FINAL JEE-MAIN EXAMINATION - SEPTEMBER, 2020
Held On Sunday, 6 September 2020
TIME : 9: 00 AM to 12 : 00 PM

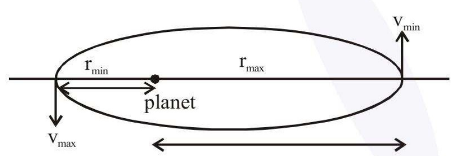
1. A satellite is in an elliptical orbit around a planet P. It is observed that the velocity of the satellite when it is farthest from the planet is 6 times less than that when it is closest to the planet. The ratio of distances between the satellite and the planet at closest and farthest points is :

(1) 1 : 6 (2) 3 : 4

(3) 1 : 3 (4) 1 : 2

Official Ans. by NTA (1)

Sol.



By angular momentum conservation

$r_{min}v_{max} = r_{max}v_{min}$ ... (i)

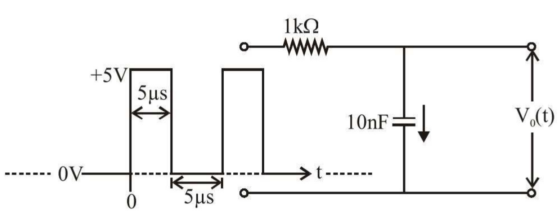
Given $v_{min} = \frac{v_{max}}{6}$

from equation (i)

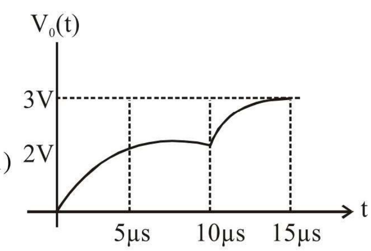
$\frac{r_{min}}{r_{max}} = \frac{v_{min}}{v_{max}} = \frac{1}{6}$

Ans. (1)

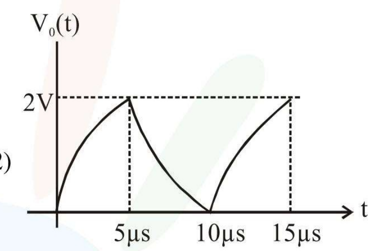
2. For the given input voltage waveform $V_{in}(t)$, the output voltage waveform $V_D(t)$, across the capacitor is correctly depicted by:



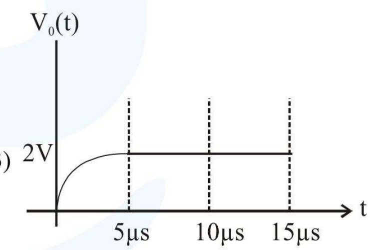
(1)



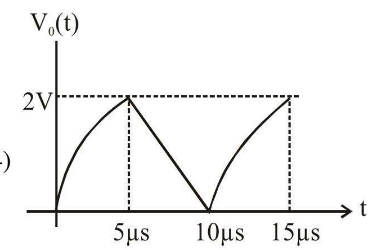
(2)



(3)

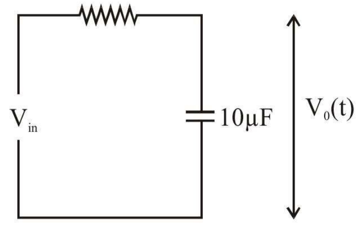


(4)



Official Ans. by NTA (1)

Sol.



$$V_0(t) = V_{in}\left(1 - e^{-\frac{t}{RC}}\right)$$

at t = 5μs

$$V_0(t) = 5\left(1 - e^{-\frac{5\times10^{-6}}{10^3\times10\times10^{-9}}}\right)$$

$= 5\,(1 - e^{-0.5}) = 2V$

Now $V_{in} = 0$ means discharging

$$V_0(t) = 2e^{-\frac{t}{RC}} = 2e^{-0.5}$$

= 1.21 V

Now for next 5 μs

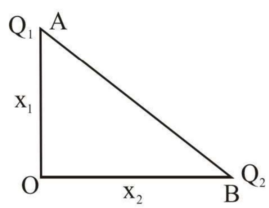
$$V_0(t) = 5 - 3.79e^{-\frac{t}{RC}}$$

after 5 μs again

$V_0(t) = 2.79$ Volt ≈ 3V

Most approperiate Ans. (1)

