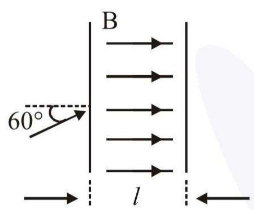
FINAL JEE-MAIN EXAMINATION - SEPTEMBER, 2020
Held On Wednesday, 2 September 2020
TIME : 3: 00 PM to 6 : 00 PM

1. The figure shows a region of length 'l' with a uniform magnetic field of 0.3 T in it and a proton entering the region with velocity 4×10^5 ms^{-1} making an angle 60° with the field. If the proton completes 10 revolution by the time it cross the region shown, 'l' is close to (mass of proton $= 1.67 \times 10^{-27}$ kg, charge of the proton $= 1.6 \times 10^{-19}$ C)



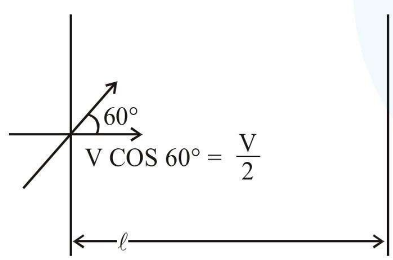
(1) 0.11 m (2) 0.22 m

(3) 0.44 m (4) 0.88 m

Official Ans. by NTA (3)

Sol. $T = \frac{2\pi m}{qB}$

total time t = 10 T



Kinematics

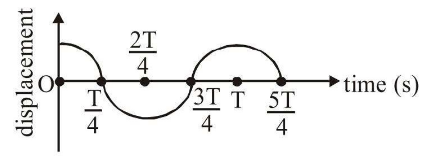
$\ell = \frac{V}{2} t$

$\ell = \frac{V}{2} 10 \times \frac{2\pi m}{qB}$

$= 4 \times 10^5 \times 10 \times \frac{3.14 \times 1.67 \times 10^{-27}}{1.6 \times 10^{-19} \times 0.3}$

= 0.439

2. The displacement time graph of a particle executing S.H.M. is given in figure : (sketch is schematic and not to scale)



Which of the following statements is/are true for this motion ?

(A) The force is zero $t = \frac{3T}{4}$

(B) The acceleration is maximum at t = T

(C) The speed is maximum at $t = \frac{T}{4}$

(D) The P.E. is equal to K.E. of the oscillation at $t = \frac{T}{2}$

(1) (A), (B) and (D)

(2) (B), (C) and (D)

(3) (A) and (D)

(4) (A), (B) and (C)

Official Ans. by NTA (4)

Sol. (A) F = ma $\quad a = -\omega^2 x$

at $\frac{3T}{4}$ displacement zero (x = 0), so a = 0

F = 0

(B) at t = T $\quad$ displacement (x) = A

x maximum, So acceleration is maximum.

(C) $V = \omega\sqrt{A^2 - x^2}$

V_{max} at x = 0

$V_{max} = A\omega$

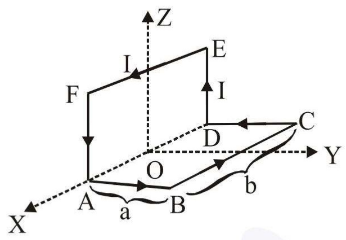
at $t = \frac{T}{4}$, x = 0, So $V_{max.}$

(D) KE = PE

$\therefore$ at $x = \frac{A}{\sqrt{2}}$.

at $t = \frac{T}{2}$ $\quad x = -A$ $\quad$ (So not possible)

3. A wire carrying current I is bent in the shape ABCDEFA as shown, where rectangle ABCDA and ADEFA are perpendicular to each other. If the sides of the rectangles are of lengths a and b, then the magnitude and direction of magnetic moment of the loop ABCDEFA is :



(1) $\sqrt{2}abI$, along $\left(\frac{\hat{j}}{\sqrt{2}}+\frac{\hat{k}}{\sqrt{2}}\right)$

(2) $\sqrt{2}abI$, along $\left(\frac{\hat{j}}{\sqrt{5}}+\frac{2\hat{k}}{\sqrt{5}}\right)$

(3) abI, along $\left(\frac{\hat{j}}{\sqrt{2}}+\frac{\hat{k}}{\sqrt{2}}\right)$

(4) abI, along $\left(\frac{\hat{j}}{\sqrt{5}}+\frac{2\hat{k}}{\sqrt{5}}\right)$

Official Ans. by NTA (1)

