec{c}|$