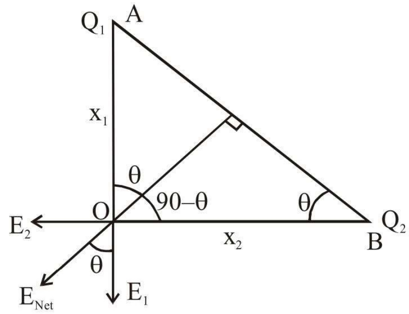
3. Charges Q_1 and Q_2 arc at points A and B of a right angle triangle OAB (see figure). The resultant electric field at point O is perpendicular to the hypotenuse, then Q_1/Q_2 is proportional to :



(1) $\frac{x_2^2}{x_1^2}$ (2) $\frac{x_1^3}{x_2^3}$ (3) $\frac{x_1}{x_2}$ (4) $\frac{x_2}{x_1}$

Official Ans. by NTA (3)

Sol.



E_2 = electric field due to Q_2

$$= \frac{kQ_2}{x_2^2}$$

$$E_1 = \frac{kQ_1}{x_1^2}$$

From diagram

$$\tan\theta = \frac{E_2}{E_1} = \frac{x_1}{x_2}$$

$$\frac{kQ_2}{x_2^2 \times \frac{kQ_1}{x_1^2}} = \frac{x_1}{x_2}$$

$$\frac{Q_2 x_1^2}{Q_1 x_2^2} = \frac{x_1}{x_2}$$

$$\frac{Q_2}{Q_1} = \frac{x_2}{x_1}$$

$$\frac{Q_1}{Q_2} = \frac{x_1}{x_2}$$

Ans. (3)

4. A screw gauge has 50 divisions on its circular scale. The circular scale is 4 units ahead of the pitch scale marking, prior to use. Upon one complete rotation of the circular scale, a displacement of 0.5 mm is noticed on the pitch scale. The nature of zero error involved, and the least count of the screw gauge, are respectively :

(1) Negative, 2 μm

(2) Positive, 10 μm

(3) Positive, 0.1 μm

(4) Positive, 0.1 mm

Official Ans. by NTA (2)

Sol. Least count of screw gauge

$$= \frac{\text{Pitch}}{\text{no. of division on circular scale}}$$

$$= \frac{0.5}{50} \text{mm} = 1 \times 10^{-5} \text{m}$$

= 10 μm

Zero error in positive

Ans. (2)

5. An object of mass m is suspended at the end of a massless wire of length L and area of cross-section, A. Young modulus of the material of the wire is Y. If the mass is pulled down slightly its frequency of oscillation along the vertical direction is:

(1) $f = \frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$ (2) $f = \frac{1}{2\pi}\sqrt{\frac{YL}{mA}}$

(3) $f = \frac{1}{2\pi}\sqrt{\frac{mA}{YL}}$ (4) $f = \frac{1}{2\pi}\sqrt{\frac{mL}{YA}}$

Official Ans. by NTA (1)

Sol. An elastic wire can be treated as a spring with

$$k = \frac{YA}{\ell}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{YA}{m\ell}}$$

Ans. (1)

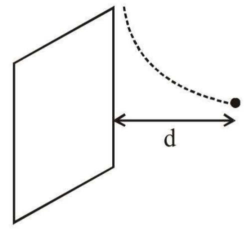
6. A particle of charge q and mass m is moving with a velocity $-\upsilon\hat{i}(\upsilon \neq 0)$ towards a large screen placed in the Y-Z plane at a distance d. If there is a magnetic field $\vec{B} = B_0\hat{k}$, the minimum value of υ for which the particle will not hit the screen is:

(1) $\frac{qdB_0}{2m}$ (2) $\frac{qdB_0}{m}$

(3) $\frac{2qdB_0}{m}$ (4) $\frac{qdB_0}{3m}$

Official Ans. by NTA (2)

Sol.