Sol. M = NIA

N = 1

For ABCD

$$\vec{M}_1 = abI\ \hat{K}.$$

For DEFA

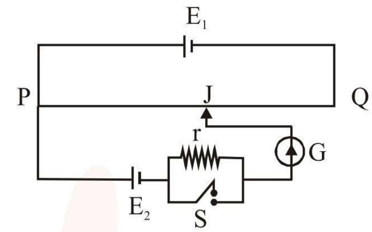
$$\vec{M}_2 = abI\ \hat{j}$$

$$\vec{M} = \vec{M}_1 + \vec{M}_2$$

$= ab\ I\ \left(\hat{k}+\hat{j}\right)$

$= ab\ I\ \sqrt{2}\ \left(\frac{\hat{j}}{\sqrt{2}}+\frac{\hat{k}}{\sqrt{2}}\right)$

4. A potentiometer wire PQ of 1 m length is connected to a standard cell E_1. Another cell E_2 of emf 1.02 V is connected with a resistance 'r' and switch S (as shown in figure). With switch S open, the null position is obtained at a distance of 49 cm from Q. The potential gradient in the potentiometer wire is :



(1) 0.02 V/cm (2) 0.04 V/cm

(3) 0.01 V/cm (4) 0.03 V/cm

Official Ans. by NTA (1)

Sol. Balancing length is measured from P.

So 100 – 49 = 51 cm

$E_2 = \phi \times 51$

Where ϕ = Potential gradient

$1.02 = \phi \times 51$

$\phi = 0.02$ V/cm

5. A heat engine is involved with exchange of heat of 1915 J, –40 J, +125 J and QJ, during one cycle achieving an efficiency of 50.0%. The value of Q is:

(1) 640 J (2) 400 J

(3) 980 J (4) 40 J

Official Ans. by NTA (3)

Sol. $\eta = \frac{\text{Work done}}{\text{Heat supplied}}$

$$\frac{1}{2} = \eta = \frac{1915-40+125-Q}{1915+125}$$

$$\frac{1}{2} = \frac{2000-Q}{2040}$$

2040 = 4000 – 2Q

2Q = 1960

Q = 980 J

6. In a Young's double slit experiment, 16 fringes are observed in a certain segment of the screen when light of wavelength 700 nm is used. If the wavelength of light is changed to 400 nm, the number of fringes observed in the same segment of the screen would be :

(1) 28 (2) 24 (3) 18 (4) 30

Official Ans. by NTA (1)

Sol. Let the length of segment is "ℓ"

Let N is the no. of fringes in "ℓ"

and w is fringe width.

→ We can write

$$N w = \ell$$

$$N\left(\frac{\lambda D}{d}\right) = \ell$$

$$\frac{N_1\lambda_1 D}{d} = \ell$$

$$\frac{N_2\lambda_2 D}{d} = \ell$$

$N_1\lambda_1 = N_2\lambda_2$

$16 \times 700 = N_2 \times 400$

$N_2 = 28$

7. In a hydrogen atom the electron makes a transition from (n + 1)th level to the nth level. If n>>l, the frequency of radiation emitted is proportional to :

(1) $\frac{1}{n^4}$ (2) $\frac{1}{n^3}$ (3) $\frac{1}{n^2}$ (4) $\frac{1}{n}$

Official Ans. by NTA (2)

Sol. In hydrogen atom,

$$E_n = \frac{-E_0}{n^2}$$

Where E_0 is Ionisation Energy of H.

→ For transition from (n + 1) to n, the energy of emitted radiation is equal to the difference in energies of levels.

$$\Delta E = E_{n+1} - E_n$$

$$\Delta E = E_0\left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right)$$

$$\Delta E = h\nu = E_0\left(\frac{(n+1)^2 - n^2}{n^2(n+1)^2}\right)$$

$$h\nu = E_0\left[\frac{2n+1}{n^4\left(1+\frac{1}{n}\right)^2}\right]$$

$$h\nu = E_0\left[\frac{n\left(2+\frac{1}{n}\right)}{n^4\left(1+\frac{1}{n}\right)^2}\right]$$

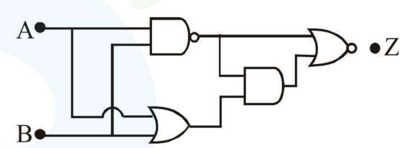
Since n >>> 1

Hence, $\frac{1}{n} \simeq 0$

$$h\nu = E_0\left[\frac{2}{n^3}\right]$$

$$\nu \propto \frac{1}{n^3}$$

8. In the following digital circuit, what will be the output at 'Z', when the input (A, B) are (1,0), (0,0), (1,1), (0,1):

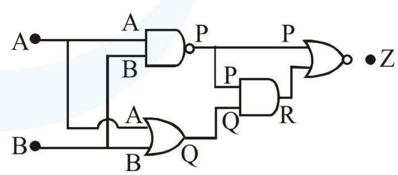


(1) 1, 0, 1, 1 (2) 0, 1, 0, 0

(3) 0, 0, 1, 0 (4) 1, 1, 0, 1

Official Ans. by NTA (3)

Sol.



$Z = \overline{(P + R)}$

$Z = \overline{(P + PQ)}$

$Z = \overline{(P(1 + Q))}$

$Z = (\overline{P})$ [Using Identity $(1 + A) = 1$]

$Z = (\overline{\overline{AB}})$

Z = AB

Truth table for Z = AB

A	B	Z
1	0	0
0	0	0
1	1	1
0	1	0

9. If momentum (P), area (A) and time (T) are taken to be the fundamental quantities then the dimensional formula for energy is :

(1) $[PA^{-1}T^{-2}]$ (2) $[PA^{1/2}T^{-1}]$

(3) $[P^2AT^{-2}]$ (4) $[P^{1/2}AT^{-1}]$

Official Ans. by NTA (2)

Sol. Let $[E] = [P]^x [A]^y [T]^z$

$ML^2T^{-2} = [MLT^{-1}]^x [L^2]^y [T]^z$

$ML^2T^{-2} = M^x L^{x+2y} T^{-x+z}$

$\rightarrow x = 1$

$\rightarrow x + 2y = 2$

$1 + 2y = 2$

$y = \frac{1}{2}$

$\rightarrow -x + z = -2$

$-1 + z = -2$

$z = -1$

$[E] = [PA^{1/2}T^{-1}]$

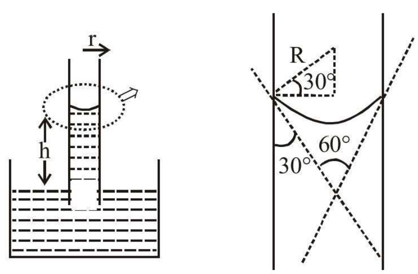
10. A capillary tube made of glass of radius 0.15 mm is dipped vertically in a beaker filled with methylene iodide (surface tension = 0.05 Nm^{-1}, density = 667 kg m^{-3}) which rises to height h in the tube. It is observed that the two tangents drawn from liquid-glass interfaces (from opp. sides of the capillary) make an angle of 60° with one another. Then h is close to ($g = 10\ ms^{-2}$).

(1) 0.137 m (2) 0.172 m

(3) 0.087 m (4) 0.049 m

Official Ans. by NTA (3)

Sol.



r → radius of capillary

R → Radius of meniscus.

From figure, $\frac{r}{R} = \cos 30°$

$$R = \frac{2r}{\sqrt{3}} = \frac{2 \times 0.15 \times 10^{-3}}{\sqrt{3}}$$

$$= \frac{0.3}{\sqrt{3}} \times 10^{-3}\,m$$

Height of capillary

$$h = \frac{2T}{\rho g R} = 2\sqrt{3}\,T$$

$$h = \frac{2 \times 0.05}{667 \times 10 \times \left(\frac{0.3 \times 10^{-3}}{\sqrt{3}}\right)}$$

h = 0.087 m

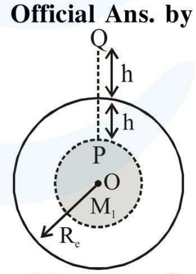
11. The height 'h' at which the weight of a body will be the same as that at the same depth 'h' from the surface of the earth is (Radius of the earth is R and effect of the rotation of the earth is neglected) :

(1) $\frac{\sqrt{5}R - R}{2}$ (2) $\frac{\sqrt{5}}{2}R - R$

(3) $\frac{R}{2}$ (4) $\frac{\sqrt{3}R - R}{2}$

Official Ans. by NTA (1)

Sol.