In uniform magnetic field particle moves in a circular path, if the radius of the circular path is 'd', particle will not hit the screen.

$$d = \frac{mv}{qB_0}$$

$$v = \frac{qB_0 d}{m}$$

∴ correct option is (2)

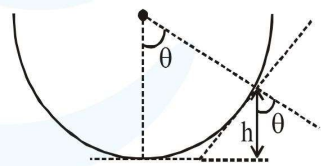
7. An insect is at the bottom of a hemispherical ditch of radius 1 m. It crawls up the ditch but starts slipping after it is at height h from the bottom. If the coefficient of friction between the ground and the insect is 0.75, then h is : ($g = 10 ms^{-2}$)

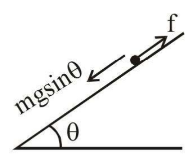
(1) 0.80 m (2) 0.60 m

(3) 0.45 m (4) 0.20 m

Official Ans. by NTA (4)

Sol.



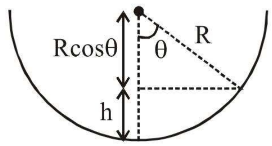


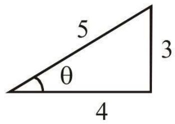
For balancing mgsinθ = f

mgsinθ = μmgcosθ

tanθ = μ

$$\tan\theta = \frac{3}{4}$$





$h = R - R\cos\theta$

$= R - R\left(\frac{4}{5}\right) = \frac{R}{5}$

$h = \frac{R}{5} = 0.2\text{m}$

$\therefore$ correct option is (4)

8. A clock has a continuously moving second's hand of 0.1 m length. The average acceleration of the tip of the hand (in units of ms^{-2}) is of the order of:

(1) 10^{-3} (2) 10^{-2}

(3) 10^{-4} (4) 10^{-1}

Official Ans. by NTA (1)

Sol. R = 0.1 m

$\omega = \frac{2\pi}{T} = \frac{2\pi}{60}$ = 0.105 rad/sec

$a = \omega^2 R$

$= (0.105)^2\ (0.1)$

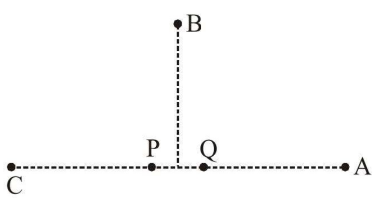
$= 0.0011$

$= 1.1 \times 10^{-3}$

Average acceleration is of the order of 10^{-3}

$\therefore$ correct option is (1)

9. In the figure below, P and Q are two equally intense coherent sources emitting radiation of wavelength 20 m. The separation between P and Q is 5 m and the phase of P is ahead of that of Q by 90°. A, B and C are three distinct points of observation, each equidistant from the midpoint of PQ. The intensities of radiation at A, B, C will be in the ratio:

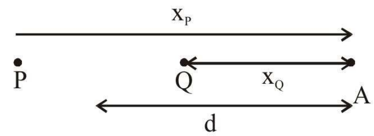


(1) 0 : 1 : 2 (2) 4 : 1 : 0

(3) 0 : 1 : 4 (4) 2 : 1 : 0

Official Ans. by NTA (4)

Sol. For (A)



$x_P - x_Q = (d + 2.5) - (d - 2.5)$

= 5m

$\Delta\phi$ due to path difference = $\frac{2\pi}{\lambda}(\Delta x) = \frac{2\pi}{20}(5)$

$= \frac{\pi}{2}$

At A, Q is ahead of P by path, as wave emitted by Q reaches before wave emitted by P.

Total phase difference at A

$= \frac{\pi}{2} - \frac{\pi}{2}$ (due to P being ahead of Q by 90°)

= 0

$I_A = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\Delta\phi$

$= I + I + 2\sqrt{I}\sqrt{I}\cos(0)$

= 4I

For C

$x_Q - x_P = 5$ m

$\Delta\phi$ due to path difference $= \frac{2\pi}{\lambda}(\Delta x)$

$= \frac{2\pi}{20}(5) = \frac{\pi}{2}$

Total phase difference at C = $\frac{\pi}{2} + \frac{\pi}{2} = \pi$

$I_{net} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos(\Delta\phi)$

$= I + I + 2\sqrt{I}\sqrt{I}\cos(\pi) = 0$

For B

$x_P - x_Q = 0$,

$\Delta\phi = \frac{\pi}{2}$ (Due to P being ahead of Q by 90°)

$I_B = I + I + 2\sqrt{I}\sqrt{I}\cos\frac{\pi}{2} = 2I$

$I_A : I_B : I_C = 4I : 2I : 0$

$= 2 : 1 : 0$

$\therefore$ correct option is (4)