• M = mass of earth

M_1 = mass of shaded portion

R = Radius of earth

• $M_1 = \frac{M}{\frac{4}{3}\pi R^3} \cdot \frac{4}{3}\pi (R-h)^3$

$$= \frac{M(R-h)^3}{R}$$

⬧ Weight of body is same at P and Q

i.e. $mg_P = mg_Q$

$g_P = g_Q$

$$\frac{GM_1}{(R-h)^2} = \frac{GM}{(R+h)^2}$$

$$\frac{GM(R-h)^3}{(R-h)^2 R^3} = \frac{GM}{(R+h)^2}$$

$(R - h)(R + h)^2 = R^3$

$R^3 - hR^2 - h^2R - h^3 + 2R^2 h - 2Rh^2 = R^3$

$R^2 - Rh^2 - h^3 = 0$

$R^2 - Rh - h^2 = 0$

$$h^2 + Rh - R^2 = 0 \Rightarrow h = \frac{-R \pm \sqrt{R^2 + 4R^2}}{2}$$

$$\text{ie } h = \frac{-R + \sqrt{5}R}{2} = \left(\frac{\sqrt{5}-1}{2}\right)R$$

12. An ideal gas in a closed container is slowly heated. As its temperature increases, which of the following statements are true ?

(A) the mean free path of the molecules decreases.

(B) the mean collision time between the molecules decreases.

(C) the mean free path remains unchanged.

(D) the mean collision time remains unchanged.

(1) (C) and (D) (2) (A) and (B)

(3) (A) and (D) (4) (B) and (C)

Official Ans. by NTA (4)

Sol. The mean free path of molecules of an ideal gas is given as:

$$\lambda = \frac{V}{\sqrt{2}\pi d^2 N}$$

where : V = Volume of container
N = No of molecules

Hence with increasing temp since volume of container does not change (closed container), so mean free path is unchanged.

Average collision time

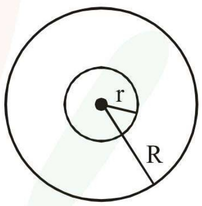
$$= \frac{\text{mean free path}}{V_{av}} = \frac{\lambda}{(\text{avg speed of molecules})}$$

$\because$ avg speed $\alpha \sqrt{T}$

$\therefore$ Avg coll. time $\alpha \frac{1}{\sqrt{T}}$

Hence with increase in temperature the average collision time decreases.

13. A charge Q is distributed over two concentric conducting thin spherical shells radii r and R (R > r). If the surface charge densities on the two shells are equal, the electric potential at the common centre is :



(1) $\frac{1}{4\pi\varepsilon_0}\frac{(R+2r)Q}{2(R^2+r^2)}$

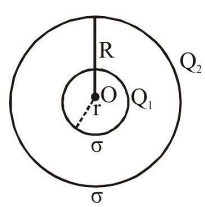
(2) $\frac{1}{4\pi\varepsilon_0}\frac{(R+r)}{2(R^2+r^2)}Q$

(3) $\frac{1}{4\pi\varepsilon_0}\frac{(R+r)}{(R^2+r^2)}Q$

(4) $\frac{1}{4\pi\varepsilon_0}\frac{(2R+r)}{(R^2+r^2)}Q$

Official Ans. by NTA (3)

Sol. Let the charges on inner and outer spheres are Q_1 and Q_2.



Since charge density 'σ' is same for both spheres, so

$$\sigma = \frac{Q_1}{4\pi r^2} = \frac{Q_2}{4\pi R^2} \Rightarrow \frac{Q_1}{Q_2} = \frac{r^2}{R^2}$$

$$Q_1 + Q_2 = Q \Rightarrow \frac{Q_2 r^2}{R^2} + Q_2 = Q$$

$$\Rightarrow Q_2 = \frac{QR^2}{(r^2 + R^2)}$$

$$Q_1 = \frac{r^2}{R^2} \cdot \frac{QR^2}{(R^2 + r^2)} = \frac{Qr^2}{(R^2 + r^2)}$$

Potential at centre 'O' = $\frac{kQ_1}{r} + \frac{kQ_2}{R}$

$$= k\left[\frac{Qr^2}{r(R^2 + r^2)} + \frac{QR^2}{R(R^2 + r^2)}\right]$$

$$= \frac{kQ(r + R)}{(R^2 + r^2)} = \frac{1}{4\pi \epsilon_0} \frac{(R + r)}{(R^2 + r^2)} Q$$

14. An inductance coil has a reactance of 100 Ω. When an AC signal of frequency 1000 Hz is applied to the coil, the applied voltage leads the current by 45°. The self-inductance of the coil is :

(1) 1.1×10^{-2} H (2) 1.1×10^{-1} H

(3) 5.5×10^{-5} H (4) 6.7×10^{-7} H

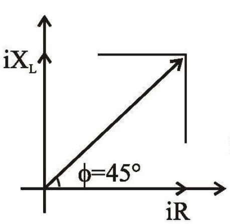
Official Ans. by NTA (1)

Sol. 

- Reactance of inductance coil

$$= \sqrt{R^2 + x_L^2} = 100 \quad .....(i)$$

- f = 1000 Hz of applied AC signal
- Voltage leads current by 45°

ie 

$\tan 45° = \frac{iX_L}{iR} = \frac{\omega L}{R}$

ie $R = X_L = \omega L$

Putting in eqn (i) : $\sqrt{X_L^2 + X_L^2} = 100$

$$\sqrt{2} X_L = 100 \Rightarrow X_L = 50\sqrt{2}$$

ie $\omega L = 50\sqrt{2}$

$$L = \frac{50\sqrt{2}}{\omega} = \frac{50\sqrt{2}}{2\pi f} = \frac{25\sqrt{2}}{\pi \times 1000} \text{ H}$$

$= 1.125 \times 10^{-2}$ H

15. Two uniform circular discs are rotating independently in the same direction around their common axis passing through their centres. The moment of inertia and angular velocity of the first disc are 0.1 kg-m^2 and 10 rad s^{-1} respectively while those for the second one are 0.2 kg-m^2 and 5 rad s^{-1} respectively. At some instant they get stuck together and start rotating as a single system about their common axis with some angular speed. The Kinetic energy of the combined system is :

(1) $\frac{10}{3}$J (2) $\frac{2}{3}$J (3) $\frac{5}{3}$J (4) $\frac{20}{3}$J

Official Ans. by NTA (4)

Sol.
- Both discs are rotating in same sense
- Angular momentum conserved for the system

i.e. $L_1 + L_2 = L_{final}$

$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega_f$

$0.1 \times 10 + 0.2 \times 5 = (0.1+0.2) \times \omega_f$

$$\omega_f = \frac{20}{3}$$

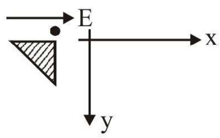
- Kinetic energy of combined disc system

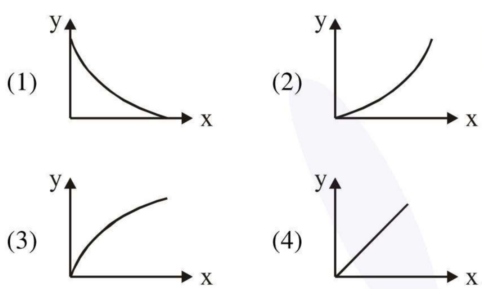
$$\Rightarrow \frac{1}{2}(I_1 + I_2)\omega_f^2$$

$$= \frac{1}{2}(0.1 + 0.2).\left(\frac{20}{3}\right)^2$$

$$= \frac{0.3}{2} \times \frac{400}{9} = \frac{120}{18} = \frac{20}{3} \text{J}$$

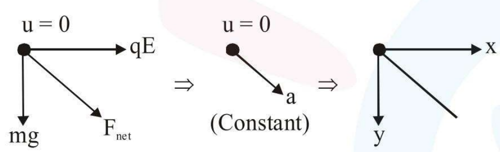
16. A small point mass carrying some positive charge on it, is released from the edge of a table. There is a uniform electric field in this region in the horizontal direction. Which of the following options then correctly describe the trajectory of the mass ? (Curves are drawn schematically and are not to scale).





Official Ans. by NTA (4)

Sol.



Since initial velocity is zero and acceleration of particle will be constant, so particle will travel on a straight line path.