10. An electron, a doubly ionized helium ion (He^{++}) and a proton are having the same kinetic energy. The relation between their respective de-Broglie wavelengths λ_e, $\lambda_{He^{++}}$ and λ_P is:

(1) $\lambda_e < \lambda_P < \lambda_{He^{++}}$ (2) $\lambda_e < \lambda_{He^{++}} = \lambda_P$

(3) $\lambda_e > \lambda_{He^{++}} > \lambda_P$ (4) $\lambda_e > \lambda_P > \lambda_{He^{++}}$

Official Ans. by NTA (4)

Sol. $\lambda = \frac{h}{P} = \frac{h}{\sqrt{2m(KE)}}$

$\lambda \propto \frac{1}{\sqrt{m}} \Rightarrow \lambda = \frac{C}{\sqrt{m}}$

$m_{He^{++}} > m_P > m_e$

$\therefore \lambda_{He^{++}} < \lambda_P < \lambda_e$

$\therefore$ correct option is (4)

11. An electron is moving along + x direction with a velocity of 6×10^6 ms^{-1}. It enters a region of uniform electric field of 300 V/cm pointing along + y direction. The magnitude and direction of the magnetic field set up in this region such that the electron keeps moving along the x direction will be:

(1) 5×10^{-3} T, along +z direction

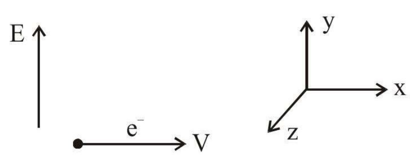
(2) 3×10^{-4} T, along –z direction

(3) 3×10^{-4} T, along +z direction

(4) 5×10^{-3} T, along –z direction

Official Ans. by NTA (1)

Sol.



$\vec{B}$ must be in +z axis.

$\vec{V} = 6 \times 10^6 \hat{i}$

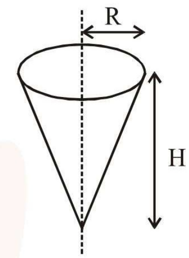
$\vec{E} = 300\hat{j}$ V/cm = 3×10^4 V/m

$q\vec{E} + q\vec{V} \times \vec{B} = 0$

$E = VB$

$B = \frac{E}{V} = \frac{3 \times 10^4}{6 \times 10^6} = 5 \times 10^{-3}$ T

12. Shown in the figure is a hollow icecream cone (it is open at the top). If its mass is M, radius of its top, R and height, H, then its moment of inertia about its axis is:

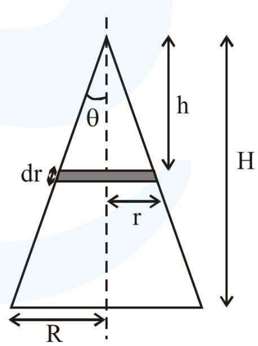


(1) $\frac{MR^2}{2}$ (2) $\frac{MH^2}{3}$

(3) $\frac{MR^2}{3}$ (4) $\frac{M(R^2+H^2)}{4}$

Official Ans. by NTA (1)

Sol.



Area = $\pi R\ell = \pi R\left(\sqrt{H^2+R^2}\right)$

Area of element dA = $2\pi r d\ell$ = $= 2\pi r \frac{dh}{\cos\theta}$

mass of element $dm = \frac{M}{\pi R\sqrt{H^2+R^2}} \times \frac{2\pi r dh}{\cos\theta}$

$$dm = \frac{2Mh\tan\theta dh}{R\sqrt{H^2+R^2}\cos\theta} \quad (\text{here } r = h\tan\theta)$$

$$I = \int (dm) r^2 = \int \frac{h^2\tan^2\theta}{\cos\theta}\left(\frac{2m}{R}\frac{h\tan\theta}{\sqrt{R^2+H^2}}\right)dh$$

$$= \frac{2M}{\cos\theta R}\frac{\tan^3\theta}{\sqrt{R^2+H^2}}$$

$$\int_0^H h^3 dh = \frac{MR^2H^4}{2RH^3\sqrt{R^2+H^2}\cos\theta}$$

$$= \frac{MR^2H\sqrt{R^2+H^2}}{2\sqrt{R^2+H^2}\times H}$$

$$= \frac{MR^2}{2}$$

13. An AC circuit has R = 100 Ω, C = 2 μF and L = 80 mH, connected in series. The quality factor of the circuit is :

(1) 0.5 (2) 2
(3) 20 (4) 400

Official Ans. by NTA (2)