17. A 10 μF capacitor is fully charged to a potential difference of 50 V. After removing the source voltage it is connected to an uncharged capacitor in parallel. Now the potential difference across them becomes 20 V. The capacitance of the second capacitor is:

(1) 10 μF

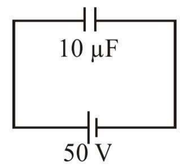
(2) 15 μF

(3) 20 μF

(4) 30 μF

Official Ans. by NTA (2)

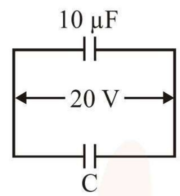
Sol. Initially



- Charge on capacitor 10 μF

$Q = CV = (10\ \mu F)\ (50V)$

$Q = 500\ \mu C$



- Final Charge on 10 μF capacitor

$Q = CV = (10\ \mu F)\ (20V)$

$Q = 200\ \mu C$

- From charge conservation,

Charge on unknown capacitor

$C = 500\ \mu C - 200\ \mu C = 300\ \mu C$

$\Rightarrow \text{Capacitance (C)} = \frac{Q}{V} = \frac{300\ \mu C}{20\ V} = 15\ \mu F$

18. When the temperature of a metal wire is increased from 0°C to 10°C, its length increases by 0.02%. The percentage change in its mass density will be closest to:

(1) 0.008 (2) 0.06

(3) 0.8 (4) 2.3

Official Ans. by NTA (2)

Sol. Given $\frac{\Delta L}{L} = 0.02\%$

$\therefore \Delta L = L\alpha\Delta T \Rightarrow \frac{\Delta L}{L} = \alpha\Delta T = 0.02\%$

$\therefore\ \beta = 2\alpha$ (Areal coefficient of expansion)

$\Rightarrow \beta\Delta T = 2\alpha\Delta T = 0.04\%$

Volume = Area × Length

$\text{Density}(\rho) = \frac{\text{Mass}}{\text{Volume}} = \frac{\text{Mass}}{\text{Area} \times \text{Length}} = \frac{M}{AL}$

$\Rightarrow \frac{\Delta\rho}{\rho} = \cancelto{0}{\frac{\Delta M}{M}} - \frac{\Delta A}{A} - \frac{\Delta L}{L}$ (Mass remains constant)

$\Rightarrow \left(\frac{\Delta\rho}{\rho}\right) = \frac{\Delta A}{A} + \frac{\Delta L}{L} = \beta\Delta T + \alpha\Delta T$

$= 0.04\% + 0.02\%$

$= 0.06\%$

19. In a plane electromagnetic wave, the directions of electric field and magnetic field are represented by $\hat{k}$ and $2\hat{i}-2\hat{j}$, respectively. What is the unit vector along direction of propagation of the wave.

(1) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$ (2) $\frac{1}{\sqrt{5}}(\hat{i}+2\hat{j})$

(3) $\frac{1}{\sqrt{5}}(2\hat{i}+\hat{j})$ (4) $\frac{1}{\sqrt{2}}(\hat{j}+\hat{k})$

Official Ans. by NTA (1)

Sol. $\hat{E}=\hat{k}$

$$\vec{B}=2\hat{i}-2\hat{j} \Rightarrow \hat{B}=\frac{\vec{B}}{|B|}=\frac{2\hat{i}-2\hat{j}}{2\sqrt{2}}$$

$$\Rightarrow \hat{B}=\frac{1}{\sqrt{2}}(\hat{i}-\hat{j})$$

Direction of wave propagation $=\hat{C}=\hat{E}\times\hat{B}$

$$\hat{C}=\hat{k}\times\left[\frac{1}{\sqrt{2}}(\hat{i}-\hat{j})\right]$$

$$\hat{C}=\frac{1}{\sqrt{2}}(\hat{k}\times\hat{i}-\hat{k}\times\hat{j})$$

$$\hat{C}=\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$$

20. A particle is moving 5 times as fast as an electron. The ratio of the de-Broglie wavelength of the particle to that of the electron is 1.878×10^{-4}. The mass of the particle is close to :

(1) 4.8×10^{-27} kg

(2) 1.2×10^{-28} kg

(3) 9.1×10^{-31} kg

(4) 9.7×10^{-28} kg

Official Ans. by NTA (4)

Sol. Let mass of particle = m

Let speed of e^- = V

$\Rightarrow$ speed of particle = 5V

Debroglie wavelength $\lambda_d=\frac{h}{P}=\frac{h}{mv}$

$$\Rightarrow (\lambda_d)_P=\frac{h}{m(5V)} \quad .....(1)$$

$$\Rightarrow (\lambda_d)_e=\frac{h}{m_e.V} \quad .....(2)$$

According to question

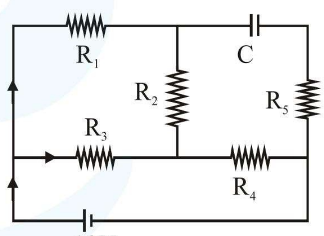
$$\frac{(1)}{(2)}=\frac{m_e}{5m}=1.878\times10^{-4}$$