Sol. $Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{100}\sqrt{\frac{80\times10^{-3}}{2\times10^{-6}}}$

$$= \frac{1}{100}\sqrt{40\times10^3}$$

$$= \frac{200}{100} = 2$$

14. If the potential energy between two molecules is given by $U = \frac{A}{r^6} + \frac{B}{r^{12}}$, then at equilibrium, separation between molecules, and the potential energy are :

(1) $\left(\frac{B}{A}\right)^{1/6}, 0$ (2) $\left(\frac{B}{2A}\right)^{1/6}, -\frac{A^2}{2B}$

(3) $\left(\frac{2B}{A}\right)^{1/6}, -\frac{A^2}{4B}$ (4) $\left(\frac{2B}{A}\right)^{1/6}, -\frac{A^2}{2B}$

Official Ans. by NTA (3)

Sol. $U = \frac{-A}{r^6} + \frac{B}{r^{12}}$

$$F = -\frac{dU}{dr} = -\left(A\left(-6r^{-7}\right)\right) + B\left(-12r^{-13}\right)$$

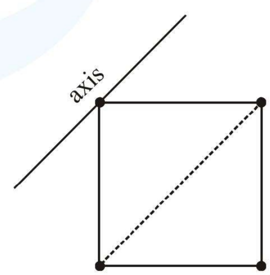
$$0 = \frac{6A}{r^7} - \frac{12B}{r^{13}}$$

$$\frac{6A}{12B} = \frac{1}{r^6} \Rightarrow r = \left(\frac{2B}{A}\right)^{1/6}$$

$$U\left(r = \left(\frac{2B}{A}\right)^{1/6}\right) = -\frac{A}{2B/A} + \frac{B}{4B^2/A^2}$$

$$= \frac{-A^2}{2B} + \frac{A^2}{4B} = \frac{-A^2}{4B}$$

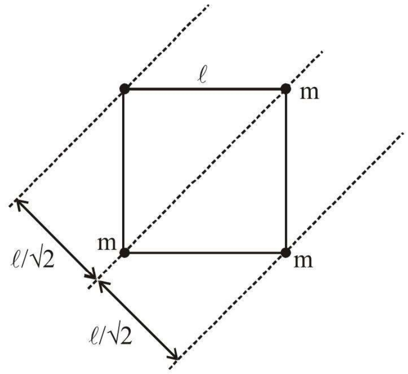
15. Four point masses, each of mass m, are fixed at the corners of a square of side ℓ. The square is rotating with angular frequency ω, about an axis passing through oneof the corners of the square and parallel to its diagonal, as shown in the figure. The angular momentum of the square about this axis is:



(1) $2m\ell^2\omega$ (2) $3m\ell^2\omega$
(3) $m\ell^2\omega$ (4) $4m\ell^2\omega$

Official Ans. by NTA (2)

Sol.



$$I = m(0)^2 + m\left(\frac{\ell}{\sqrt{2}}\right)^2 \times 2 + m\left(\sqrt{2}\ell\right)^2$$

$$= \frac{2m\ell^2}{2} + 2m\ell^2 = 3m\ell^2$$

Angular momentum L = Iω

= $3m\ell^2\omega$

16. You are given that Mass of ${}^{7}_{3}Li$ = 7.0160 u,

Mass of ${}^{4}_{2}He$ = 4.0026 u

and Mass of ${}^{1}_{1}H$ = 1.0079 u.

When 20 g of ${}^{7}_{3}Li$ is converted into ${}^{4}_{2}He$ by proton capture, the energy liberated, (in kWh), is: [Mass of nudeon = 1 GeV/c^2]

(1) 8×10^6 (2) 1.33×10^6

(3) 6.82×10^5 (4) 4.5×10^5

Official Ans. by NTA (2)

Sol. ${}^{7}_{3}Li + {}^{1}_{1}H \rightarrow 2\left({}^{4}_{2}He\right)$

$$\Delta m \Rightarrow [m_{Li} + m_H] - 2[M_{He}]$$

Energy released in 1 reaction ⇒ Δmc^2.

In use of 7.016 u Li energy is Δmc^2

In use of 1gm Li energy is $\frac{\Delta mc^2}{m_{Li}}$

In use of 20 gm energy is $\Rightarrow \frac{\Delta mc^2}{m_{Li}} \times 20gm$

$$\Rightarrow \frac{[(7.016 + 1.0079) - 2 \times 4.0026]\,u \times c^2}{7.016 \times 1.6 \times 10^{-24}\,gm} \times 20gm$$

$$\Rightarrow \left(\frac{0.0187 \times 1.6 \times 10^{-19} \times 10^9}{7.016 \times 1.6 \times 10^{-24}\,gm} \times 20gm\right) \text{Joule}$$

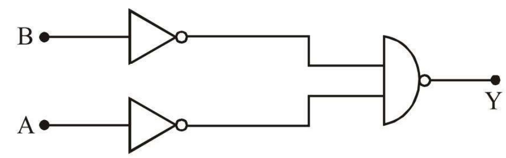
⇒ $0.05 \times 10^{+14}$ J

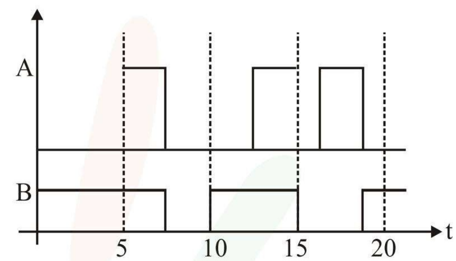
⇒ $1.4 \times 10^{+6}$ kwh

[1 J ⇒ 2.778×10^{-7} kwh]

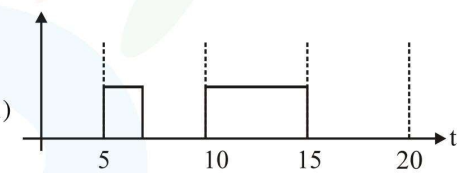
Ans. (2)

17. Identify the correct output signal Y in the given combination of gates (as shown) for the given inputs A and B.

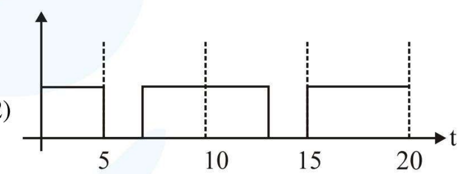




(1)



(2)



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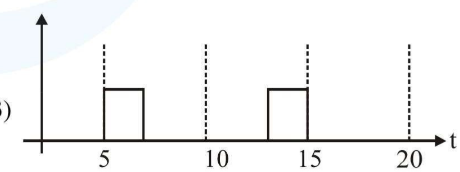
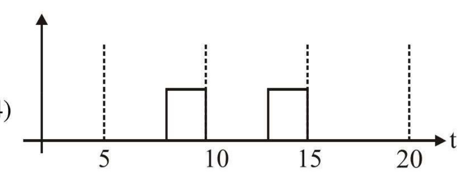
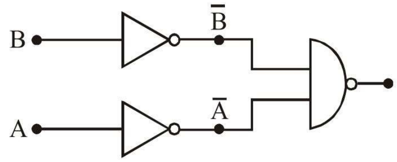


(4)

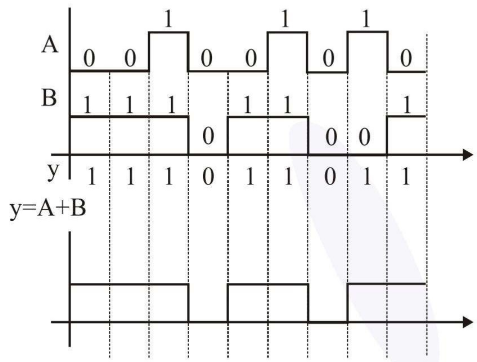


Official Ans. by NTA (3)

Sol.