$$\Rightarrow m=\frac{m_e}{5\times1.878\times10^{-4}}$$

$$\Rightarrow m=\frac{9.1\times10^{-31}}{5\times1.878\times10^{-4}}$$

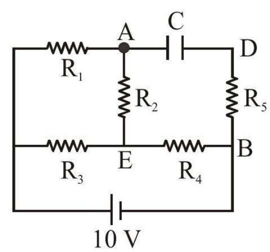
$\Rightarrow$ m = 9.7×10^{-28} kg

21. An ideal cell of emf 10 V is connected in circuit shown in figure. Each resistance is 2 Ω. The potential difference (in V) across the capacitor when it is fully charged is ________.

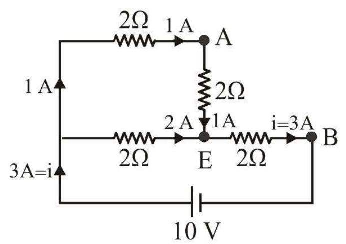


Official Ans. by NTA (8.00)

Sol.



- R_1 to $R_5 \rightarrow$ each 2Ω
- Cap. is fully charged
- So no current is there in branch ADB
- Effective circuit of current flow :



$$R_{eq} = \left(\frac{4\times 2}{4+2}\right) + 2$$

$$R_{eq} = \frac{4}{3} + 2 = \frac{10}{3}\Omega$$

$$i = \frac{10}{10/3} = 3A$$

So potential different across AEB

$\Rightarrow 2 \times 1 + 2 \times 3 = 8V$

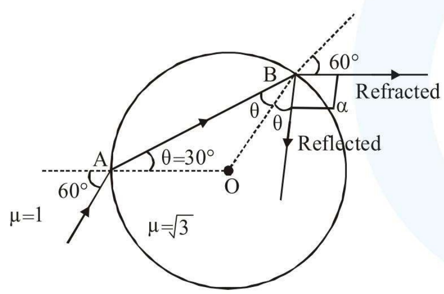
Hence potential difference across

Capacitor = $\Delta V = V_{AEB} = 8V$

22. A light ray enters a solid glass sphere of refractive index $\mu = \sqrt{3}$ at an angle of incidence 60°. The ray is both reflected and refracted at the farther surface of the sphere. The angle (in degrees) between the reflected and refracted rays at this surface is______.

Official Ans. by NTA (90.00)

Sol.



By Snell's law at A :

$1 \times \sin 60° = \sqrt{3} \times \sin\theta$

$$\frac{\sqrt{3}}{2} = \sqrt{3}\sin\theta$$

$$\sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$$

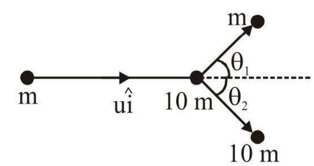
So at B :

$\theta + 60° + \alpha = 180°$

$30° + 60° + \alpha = 180°$

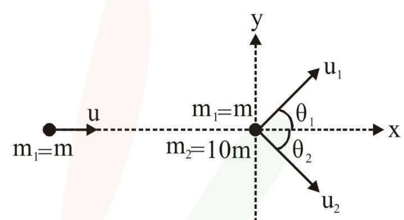
$\alpha = 90°$

23. A particle of mass m is moving along the x-axis with initial velocity $u\hat{i}$. It collides elastically with a particle of mass 10 m at rest and then moves with half its initial kinetic energy (see figure). If $\sin\theta_1 = \sqrt{n}\sin\theta_2$ then value of n is _______.



Official Ans. by NTA (10.00)

Sol.



By momentum conservation along y :

$m_1u_1\sin\theta_1 = m_2u_2\sin\theta_2$

i.e. $mu_1\sin\theta_1 = 10mu_2\sin\theta_2$

$$\Rightarrow \boxed{u_1\sin\theta_1 = 10u_2\sin\theta_2} \quad .....(i)$$

$$kf_{m_1} = \frac{1}{2}ki_{m_1} \quad \text{i.e. } \frac{1}{2}mu_1^2 = \frac{1}{2}\times\frac{1}{2}mu^2$$