$y = \overline{\overline{A}.\overline{B}} = \overline{\overline{A}} + \overline{\overline{B}} = A + B$



18. Molecules of an ideal gas are known to have three translational degrees of freedom and two rotational degrees of freedom. The gas is maintained at a temperature of T. The total internal energy, U of a mole of this gas, and the value of $\gamma\left(=\frac{C_P}{C_v}\right)$ given, respectively, by:

(1) $U = \frac{5}{2}RT$ and $\gamma = \frac{6}{5}$

(2) U = 5RT and $\gamma = \frac{7}{5}$

(3) U = 5RT and $\gamma = \frac{6}{5}$

(4) $U = \frac{5}{2}RT$ and $\gamma = \frac{7}{5}$

Official Ans. by NTA (4)

Sol. Total degree of freedom = 3 + 2 = 5

$U = \frac{nfRT}{2} \Rightarrow \frac{5RT}{2}$

$\gamma \Rightarrow \frac{C_P}{C_V} \Rightarrow 1 + \frac{2}{f} \Rightarrow 1 + \frac{2}{5} \Rightarrow \frac{7}{5}$

Ans. (4)

19. A point like object is placed at a distance of 1m in front of a convex lens of focal length 0.5 m. A plane mirror is placed at a distance of 2 m behind the lens. The position and nature of the final image formed by the system is :

(1) 1 m from the mirror, virtual

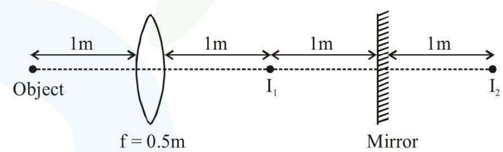
(2) 1 m from the mirror, real

(3) 2.6 m from the mirror, real

(4) 2.6 m from the mirror, virtual

Official Ans. by NTA (1,4)

Sol.



Object is at 2f. So image will also be at '2f'. (I_1).

Image of I_1 will be 1m behind mirror.

i.e. $\Rightarrow I_2$

Now I_2 will be object for lens.

$\therefore$ u $\Rightarrow$ – 3m

f $\Rightarrow$ +0.5 m

$\frac{1}{v} \Rightarrow \frac{1}{f} + \frac{1}{u}$

$\Rightarrow \frac{1}{+0.5} + \frac{1}{-3}$

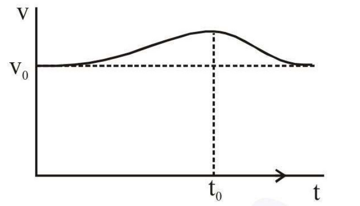
$v \Rightarrow \frac{3}{5} \Rightarrow 0.6m$

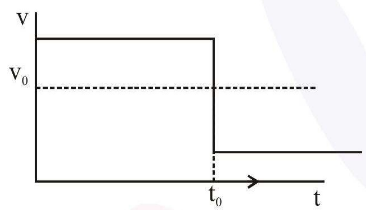
So total distance from mirror $\Rightarrow$ 2 + 0.6

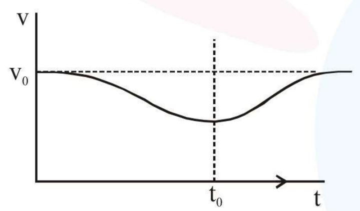
$\Rightarrow$ 2.6 m and real image

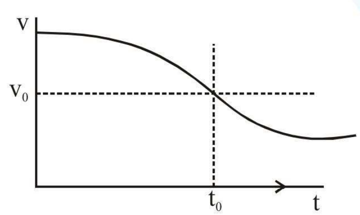
Ans. (3)

20. A sound source S is moving along a straight track with speed v, and is emitting sound of frequency v_0 (see figure). An observer is standing at a finite distance, at the point O, from the track. The time variation of frequency heard by the observer is best represented by :
(t_0 represents the instant when the distance between the source and observer is minimum)

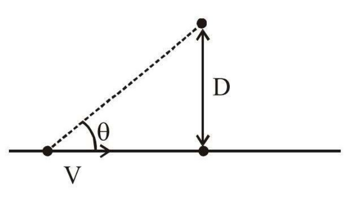
(1)


(2)


(3)


(4)


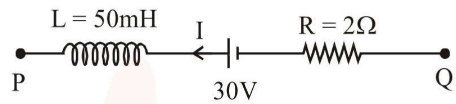
Official Ans. by NTA (4)

Sol.


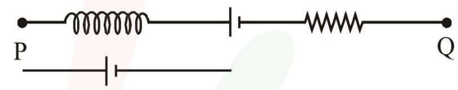
$$f_{observed} \Rightarrow \left(\frac{v_{sound}}{v_{sound} - v\cos\theta}\right) f_0$$

initially θ will be less ⇒ cosθ more
∴ $f_{observed}$ more, then it will decrease.
∴ Ans. (4)

21. A part of a complete circuit is shown in the figure. At some instant, the value of current I is 1 A and it is decreasing at a rate of 10^2 A s^{-1}. The value of the potential difference $V_P - V_Q$, (in volts) at that instant, is.



Official Ans. by NTA (33.00)

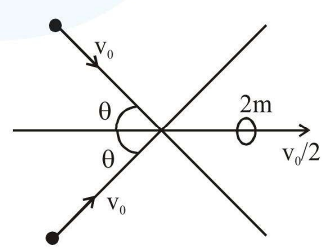
Sol.


$$\frac{Ldi}{dt} = 5$$

$V_P - 5 - 30 + 2 \times 1 = VQ$
$V_P - V_Q = 33$ volt
Ans. 33.00

22. Two bodies of the same mass are moving with the same speed, but in different directions in a plane. They have a completely inelastic collision and move together thereafter with a final speed which is half of their initial speed. The angle between the initial velocities of the two bodies (in degree) is.

Official Ans. by NTA (120.00)

Sol.


Momentum conservation along x

$$2mv_0\cos\theta = 2m\frac{v_0}{2}$$

$\cos\theta = \frac{1}{2}$

$\theta = 60$

Angle is $2\theta = 120$

Ans. 120.00

23. Suppose that intensity of a laser is $\left(\frac{315}{\pi}\right) W/m^2$. The rms electric field, in units of V/m associated with this source is close to the nearest integer is

($\in_0 = 8.86 \times 10^{-12}\ C^2\ Nm^{-2}$; $c = 3 \times 10^8\ ms^{-1}$)

Official Ans. by NTA (275.00)

Sol. $I = \in_0 E_{rms}^2 C$

$$E_{rms}^2 = \frac{I}{\in_0 C}$$

$$= \frac{315}{\pi \in_0} \times \frac{1}{C}$$

$$= \frac{4 \times 315}{4\pi \in_0} \times \frac{1}{3 \times 10^8}$$

$$= \frac{4 \times 315 \times 9 \times 10^9}{3 \times 10^8}$$

$E_{rms}^2 = 4 \times 315 \times 30$

$E_{rms} = 2\sqrt{315 \times 30}$

$= 194.42$

Ans. 194.00

24. The density of a solid metal sphere is determined by measuring its mass and its diameter. The maximum error in the density of the sphere is $\left(\frac{x}{100}\right)\%$. If the relative errors in measuring the mass and the diameter are 6.0% and 1.5% respectively, the value of x is .

Official Ans. by NTA (1050.00)

Sol. $\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi\left(\frac{D}{2}\right)^3}$

$$\rho = \frac{6}{\pi} M\ D^{-3}$$

taking log

$$\ell n\rho = \ell n\left(\frac{6}{\pi}\right) + \ell nM - 3\ell mD$$

Differentiates

$$\frac{d\rho}{\rho} = 0 + \frac{dM}{M} - 3\frac{d(D)}{D}$$

for maximum error

$$100 \times \frac{d\rho}{\rho} = \frac{dM}{M} \times 100 + \frac{3dD}{D} \times 100$$

$= 6 + 3 \times 1.5$

$= 10.5\ \%$

$= \frac{1050}{100}\%$ so $x = 1050.00$

25. Initially a gas of diatomic molecules is contained in a cylinder of volume V_1 at a pressure P_1 and temperature 250 K. Assuming that 25% of the molecules get dissociated causing a change in number of moles. The pressure of the resulting gas at temperature 2000 K, when contained in a volume $2V_1$ is given by P_2. The ratio P_2/P_1 is.

Official Ans. by NTA (5.00)

Sol. $PV = nRT$

$P_1V_1 = nR\ 250$

$$P_2(2V_1) = \frac{5n}{4} R \times 2000$$

Divide

$$\frac{P_1}{2P_2} = \frac{4 \times 250}{5 \times 2000}$$

$$\frac{P_1}{P_2} = \frac{1}{5}$$

$$\frac{P_2}{P_1} = 5$$

Ans. 5.00