$$\text{i.e. } \boxed{u_1 = \frac{u}{\sqrt{2}}} \quad .....(ii)$$

Also collision is elastic : $k_i = k_f$

$$\frac{1}{2}mu^2 = \frac{1}{2}mu_1^2 + \frac{1}{2}.10m.u_2^2$$

$$\frac{1}{2}mu^2 = \frac{1}{2}\times\frac{1}{2}mu^2 + \frac{1}{2}\times 10m.u_2^2$$

$$\frac{1}{4}mu^2 = \frac{1}{2}\times 10 \times mu_2^2$$

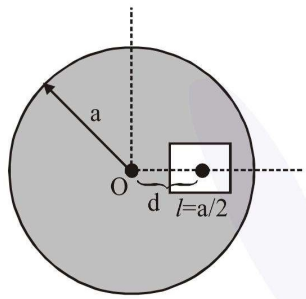
$$\boxed{u_2 = \frac{u}{\sqrt{20}}} \quad ....(iii)$$

Putting (ii) & (iii) in (i)

$$\frac{u}{\sqrt{2}}\sin\theta_1 = 10.\frac{u}{\sqrt{20}}\sin\theta_2$$

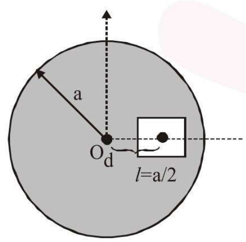
$\boxed{\sin\theta_1 = \sqrt{10}\sin\theta_2}$ → Hence n = 10

24. A square shaped hole of side $l = \frac{a}{2}$ is carved out at a distance $d = \frac{a}{2}$ from the centre 'O' of a uniform circular disk of radius a. If the distance of the centre of mass of the remaining portion from O is $-\frac{a}{X}$, value of X (to the nearest integer) is _____.



Official Ans. by NTA (23.00)

Sol.



$$X_{com} = \frac{m_1x_1 - m_2x_2}{m_1 - m_2}$$

where :
- m_1 = mass of complete disc
- m_2 = removed mass
- Let σ = surface mass density of disc material

$$\text{wrt 'O' : } X_{com} = \frac{\sigma\pi a^2(O) - \sigma.\frac{a^2}{4}.d}{\sigma\pi a^2 - \sigma\frac{a^2}{4}} = \frac{-\frac{a^2}{4}d}{\pi a^2 - \frac{a^2}{4}}$$

$$= \frac{-d}{4\pi - 1} = -\frac{a}{2(4\pi - 1)}$$

So, X = 2(4π–1) = (8π–2) = 23.12

So, nearest integer value of X = 23

25. A wire of density 9×10^{-3} kg cm^{-3} is stretched between two clamps 1 m apart. The resulting strain in the wire is 4.9×10^{-4}. The lowest frequency of the transverse vibrations in the wire is (Young's modulus of wire $Y = 9 \times 10^{10}$ Nm^{-2}), (to the nearest integer),__________.

Official Ans. by NTA (35.00)

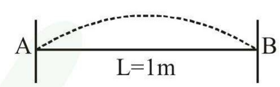
Sol. $\rho_{wire} = 9 \times 10^{-3} \frac{kg}{cm^3} = \frac{9 \times 10^{-3}}{10^{-6}} kg/m^3$

$= 9000$ kg/m^2

(A = CSA of wire)

($Y = 9 \times 10^{10}$ Nm^2)

(Strain = 4.9×10^{-4})



$$\Rightarrow L = 1m = \frac{\lambda}{2} \Rightarrow \lambda = 2m$$

$$\Rightarrow v = f\lambda \Rightarrow \sqrt{\frac{T}{\mu}} = f\,\lambda$$

Where $Y = \frac{T/A}{strain} \Rightarrow T = Y.A.strain$

$$\Rightarrow \sqrt{\frac{Y.A.strain}{m/L}} = f \times 2 \Rightarrow \sqrt{\frac{Y.A.L.strain}{M}} = f \times 2$$

$$\Rightarrow \sqrt{\frac{Y \times V \times strain}{M}} = f \times 2 \Rightarrow \sqrt{\frac{Y \times strain}{\rho}} = f \times 2$$

$$f = \frac{1}{2}.\sqrt{\frac{Y \times strain}{\rho}} = \frac{1}{2}\sqrt{\frac{9 \times 10^{10} \times 4.9 \times 10^{-4}}{9000}}$$

$$f = \frac{1}{2}.\sqrt{\frac{9 \times 10^3}{9} \times 4.9} = \frac{1}{2}\sqrt{4900} = \frac{70}{2} = 35 \text{ Hz}$$